Exploring the Complexity
in an Oil Jet Falling into an Oil Bath

by

Chao Ieong

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This is to certify that I have examined the above MPhil thesis and have found that it is complete and satisfactory in all respects, and that any and all revisions required by the thesis examination committee have been made.

[Signature]
Prof. Wing Yim Tam, Thesis Supervisor

[Signature]
Prof. Ping Sheng, Department Head

Department of Physics
August 31, 2007
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Department of Physics
The Hong Kong University of Science and Technology

Abstract

An oil stream falling vertically into a stationary bath of the same liquid, if its flow speed is above a critical threshold, because of its viscous property, will induce a transition where a “cusp” on the free surface is “cracked” open by the jet. This industrially important phenomenon has been well studied. However, results from the present study indicate that there is still some puzzling issue of this phenomenon, concerning the stability of the “cusp” before the transition. And this issue becomes more complicated when a constant bath motion is introduced. New “dynamical states” that are not possible when the bath is stationary can arise when energy is continuously supplied into the system by the externally driven constant flow of the bath liquid. Multiple “dynamical states” can exist for the same experimental conditions. The system can exhibit slight hysteretic behavior.
Chapter 1: Overview

1.1 Introduction

We are all, presumably, familiar with the behaviors of water (even though much of it is still not well understood), as our survivals depend on it and the majority of the Earth’s surface is covered with it. We use it in various ways in our everyday life and we can observe its behavior when we go to a river or waterfall, a swimming pool or beach, or when we are out in the sea or ocean on a boat or ship, or when we stare out to the falling rains as they make their ways to various places on Earth, or when we just pass by the water fountains in public areas. But we are less familiar with the behaviors of fluids that are more viscous than water – one ready example of which is oil, whether it is the ones used in cooking or those used in the industries. And indeed, the behaviors of liquids that are more viscous than water are sometimes quite counter-intuitive and perplexing to most people, as the present work may demonstrate. Every fluid – when it is flowing smoothly and steadily (so called laminar flow and is characterized by a low Reynolds Number associated with the problem) – has a measurable physical property called the viscosity. Fluid that is more viscous is more resistant to change of shape or flow when subjected to a shear force. It is like, if you would, the constituents of viscous fluids are standing much stronger together as a “community” against certain outside force.

This thesis will study the behaviors of a vertical viscous jet falling into a horizontal bath of the same liquid that is either stationary or moving relative to the vertical jet (with in mind the bigger context of both viscous and non-viscous jets).
1.2 Air-entraining jets

![Fig.1.1. Qualitative difference of air entrainment by viscous and water streams plunging into a stationary bath of the same liquid. (a) The liquid is silicone oil of nominal kinematics viscosity 150 cSt and density 0.86 g/cm$^3$. A thin film of air is “entrained” into the bath wrapping around the jet. The length of this film can vary as shown. The length of the shorter air film is about 2 cm. Bubbles are formed out of the air in this thin film. (The shape of the jet is distorted optically by the surface roughness of the wall of the container.) (b) The liquid is water. No air film is observed but only short-lived air bubbles. The mechanism of bubble formation cannot be resolved by eyes. The diameter of the beaker is about 10 cm.]

If one has chance to play or work with pouring oil or other viscous liquid into a stationary bath of the same liquid, one may notice that it is very different from pouring water into water. Figure 1.1 shows the qualitative difference of the two cases. In both cases, air from above the liquid-air interface is forced into the liquid bath, forming air bubbles in the bath. But in the case of a viscous liquid, the jet seems to be able to “push” the bath liquid away and dive further down into the bath before combining or merging with the bath liquid. The oil jet in the bath is actually surrounded by a thin film of air, preventing the jet from coalescing with the bath or providing lubrication between them. This film is sustained for as long as the oil stream’s speed is greater than a threshold (as will be learned later). But the length of this film may vary, and air bubbles are formed out of this film at its lower end. These bubbles usually float back to the bath surface and stay there for some time before
bursting. On the other hand, in the case of water, the behavior is dramatically different, no air film is observed and the bubbles burst almost right after they are formed in the water bath. Such differences will be explained more and addressed in Chapter 2.

This phenomenon is called the “air entrainment by plunging liquid jets”. It has been well studied before because of its wide relevance in a lot of industrial processes, where it is either a nuisance or a desired thing [3]. As will be learned in Chapter 2, this phenomenon has been understood quite well for the case of viscous liquids: when the oil bath is stationary, only two states are possible, namely, the “merge” and “plunge” states (see Figure 1.2), and all the previous studies on viscous jets have shown that there exists a “critical entrainment velocity $V_c$” (at the entry point into the bath) for a viscous jet plunging into a stationary bath of the same liquid; below the critical jet velocity $V_c$ the merge state is stable, and at and above the critical velocity $V_c$ the merge state will spontaneously change to the plunge state [1, 3, 6, 8].

However, as the results in Chapter 4 will show, this is not quite what was observed in the present study; right below the critical entrainment jet velocity $V_c$, the merge state was not absolutely stable but the plunge state was very stable – the stability of the merge state seemed to increase exponentially, and that of the plunge state decreased, as the jet speed was decreased away from the critical entrainment speed $V_c$. These observations have not been mentioned at all in all the previous studies related to this topic.
Fig. 1.2. **The different states of a viscous jet falling into a stationary bath of the same liquid.** The liquid is a silicone oil of nominal kinematics viscosity 150 cSt and density 0.86 g/cm$^3$. (a) The “merge” state: an axis-symmetric dip is formed around the impinging jet on the bath surface. (b) The “plunge” state: a thin film of air, wrapping around the jet, is being “entrained” into the bath by the plunging jet.
The different states of a viscous jet falling into a moving bath of the same liquid. The liquid is a silicone oil of nominal kinematics viscosity 150 cSt and density 0.86 g/cm³. The oil bath is moving from right to left with a speed less than that of the jet. These different states are observed at different control parameter values of the experiment.

(a) The “plunge” state: the oil jet is plunging into the bath and a thin film of air is wrapping around the jet in the bath; the air layer is being dragged along by the moving bath. Air bubbles of varied sizes will pinch off the end of the air film continually (compare to Figure 1.2b).

(b) The “half-plunge” state: one side of the jet is entraining air into the bath, as can be seen by the tiny air bubbles streaking off the end of the length of air lubricating between the jet and bath; the other side is not entraining air but deforming the bath surface and merging with it.

(c) The “bounce” state: the jet is making an indentation on the bath surface and leaping off it and merging into the bath afterwards; an air layer is preventing the jet and bath from coalescing or combining together.

(d) The “trailing jet” or “slide” state: similar to that in (c), except that the jet is lying almost flat on the bath surface; this state can be obtained directly from the bounce state by increasing the bath speed.

(e) The “merge” state: the jet is dragging the bath liquid close to the surface further down into the bath, creating a sharp dip wrapping around the impinging jet on the bath surface; the dip is not symmetric due to the motion of the bath liquid (compare to Figure 1.2a).
1.3 The “bouncing jet” and more

The majority of the previous studies on liquid jets falling into a bath of the same liquid (with air as the medium above the bath liquid) have been confined to the situation where the bath is stationary. But what if there is a relative translational motion between the vertical jet and the horizontal bath? For viscous jets, the short answer is that the behaviors of the system can become much more complex and intriguing.

Figure 1.3 (c) shows one of the behaviors called the “bouncing jet”. The “bouncing jet” phenomenon in the case of shear-thinning non-Newtonian fluids – known as the “Kaye effect” after its discoverer – has been studied before [11, 12], and in greater depth in a more illuminating way recently [13]. But for a stream of viscous Newtonian fluid bouncing off the surface of a bath of the same liquid, it has just been discovered recently, by accident, by Matt Thrasher [9] and was studied for the first time by him and his collaborators (one of whom is my colleague YK Pang) [10]. They focused on finding out how the flow rate and falling height of the jet, the bath speed and the viscosity of the liquid would affect whether the bouncing jet was observed or not. And in the course of mapping out the region in a control parameter space in which the bouncing jet was “stable” or could be observed – which can be called the “bouncing jet region” – by perturbing the jet to initiate a bounce, they also noticed that at some parameter point, as many as three states could be observed: the plunge, half-plunge and bounce states (see Figure 1.3 (a) to (c)). (The details of their study will be reviewed in Chapter 2.)
In the present study, their observation of a bouncing jet region was reproduced with slightly but crucially different control parameters: the jet diameter at the entry point into the bath was held fixed in the present study; this made the mapped regions look much nicer, and enabled, in addition to the bouncing jet region, the mapping of the stable regions of the half-plunge and plunge states. In doing so, it helped to achieve two things: first, it confirmed that in some smaller region in the control parameter space, the plunge, half-plunge and bounce state could all be observed; and second, it raised the question of the relative stability of each of the states, as the half-plunge and plunge states were very stable in the their mapped stable regions, in comparison to the less stable bounce state observed in the present study. (However, whether the bounce state is inherently less stable, or it is relatively less stable due to some imperfection of the current system, is yet to be resolved.)

The stable region of the merge state – an example of this state is shown in Figure 1.2 (e) – in the control parameter space has not been completely mapped out in the present study, but the study has shown that the stability of the merge state was affected by the motion of the bath in a rather complicated but interesting way.

Moreover, the present study has shown that, at the boundaries of the mapped stable region of the half-plunge state, the half-plunge state would spontaneously become the merge, bounce or plunge state. The spontaneous transition from the half-plunge state to the bounce state is called the “auto-bounce” transition, and the curve in the control parameter space corresponding to the auto-bounce transition has been mapped out for the first time in the present study. (Thrasher et al. have also noticed
the “auto-bounce” transition; it was mentioned by them as another way to initiate a bounce state [10].)

Lastly, the mapped stable regions of the different states in the control parameter space suggested that the system could exhibit some hysteretic behavior at several places in the control parameter space; the hysteresis here is in the broad sense that the system will not retrace its path of change of state when a control parameter of the experiment is changed through a cycle. To give an example, at a particular experimental setting (or control parameter point) when the bath speed was increased, the system would change from a (stable) half-plunge state to a bounce state at a particular bath speed, but it would not change from the bounce state back to the (stable) half-plunge state at the same bath speed when the bath speed was decreased back to the starting value; instead, the reverse transition was delayed or lagged behind. And the present study tried to demonstrate this in the last part of the study.

1.4 Survey of the other chapters

In Chapter 2, basic concepts pertaining to the present study will be introduced first, and then preliminary observations and relevant previous work will be described and reviewed. Then in Chapter 3, the experimental apparatus, parameters and measurements (of the present study) will be detailed. Chapter 4 will give and explain the experimental results of the present study. Finally, a summary of the results (of the present study) and discussions on them will be given in Chapter 5.
Chapter 2: Background

2.1 Basics of a liquid jet (stream of liquid)

2.1.1 Laminar and turbulent jets

Liquid flow can be roughly partitioned into *laminar* or *turbulent* regimes according to the magnitude of the dimensionless Reynolds number that characterizes the problem. In the laminar regime, the flow is smooth and steady, whereas in the turbulent regime, the flow is both spatially and temporally chaotic. Laminar flows are characterized by low Reynolds numbers and turbulent flows are characterized by very high Reynolds numbers. Thus, a laminar jet has a low Reynolds number and a turbulent jet will have a very high Reynolds number. (A liquid jet in this study means a thin stream of liquid of circular cross-sections.) The Reynolds number associated with a liquid jet is given by

\[
Re = \frac{\rho V L}{\mu} = \frac{V L}{\nu} \quad (\nu = \frac{\mu}{\rho}),
\]

where \(V\), \(\rho\), \(\mu\) and \(\nu\) are the jet’s speed, density, dynamic and kinematics viscosity\(^1\) respectively, and \(L\) is a length that characterizes the jet (e.g., the diameter of the jet at entry point into the bath or at the outlet of a cylindrical nozzle). Then, for a flow speed of \(100 - 1000\) cm/s, and a jet diameter of \(0.1 - 1\) cm (applicable to a typical pouring experiment), water jets will have high (\(~10^3\)) to very high (\(~10^5\)) Reynolds numbers (for water at normal conditions: \(\nu = 1\) cSt), but viscous jets will have low to moderate Reynolds numbers. This is because the viscosity of water is very low while

\(^1\)In CGS units, the (dynamic) viscosity is measured in centipoises, cP, and the kinematics viscosity is measured in centistokes, cSt. 1 cP = 0.01 g cm\(^{-1}\) s\(^{-1}\); 1 cSt = 0.01 cm\(^2\) s\(^{-1}\).
those of viscous liquids are usually much higher (for the oil used in this study at normal conditions: $\nu = 150$ cSt). So, viscous jets are usually laminar and water jets can easily be turbulent.

### 2.1.2 Reynolds number and capillary number

The Reynolds number of a liquid jet falling into a stationary bath, at the point of entry into the bath, is: $Re = Vd_j/\nu = \rho Vd_j/\mu$, where $d_j$ is the jet’s diameter at the entry point. This dimensionless number $Re$ is actually a ratio of two quantities: the first is the inertial resistance force ($\sim \rho V^2 d_j^2$) the jet experiences as it tries to “push” away the bath liquid and make its way into the bath; the second is the viscous drag force ($\sim \mu Vd_j$) the jet experiences due to the resistance of the bath liquid from being strained or deformed. Hence, the value of the Reynolds number indicates the relative magnitude of the inertial (resistance) and viscous drag force.

Another dimensionless number that compares the relative dominance of the viscous drag force and surface tension is the capillary number, denoted as $Ca$. For a jet of diameter $d_j$ at the entry point, the viscous drag is $\sim \mu Vd_j$ and the surface tension is $\sim \gamma d_j$ ($\gamma$ is the surface tension per unit length). Hence,

$$Ca \equiv \mu V/\gamma.$$

### 2.1.3 Continuity condition

For a stream of incompressible fluid flowing steadily, the principle of conservation of mass leads to a simple form of continuity condition, namely,
\[(2.3) \quad A_1 V_1 = A_2 V_2,\]

where the A’s and V’s are the cross-sectional areas and average speeds (over cross-sectional areas) of the fluid stream at two points along the flow direction.

![Fig. 2.1](image)

**Fig. 2.1**   **Shape of a jet falling out of a nozzle under gravity at low flow rate.**
Taken from [9]

### 2.1.4 Shape and velocity profile

A typical shape of a liquid jet flowing smoothly out of a vertical cylindrical nozzle is shown in Figure 2.1. The jet is tapering off as it falls towards the bath below it; this is because as the jet falls it picks up speed due to gravity (i.e. \(V_2 > V_1\)), and the continuity condition (2.3) immediately leads to \(A_2 < A_1\).

When the jet’s (average) speed flowing out of the nozzle is high, the tapering off of the jet is less pronounced or obvious. This is because the increase in the jet’s speed, due to gravity after a short distance (say, \(~1\)cm) of fall, will then be small compared to the already large jet speed flowing out of the nozzle, and this will result
in a not-so-obvious change of the jet’s diameter (or cross-sectional area) along the fall, as imposed by the continuity condition. For the same argument, the velocity change of the jet in a short distance of fall a sufficient distance below the nozzle will be negligibly small: the jet’s speed, after free falling a distance of ~10 to 100 times that of the jet’s diameter (at the nozzle outlet), will be quite large compared to the speed change in a short distance of fall that is ~ the jet’s diameter (at the nozzle outlet); hence, the jet’s shape right before impacting the bath surface is essentially uniformly straight, and thus, the velocity profile of the jet at the entry point into the bath is essentially uniform or flat (so-called a plug flow). So, there is no need to distinguish the velocity of the jet at the entry point, from its average velocity (over the cross-sectional area) at the same point.

2.1.5 Jet speed and flow rate

Denoting the volumetric flow rate as $Q$, and assuming the cross-sections of the jet to be circular – which is a natural consequence of the symmetry of the problem of a truly vertical jet and cylindrical nozzle, then

\[(2.4)\quad Q = \pi r^2 V,\]

where $r$ and $V$ are the radius and speed of the jet at a particular point along the jet’s flow direction. Thus for a constant flow rate, equation (2.4) can be used to calculate the velocity of the jet at a particular point of interest along the jet, by knowing $Q$ and the jet’s diameter at that point; this is how the jet’s velocity at the entry point into the bath was obtained in the present study.
The diameter of a liquid stream falling vertically (in air) under gravity can in principle be predicted by solving the equation of motion of an inertial jet (this is so called because air has only a negligible viscosity, and a liquid stream falling in air is dominated by inertial effect) for the full shape of the stream. But this is a difficult mathematical problem in its own right, especially when the liquid is viscous. So an easier and more foolproof way is to measure the jet’s diameter directly.

2.1.6 Rayleigh instability for a free falling jet

If the flow rate is too small, a liquid jet falling out of a nozzle will become very thin quickly at a certain distance below the nozzle outlet; when the jet’s diameter is smaller than a critical value, the jet will become unstable and break up into liquid drops – this behavior is known as the Rayleigh instability. As a result, investigation of the jet’s behavior at the slow end of the flow rates is not possible because of this constraint.

2.2 Liquid jets falling into stationary bath

2.2.1 “Uncommon” behavior of viscous jets

The overall behavior of an oil jet falling into a stationary bath of the same liquid was very different from that observed for a water jet. As an oil jet impinged on the bath surface, depending on the jet’s speed (momentum) right before entry into the bath, the jet would behave differently. If the jet’s speed was low enough, the jet, being slowed down even more at the bath surface, would thicken at the entry point as a result of the steady flow and continuity condition, and merged with the bath by spreading radially outward on the bath surface upon contact with the bath. This could be inferred by the motion of tiny air bubbles introduced onto the bath surface near the
contact point. The bath surface was hardly deformed, and a meniscus curling from the bath surface up to the jet was visible because of the surface tension (air-liquid interfacial tension) of the liquid (see Figure 2.2.1a).

As the jet speed increased, the deformation or depression of the bath surface became obvious. The curled up meniscus gradually disappeared and a smoothly curved dip on the bath surface formed around the impinging jet. At a critical jet speed, the jet no longer spread out over the bath surface; instead, the jet “penetrated” the bath surface, dragging with it the liquid in the vicinity of its flow down into the bath – as could be inferred from tiny air bubbles, present on the bath surface close to the jet, being pulled towards the impinging jet, and dragged into the bath below the bath surface. The dip was no longer smoothly curved at its bottom, but instead a visually sharp dip surrounded the impinging jet (see Figure 2.2.1b). As the jet speed continued to increase and approached another critical threshold, $V_c$, this dip on the free surface became deeper (slightly) and the bottom of it became even sharper, getting even closer to being a “cusp” (see Figure 2.2.2a). (A cusp, by definition, is a sharp end of infinite or singularity curvature.)

Above the (second) critical jet velocity, $V_c$, the stationary cusp-like tip on the free surface surrounding the impinging jet ceased to exist. Instead, a new stable state (the “plunge” state) was formed, where a thin cylindrical sheet of air, entrained by the plunging jet, constantly surrounded the jet, lubricating between the jet and the rest of the bath liquid (Figure 2.2.2b). This air film was unstable at its lower end and air bubbles, pinching off from this film, were formed below it. These bubbles were dragged further down into the bath by the plunging flow of the jet, but, when viscous
drag gave way to gravity (buoyancy), they either floated back up to the surface and burst, or floated back up a distance and were dragged down again by the flow. This “peculiar” behavior of the oil or viscous jet is an example of a more general phenomenon known as “air entrainment by plunging liquid jets”.

Fig. 2.2.1. **A slow viscous jet fall into a stationary bath.** The medium above the bath liquid is air. (a) The jet speed $V$ is much below the first critical threshold. (b) The jet speed $V$ is above the first critical threshold, but below the second critical threshold $V_c$. A visually sharp tip is formed at the bottom of the dip surrounding the impinging jet.

Fig. 2.2.2. **A fast viscous jet falling into a stationary bath.** The medium above the liquid bath is air. (a) The jet’s speed is below the (second) critical threshold $V_c$ but is very close to it: a cusp-like tip (circled) is formed on the free surface. (b) The jet’s speed is above the (second) critical threshold $V_c$; $h$ denotes the thickness of the air film entrained by the plunging jet. Taken from [8].
2.2.2 Air entrainment by plunging liquid jets

Air entrainment by liquid jets plunging into a stationary bath of the same liquid was first systematically investigated (experimentally) by Tong Joe Lin and Harold G. Donnelly about forty years ago [1], and was subsequently continued by many others because the problem is industrially important [3, 4, 6, 8], as well as scientifically interesting.

Different mechanisms for high and low Reynolds number jets

As was observed by Lin and Donnelly, the air entrainment mechanisms were very different for oil and water jets (illustrated in Section 1.1). Actually, their observations were more general than this: they found that the air entrainment mechanisms were very different for liquid jets with low and high Reynolds numbers. For jets with low Reynolds numbers, air entrainment occurred when the jet’s speed at entry into the bath exceeded a critical value, and the jet was always surrounded by a thin film of air below the bath surface (Figure 2.3(a)). On the other hand, for high Reynolds number jets, there were no surrounding air films observed if the Reynolds number exceeded 2000, and for jets with Reynolds numbers exceeding 1500, entrainment was observed only when there was disturbance to or non-smoothness of the jet surface – the higher the Reynolds number of the jet, the more the disturbances to the jet’s surface, and the more the air was entrained into the liquid bath – as illustrated by a turbulent jet (right picture) in Figure 2.3(b). The critical jet speed of air entrainment observed for viscous jets became ill-defined for high Reynolds number jets.
Recently, study by Andrea Prosperetti’s group [4] has confirmed such observation by showing that, jets with Reynolds numbers well beyond 10000 would not entrain air if the surface of the jet was smooth, while on the other hand, if there was a bulge (named “positive” disturbance to the jet’s surface in Ref. [5]) on the jet’s surface, air entrainment was observed for jets with Reynolds number below 10000 (Figure 2.4 (a)). They used electronically controlled valves to suddenly increase or decrease the flow rate to generate the controllable disturbance to the jet’s surface. In another study also done by them [5], it was observed that, if the bulge on the jet’s surface was replaced by an axis-symmetric trough (see Figure 2.4 (b)), there would be no air entrainment for the water jet. Results from computer simulations have agreed well with their experimental observations. But up to date, there is still no good explanation for this mechanism of air entrainment by high Reynolds number jets. On the contrary, the air-entraining behavior of a viscous jet has been understood relatively better in comparison, as will be described below. (This may perhaps seem quite ironic as the behaviors of water are so much more common or familiar to people.)
Fig. 2.3. Sketches for air-entraining liquid jets. (a) Air entrainment at low Reynolds numbers. (b) Air entrainment at high (left) and very high (right) Reynolds numbers. Taken from [1].

Fig. 2.4. A “positive” and “negative” disturbance to the surface of a high Reynolds number jet. Frames are ordered from left to right and then from top to bottom; successive frames are separated by 5 ms. (a) A “positive” disturbance: a sudden increase in the flow rate causes a bulge in the jet that impacts the bath surface and causes air entrainment. (b) A “negative” disturbance: a sudden decrease in the flow rate causes a “collapse” in the jet that impacts the bath surface but causes no air entrainment. Taken from [5].
Factors affecting the critical entrainment speed of viscous jets

It was mentioned above that the critical entrainment velocity was ill-defined for a water jet. But for a viscous jet, the critical entrainment velocity is well-defined, and hence it can be studied systematically. This was what Lin and Donnelly exactly did in the same study mentioned above [1]: they studied how viscosity, surface tension, and the jet’s diameter at the entry point into the bath affected the critical entrainment speed (called “minimum entrainment velocity” by them) of laminar jets. First, they found that the critical entrainment velocity was very significantly affected by the viscosity; it decreased linearly from about 400 to 80 cm/s when the viscosity was increased from about 30 to 400 centipoises (while other factors were held fixed). Second, the critical entrainment velocity decreased with the lowering of surface tension by adding surfactant. Third, the jet’s diameter at the entry point also had a significant effect on the critical entrainment velocity if it was less than 5 or 6 mm – the critical entrainment velocity increased by about 50 % when the jet’s diameter was decreased from 5 or 6 to 2 mm. (For comparison purpose to the present study, note that these results were based on glycerol solutions of kinematics viscosity ~ 100 cSt, surface tension ~ 10 dynes/cm and density ~ 1g/cm$^3$.)

2.2.3 “Prediction” of critical entrainment velocity of viscous jets

As described above, an impinging viscous jet falling into the bath drags the bath liquid near the bath surface surrounding the jet deep into the bath, creating a convergence of flows near the bath surface, and hence a ring of sharp dip on the free surface around the circumference of the jet. It turned out that the problem of the critical entrainment velocity was related to the stability of this sharp dip or cusp-like curvature on the free surface.
The formation of a cusp on the free surface of a liquid (liquid-air interface) will be something quite remarkable – if it is at all possible – as intuition expects that surface tension will oppose the formation of such cusps. (A cusp is by definition a sharp end with singularity curvature, but in real fluids, the smallest dimension possible is the molecular length scale, so a physical cusp cannot be infinitely sharp without limit.) It turned out that the formation of a cusp on a free surface by converging flows at low Reynolds number was indeed possible, despite the opposition of surface tension, as solved by Jeong and Moffatt (about fifteen years ago) in a strictly two-dimensional problem [2] – that is, the flow pattern in a plane cut at any position along the third dimension is just a “carbon copy” of one another.

They solved a problem in which two counter-rotating cylinders were submerged a distance (~ d_c, diameter of the two identical cylinders) below the surface of a bath of viscous (Newtonian) liquid. The parallel axes of the cylinders, separated by a distance of about 2d_c, lied on a horizontal plane parallel to the otherwise flat bath surface (if the cylinders had not been rotated). The counter-rotating cylinders dragged the viscous liquid with them and created a convergence of flows near the free (bath) surface. When the angular velocity ω was large enough, a visually observable sharp tip was formed on the free surface.

For the theoretical part of their work, they idealized the problem by replacing the two counter-rotating cylinders with a vortex dipole at the mid-point between the cylinders, and then solved the Navier-Stokes equation at low Reynolds number by complex variable techniques and appropriate boundary conditions, including the
effect of surface tension but neglecting that of the gravity, which was insignificant to their main result. They showed that, as long as the capillary number \( \text{Ca} = \frac{\mu \omega d_c}{2 \gamma} \) (similar to equation (2.2) above but with \( V \) replaced by \( \frac{\omega d_c}{2} \)) was large enough – greater than 0.1 for their problem – the curvature \( \kappa \) at the tip was exponentially sharp, that is,

\[
(2.5) \quad \kappa = c_1 \exp [c_2 (\text{Ca})],
\]

where \( c_1 \) and \( c_2 \) were constants specific to the problem. (The curvature has dimension of inverse length – the inverse of curvature is the radius of curvature – and is taken as positive if the center of curvature is in the air side.) In the limit \( \text{Ca} \to \infty \), a cusp was indeed possible, but in real fluids, they estimated for their specific problem that a physical cusp could easily be formed for \( \text{Ca} \geq 0.25 \). Equation (2.5) has been experimentally verified by Lorenceau et al. recently [7]. (It should be emphasized in advance that all the theoretical solutions described so far were derived under the assumption that the problem was only two-dimensional in nature.)

In reality, however, a physical cusp can never be formed due to the presence of air above the interface, which has a small but non-vanishing viscosity that was not taken into account in Jeong and Moffatt’s solution. The non-vanishing viscosity of air means that air immediate above the interface is very likely swept along and drawn into the narrow cusp-like tip by the converging flows continuously, and forced back out. Hence, the air will exert a lubrication pressure between the converging flows. When the cusp-like tip has become so sharp (narrow) that the lubrication pressure is too big a stress for the tip to bear (which is possible if the capillary number is large
enough), the stationary cusp-like tip “cracks” before a physical cusp can be reached, and air is entrained into the bath below the free surface. This is a simplified explanation of Eggers’ solution [6], and is indeed what is observed in reality.

Noticing from experimental observations that the entrainment transition was similar to the “cracking” of the stationary cusp-like tip, and combining Reynolds’ lubrication theory with result obtained from linear elasticity theory, Eggers arrived at a critical curvature for the cusp-like tip

\[ \kappa_c \sim (\mu/\mu_0)^{4/3} \]  

above which the sharp tip could not be stationary – that is, above which fluid (of viscosity \( \mu_0 \)) from the upper half of the interface would be entrained into the liquid (of viscosity \( \mu \)) in the lower half; the constant of proportionality would be determined by the parameters of a specific problem. Equation (2.6) immediately implies, because of equations (2.2) and (2.5), that there is a corresponding critical capillary number, and hence a critical speed of flow for every specific problem. Combining equations (2.5) and (2.6) leads to a “prediction” (not exact, just order of magnitude prediction)

\[ C_a_c \sim \ln(\mu/\mu_0) \quad \text{or} \quad V_c \sim (\gamma/\mu) \ln(\mu/\mu_0). \]

This has been verified experimentally by Lorenceau et al. [7] in an experiment similar to that carried out by Jeong and Moffatt described above, which was strictly a two-dimensional flow problem.
Eggers further pointed out that the local solution (of the shape of the free surface) near a cusp was self-similar and universal, regardless of the type of flow (whether in 2-D or 3-D) that generated the cusp. Hence, equations (2.5) and (2.6), derived based on such solution, were equally generic for all problems that share the same cusp solution, including the viscous jet diving into a stationary bath, and the counter-rotating cylinders submerged in a bath of viscous liquid.

For a viscous jet falling into a stationary bath, he further argued that the problem could be treated as a two-dimensional flow. The justification was that the cusp-like tip was very narrow, ~ 1 μm, hence it was much smaller than, say, the jet’s diameter at the entry point, ~ 1 mm, so the circular nature of the dip around the jet could be neglected, and the flow could be treated as two-dimensional locally near the cusp. Therefore, equation (2.7) would be a good quantitative explanation for the critical entrainment velocity of a viscous jet falling into a stationary bath. However, it should be noted that equation (2.7) has not been verified experimentally for the problem of a plunging viscous jet. Nevertheless, it is consistent with the experimental results of Lin and Donnelly, even though there is an extra logarithmic factor in the expression for \( V_c \); the dependence of \( V_c \) on the liquid viscosity in the logarithmic term is weak because \( \mu/\mu_0 \sim 10^4 \) to \( 10^5 \) (assuming upper half of free surface was air and lower half was a liquid of typical viscosity ~ 100 centipoises), even if \( \mu \rightarrow 10\mu \), \( \ln(10\mu/\mu_0) = \ln(10^4) \) [or \( \ln(10^5) \)] + \( \ln(10) \), the increase of \( V_c \) due to the 2\(^{nd} \) term in this expression paled in comparison to the decrease of \( V_c \) in the factor \( \gamma/\mu \), so the logarithmic correction to \( V_c \) was hard to detect.
Equation (2.7) also helps to offer some explanation for the difference observed for high and low Reynolds number jets, as pointed out by Eggers. For high Reynolds number jets (such as water jets), the critical capillary number for air entrainment cannot be reached before the jets become turbulent, due to the low viscosity of the liquid; low viscosity of the liquid does indeed lower the critical curvature (see equation (2.6)), and hence the critical capillary number, but this advantage is not as great as the disadvantage of a low viscosity apparent in equation (2.2), because of the logarithmic form of equation (2.5).

2.3 Viscous jets falling into moving bath

It was quite interesting to learn (for the case of a stationary bath) that the peculiar way of air entrainment by a plunging viscous jet, in contrast to that of a high Reynolds number jet, was due to the viscous property of the liquid. For the case of a moving bath, a viscous jet continues to offer intriguing behaviors that are not observable for its counterpart in the high Reynolds number case.

2.3.1 The “bouncing jet”

There is no reason why the viscous bath cannot be moved when a viscous jet is falling into it. Just to see what will happen, one may start with a small jet speed (below the critical entrainment threshold) and a slow constant bath motion. Then what one will see is about the same as when the bath was stationary. The jet will merge smoothly with the bath, but in this case the dip on the free surface wrapping around the impinging jet is no longer symmetric – the dip on the side facing the incoming flow of the bath liquid will be less deep (see Figure 1.3 (e)). This is perhaps hardly surprising. But if one tries to perturb the falling jet by passing a stick
through it, then to one’s surprise, the jet bounces off the moving bath surface and can stay bouncing for minutes (depending on the exact experimental conditions)! Figure 2.5 shows an example of this “bouncing jet” phenomenon – for this particular example, the jet even leaps up from the surface twice. This phenomenon has been shown to occur in a variety of viscous liquids by Matthew Thrasher [9], including cooking oils, shampoo and laundry detergent. “Non-viscous” liquid such as water has not been able to demonstrate the same behavior.

Fig. 2.5. The “bouncing jet”. The jet and bath are silicone oil. The bath is moving to the right. The image above and below the bath surface were taken separately from different angles to avoid the meniscus of the bath surface on the wall of the container. Taken from [10].

The first observations were enough to motivate Thrasher et al. [10] to proceed to investigate the phenomenon further in a more systematic way. They chose silicone oils with different (dynamic) viscosities (52 – 349 centipoises) but similar surface tension (21.0 – 21.2 dynes/cm) and density (0.959 – 0.968 g/cm$^3$) as their experimental liquids. Several parameters, based on their importance to the phenomenon and readiness to be controlled and varied, were chosen to be studied: the (dynamic) viscosity of the liquid $\mu$, the horizontal relative velocity of the bath $V_b$, the flow rate $Q$ and the height of release of the jet $H$ above the bath surface.
Accordingly, they built an apparatus (similar to the one used for the present work) such as that sketched in Figure 2.6 to carry out the study. The liquid was circulated by a digitally controlled pump, from a central overflow reservoir to a vertical cylindrical nozzle (of fixed inner diameter: 0.52 cm) a distance H above the annular bath surface (of width ≈ 12 cm, and depth ≈ 8 cm) from which the jet was released under gravity. A constant bath speed $V_b$ was created by fixing the jet’s position and rotating the tank at a constant revolution rate $\Omega$. To make an otherwise non-bouncing or merging jet bounce, a small rod (about 0.5 cm in diameter) was used by them to perturb the falling jet by passing it through the jet – which they called “initiating” the bounce (see Figure 2.7).

Fig. 2.6. The experimental setup used by Thrasher et al. [10] (Taken from the same reference.)
As a matter of fact, a viscous jet of speed or capillary number less than that needed to entrain air, cannot bounce on its own, unless it is “initiated” by perturbing the jet (i.e. by passing a rod or stick through the jet); on the other hand, a jet, whose speed or capillary number is above that needed to entrain air, can bounce on its own when the bath velocity is faster than a critical threshold, without the need to initiate the bounce by perturbation (even though perturbing the jet in this case can also make the jet bounce). This was mentioned by Thrasher et al. [10] as another way to start the jet bouncing. (This behavior of the jet is called “auto-bounce” and will be described and explained in Chapter 4.) It seems that, from going through the recorded observation frame by frame, what perturbing the jet does is to create a disturbance to the jet’s surface (Figure 2.7 (c)) – much like those indicated in Figure 2.3 (b) or Figure 2.4 (a) for a high Reynolds number jet – before the jet impacts the bath surface. Such a disturbance to the jet’s surface can help to entrain air – just like that in the case of high Reynolds number jets – and create non-coalescence between the jet and bath that is a prior condition for a jet to bounce off the moving bath surface.
(During bouncing, the jet is separated from the bath by a thin layer of air in between them, preventing them from coalescing together, as was verified by Thrasher et al. [10].) But the bouncing jet phenomenon is not observed for the case of water. This may be due to, besides other factors, the reason that non-coalescence between the jet and bath is not possible for water, even if air is entrained when the disturbance (of the jet’s surface) impacts the bath surface. The reasoning presented here, however, remains a speculation that is yet to be verified or looked further into.

In spite of the mystery of how a disturbance to the jet helps to start the jet bouncing, perturbing the jet manually by passing a rod through it, however low the controllability or reproducibility of the action may be, often results in the jet bouncing (when the bath is moving). In this spirit, Thrasher et al. studied how the bouncing jet phenomenon depended on the bath velocity, flow rate, nozzle height and the viscosity of the liquid, i.e. $V_b, Q, H$ and $\mu$.

**Dependence on bath velocity**

First, they fixed $H$, $\mu$ and $Q$, and varied $V_b$. Their result suggested that at a particular $H$, $\mu$ and $Q$, no bouncing would be observed when $V_b$ was below a certain threshold. But they were unable to pinpoint the exact threshold; they just defined the threshold to be the bath velocity below which the bouncing jet could not last for more than 5 seconds after it had been initiated by perturbation. At the same $H$, $\mu$ and $Q$, there was a second higher threshold above which the jet’s behavior became what they called the “trailing jet”, in which the jet basically laid flat on the bath surface, prevented from coalescing by an air layer that would eventually drain out (see Figure 2.8(c)). And between the lower and higher thresholds, the jet would bounce obliquely.
All others being equal, increasing $V_b$ will make the obliqueness of the jet’s bounce more horizontal, which is perhaps not surprising, as one can speculate that the jet will pick up more horizontal momentum, from the faster motion of the bath liquid, when $V_b$ is larger. But note that when $V_b$ is larger, the jet’s vertical momentum after impact with the bath is also less – as can be seen from the decrease of the bouncing height, when $V_b$ was increased from that in Figure 2.8 (a) to that in Figure 2.8 (b). It should also be noted that the jet does not seem to lose much of its speed after impacting with the bath surface, as can be inferred from the jet’s thickness before and after impact using the continuity condition (2.3).

Fig. 2.8. Effect of changing bath velocity on bouncing. The bath velocity increases from (a) to (c). The bottom cross-sectional schematic diagram exaggerates the air layer between the jet and the bath surface. The behavior in (c) was called the “trailing jet”; the photo was taken above and below the bath surface from different angles to avoid the meniscus. Taken from [10].
Dependence on $Q$

Next, they fixed $H$, $\mu$ and $V_b$, and varied $Q$. At a particular $H$, $\mu$ and $V_b$, a larger $Q$ would result in a bounce that penetrated deeper below the bath surface level, or similarly, larger $Q$ would result in a deeper indentation of the bath surface, as illustrated in Figure 2.9.

At a fixed $H$ (and nozzle diameter), increasing $Q$ will increase both the jet’s speed and diameter at the entry point into the bath. As a result, the jet’s inertia or momentum is increased. This larger downward momentum, lubricated by an air film, “pushes” more of the viscous bath liquid away and deforms the bath surface more, resulting in a larger surface indentation upon impact. Note that even though the jet’s speed before impact is greater for higher $Q$ than for lower $Q$, the jet’s speed after impact for higher $Q$ is less than that for lower $Q$ – as can be inferred from the difference in bouncing height and range in Figure 2.9 (a) and (b). This leads to the conclusion that higher inertia or momentum of jet will incur higher loss of the jet’s energy. But it is difficult to say where the jet’s energy is lost to; it could be used to deform the bath surface, or lost in dissipation in the shear straining of the air film or the bath liquid and jet itself.
The “bouncing regime” diagram

Next, they proceeded to obtain the “bouncing regime” diagram. As explained above, if the parameters H, μ and Q were held fixed and the bath velocity was varied, one could obtain a lower and higher threshold of the bath velocity, between which a “stable” bouncing jet could be observed. And if this was carried out for different flow rates Q (at fixed H and μ), the result was a “bouncing regime” diagram, in the flow-rate and bath-velocity parameter space, for a particular viscosity μ at a particular nozzle height H. Figure 2.10 (a) gives several of this bouncing regime (or region) diagram for different viscosities μ at a fixed nozzle height H. (Notice from each of the diagrams another effect of increasing Q while fixing μ and H: the lower and higher thresholds of the bath velocity increases.) It should be noted that increasing Q while fixing H will increase both the jet’s velocity and diameter at the entry point into the bath.

In an alternative way, Thrasher et al. have kept the flow rate Q and viscosity μ fixed, and varied the nozzle height H (and bath velocity V_b), to obtain a “bouncing regime” diagram in the “jet-velocity” and bath-velocity parameter space (Figure 2.10 (b)). Note that in this case, fixing Q and increasing H will increase the jet’s velocity but decrease the jet’s diameter at the entry point into the bath.

Also note that the effect of the jet’s diameter at the entry point was not taken into account in both of these two types of bouncing regime (region) diagrams described just above.
Fig. 2.10. The “bouncing regime” diagrams. The lower boundary of each (bouncing) regime diagram corresponds to the lower threshold of the bath velocity, below which no bounce state can be observed; whereas the upper boundary of each regime diagram corresponds to the higher threshold of the bath velocity, above which the system is in the “trailing jet” state. These bouncing regimes were obtained by perturbing the jet to initiate a bounce. (a) The regime diagrams for the different viscosities are all obtained at a fixed nozzle height $H$. (b) The regime diagram, for one particular viscosity, is obtained by fixing the flow rate $Q$ and varying $H$ in the range: 1.7 to 14.1 cm. (The jet velocity is obtained from $V_{\text{jet}} = Q / \pi r_{\text{jet}}^2$, where $r_{\text{jet}}$ is the jet’s radius before entry into the bath.) Taken from [10].
Dependence on $\mu$

The dependence on the viscosity of the liquid is best illustrated by the bouncing regime diagrams in Figure 2.10a. The results showed that as $\mu$ was decreased, both the lower and higher thresholds of the bath velocity went up, but the higher ones increased more, resulting in a larger bouncing regime (region) for a liquid of lower viscosity $\mu$. This observation implies that viscosity plays an important role in determining whether the bouncing jet phenomenon can be observed or not.

2.3.2 Multiple stable states

In the course of trying to obtain the bouncing regime of the bouncing jet, using the method of perturbing the jet to initiate a bounce, Thrasher et al. also noticed that such action sometimes would result in other behaviors of the jet that were also stable\(^2\)! For example, besides the bounce state, they could also obtain two other states by perturbing the jet with the rod; these two other states were called the “plunging jet” and “half-entraining jet” by them. Figure 2.11 shows the three states that could be obtained at the same experimental conditions by perturbing the jet with the rod. Below the photograph of each state is a corresponding schematic illustration of the state.

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\(^2\) The authors did not specify how “stable” these states were. But since these behaviors of the jet were also observed in the present study, it can be said that the stability each of the three states shown in Figure 2.11 is different and may vary depending on the experimental control parameters, and that, averagely speaking, each of the observed states could be maintained for a length of time that was at least an order of magnitude longer than couple seconds.
Fig. 2.11. **Multiple “stable” states of a viscous jet falling into a moving bath of the same liquid.** The bath surface is above each photograph. Below each photograph is a schematic illustration for the corresponding state. (a) The “plunging jet”. (b) The “half-entraining jet”. (c) The “bouncing jet”. These states were obtained under the same experimental conditions by passing a rod through the falling jet: $\mu = 106$ centipoises (silicone oil), $Q = 0.35 \text{ cm}^3/\text{s}$, $H = 6.4 \text{ cm}$ and $V_b = 15.2 \text{ cm/s}$. Taken from [10]
Chapter 3: Experiment

3.1 Experimental apparatus

Fig. 3.1. The experimental setup. The outer diameter of the cylindrical tank is approximately 60 cm. The objects are not drawn to scale but their relative sizes in the drawing do give a sense of their relative sizes in reality.

An apparatus such as that shown in the schematic drawing of Figure 3.1 was used to carry out the present study in ambient (air-conditioned room) temperature and pressure conditions. A (near) cylindrical glass tank was constructed and fixedly mounted onto a horizontal base that was rotated by a computer-controlled motor; the wall of the glass tank was transparent to allow for direct observations or recordings of the jet’s behaviors. (It is hard to make a perfectly cylindrical tank, a small portion of the wall of the tank is actually flat instead of being an arc – this small imperfection actually has the advantage that images of the jet taken through this portion will not be
distorted as in the case when the wall is curved.) The tank was partitioned into a central overflow reservoir and an annular bath, of approximately uniform width, by a strip of stainless metal forming a cylindrical barrier between the inner reservoir and the annular bath. The width of the annular bath is about 7 to 8 cm, and its depth is about 5.5 cm. The cylindrical metal barrier was painted on the outer side to reduce the reflection of light, which would otherwise make the observation of the jet’s behavior in the bath difficult. The tank was properly covered to prevent dust from contaminating the oil and any wind in the air-conditioned room from disturbing the vertical jet.

A pump (not shown in Figure 3.1 but will be described below) was used to withdraw oil from the central reservoir and release it out of a vertically aligned, cylindrical Teflon nozzle, of inner diameter 1.82 mm, directly above the center of one side of the annular bath. Any excess oil of the bath would drain down the inner side of the metal barrier to the overflow reservoir, thus maintaining a constant bath surface level during experiments. Although no attempt has been made to control the oil temperature, the oil jet temperature was monitored by a thermocouple properly and fitly inserted into a T-junction (T-shaped Swagelok™ tube fitting) in which the oil was temporarily settled before being forced out of the nozzle connected to it from below; the T-junction had an inner diameter of approximately 5 mm, two to three times that of the nozzle. The entire T-junction assembly was fastened to a vertical bar (not shown in the drawing), that had a scale on it, by an adjustable slide, so that the nozzle height or release height of the jet could be adjusted and measured; the nozzle height was defined as the distance between the bath surface and the outlet of the nozzle, and indicated as $H$ in the schematic drawing. The pump was connected to the
overflow reservoir and the T-junction with some Masterflex® tubing and Swagelok™ tube fittings and adapters, and the tubing was held in fixed positions throughout the entire study.

A thin metal wire of diameter about 1 mm, connected to a fixed assembly, could be rotated freely in and out of the side of the annular bath where the jet was, crossing the jet at about 0.5 cm above the bath surface; the wire was used to slow down the flow of the jet or to perturb the jet by passing it through the jet (the purpose of these will become clear in due course).

Two CCD cameras (25 frames s⁻¹) were used to observe and record the jet’s behavior at the same time by using a beam splitter slanting at a 45 degree angle from the vertical jet; this would allow both a close-up and zoom-out image of the jet to be captured simultaneously. The jet’s behavior below the bath surface level needed a more close-up observation and was obtained by the horizontal camera; the vertical camera was used to capture the jet’s entire behavior both below and above the bath surface level. The observation obtained by the vertical camera, however, would only be a mirror image of the actual observation under this setup. The recorded observations were stored away in digital format for later analyses.

Finally, it was noted that during the continuous operation of the pump, the oil jet temperature would heat up by a measurable amount (≥ 0.1 °C) depending on the duration the pump was run continuously. For example, a continuous operation of the pump for three to six hours might cause the oil jet temperature to go up by about 1 °C. The heating up of the oil jet temperature was fast initially, and became more and
more gradual as time progressed, but seemed unable to reach an equilibrium value as the ambient environment was not controlled.

### 3.2 Experimental parameters

The experimental parameters of interest to the current study are the jet’s speed and diameter at the entry point into the bath (or right before impact with the bath surface): $V_j$ and $d_j$, and the bath velocity at the jet’s entry point: $V_b$. But these are not the parameters that are directly controlled and varied. The ones that are directly controlled and varied are: the flow rate $Q$, the nozzle height $H$, the rotation rate of the tank $\Omega$ and the radial distance $R$ of the jet from the center of the cylindrical tank. $Q$ is related to $V_j$ and $d_j$ ($Q = \pi d_j^2 V_j/4$) and $\Omega$ and $R$ are related to $V_b$ ($V_b = 2\pi R\Omega$). In the present study $R$ and $d_j$ are to be held constant – this leaves $Q$, $H$ and $\Omega$ that are directly controlled and varied, but $Q$ and $H$ cannot be varied independently of each other if $d_j$ is to be held fixed (see *Calibration for constant jet diameter* below).

The experimental liquid is a silicone oil of nominal kinematics viscosity 150 cSt and density 0.86 g/cm$^3$. Its surface tension $\gamma$ is estimated to be around 20 dynes/cm at ambient conditions. The same liquid has been used throughout the whole study.

### 3.3 Calibrations

#### 3.3.1 Pump calibration

The pump was manufactured by Cole Parmer (Model number: 75211-30). It is a digitally controlled gear pump with variable motor speed and is used with a rotary pump head (Model number: 73004-00). The manufacturer gave a nominal
volume-per-revolution value of 0.092 mL/rev for the pump based on pumping water at room temperature. This value may be different if the liquid being pumped is viscous or the tubing used is significantly different. Since the value is stored in the microprocessor of the gear pump, and used by the pump to control the pump speed (rev/min) once a desired flow rate (mL/min) is input into the pump, this value needs to be as accurate as possible. Hence, it was re-calibrated for the current study, by following the calibration instruction of the pump. After the procedures, the pump automatically figured out the new value and stored it in its memory, replacing the previously memorized one. To check whether the pump was accurate or not after the calibration, the pump was instructed to pump out a specified volume, then the volume was measured by weighing the oil pumped out and then dividing it by the oil’s density; the error was less than 0.5 % on average, taking into account both the errors of the weight measurement and oil density. The density of the oil used in this study was measured, on average, to be $0.8654 \pm 0.0003 \, \text{g/cm}^3$ in the temperature range: $22.5 \pm 1.0 \, ^\circ\text{C}$, by using a 50-mL density bottle and precise weighing balance.

3.3.2 Calibration for constant jet diameter

As mentioned above, if $d_j$ is to be held constant, $Q$ and $H$ cannot be varied independently of each other. This is because $d_j$ can only be fixed at a particular value if $Q$ and $H$ are changed correlatively; for example, if $Q$ is increased, both $V_j$ and $d_j$ will increase, and in order to maintain $d_j$, $H$ needs to be increased such that $d_j$ will decrease back to the fixed value. In order to fix $d_j$ at a particular value for the present work, a calibration was carried out to find a corresponding $H$ for each $Q$ by trial and error. The diameter of the jet was measured by using a macro lens attached to the horizontal CCD camera, at a short fixed distance from the jet. This method could
achieve a resolution of 0.0052 mm/pixel (about 5 microns per pixel). The calibration result for $d_j$ fixed at $1.011 \pm 0.005$ mm is shown in Figure 3.2 below. Since the full calibration process (for all Q) took a long time, it was only performed once; this was okay because the pump and setup were reliable – repeating the calibration process (not for all Q) several times, with weeks or months apart, could reproduce the same calibration result.

![Constant jet diameter at entry = 1.011(0.005) mm](image)

Fig. 3.2. Calibration of constant jet diameter for the present study. Note that “jet height reading” is not the same as H. H is the difference between the jet-height and zero-jet-height readings. The zero-jet-height reading is $15.60 \pm 0.03$ cm.

3.4 Measurement methodology

3.4.1 Flow rate and jet velocity

Measurements of the flow rate was done by weighing the oil collected in 60 seconds when the pump was running continuously at the set flow rate, in a pre-weighed container, and then dividing the measured oil mass by the oil density. Such
measurements were carried out at different nominal flow rates of the pump, with each nominal flow rate being measured at least two times consecutively. And the whole process of measurements has been repeated several times, with weeks or months apart, to monitor the performance of the pump. The conclusion was that the pump’s performance was reliable; the error of the nominal flow rates of the pump was small, only about 0.1 to 0.2 % of the nominal value. Hence, the nominal flow rates could be taken as the “measured” flow rates with an error of no bigger than 0.2 %. With such practice, the jet velocity $V_j$ (at the entry point) could be calculated using equation (2.4) with $d_j = 1.011 \pm 0.005$ mm and $Q$ from the flow rate readings of the pump.

3.4.2 Tank rotation rate and bath velocity

The tank rotation rate $\Omega$ was controlled and varied by a computer-interfaced motor, with precision down to 0.001 rev/sec. The range of $\Omega$ used in this study was between zero and 0.3 rev/sec. For $\Omega$ values higher than that, the bath surface would become too much curved due to centrifugal forces. The tank rotation rate could be measured by fixing a reference marker on the side of the tank, and using the CCD camera to measure the average period of rotation at a particular $\Omega$. Repeated measurements of $\Omega$ this way, at different nominal $\Omega$ values, separated by weeks or months, gave measured $\Omega$ values that were essentially the same as the nominal ones, with a typical error of about 0.0005 rev/sec. Hence, the nominal $\Omega$ values could be taken as the “measured” ones. With this practice, the bath velocity at the jet’s entry point $V_b$ was calculated from $V_b = 2\pi R \Omega$, where $R$ was the fixed radial distance of the jet from the center of the tank ($R = 25.80 \pm 0.05$ cm in the current study), assuming that the bath has reached the same rotation rate as that of the tank.
The assumption that the oil bath’s rotation rate is the same as that of the tank is justified as the bath is viscous, and it can easily reach a “solid-body” rotation when enough time is given for it to change the rotation rate. The response time of the oil bath to the change of its rotation rate is almost negligible if the rotation rate has been changed slowly – using an acceleration or deceleration rotation rate of 0.001 (rev/sec^2), for example. This has been tested using a method that makes use of a behavior of the jet that is dependent on the bath velocity (the “auto-bounce” mentioned in Section 2.3.1) which will be described in Chapter 4. So the description of this method will be postponed till after the “auto-bounce” has been explained.

3.4.3 Oil jet temperature

As described above, the temperature of the oil jet was monitored by a thermocouple inserted into the T-junction, from which the oil was forced out of the connected nozzle. To verify the accuracy of this temperature measurement, oil flowing out of the nozzle was collected in 50-mL glass bottle, and a second accurate thermocouple was quickly inserted into the collected oil to measure its temperature. Within the precision of the measurements (0.1 °C), the two temperature readings were essentially the same. During this verification process, care was taken to ensure that the oil was not heated up or cooled down quickly by any external sources. This process was repeated at least once.

3.4.4 Stability time and other characterizations

The recorded results of the experiment can be used to obtain other measurements. For example, the stability time of the merge or bounce state can be
obtained by analyzing the video files. The calibrated images of the jet can also be used to obtain other length or angle measurements of the jet.
Chapter 4: Results

4.1 Conventions

First, since the jet diameter \( d_j \) (cm) at the entry point into the oil bath was held constant to be \( 0.1011 \pm 0.0005 \) cm (see Chapter 3 about the constant jet diameter calibration), \( Q = \pi r^2 V \) implies that the jet speed \( V_j \) (cm/s) at that point is proportional to the volumetric flow rate \( Q \) (cm\(^3\)/min) of the pump such that:

\[
Q = 0.05055^2 \pi (60) V_j.
\]

(4.1)

Second, since the radial distance \( R \) (cm) of the vertical jet from the center of the rotation tank was also held fixed to be \( 25.80 \pm 0.05 \) cm, the bath velocity \( V_b \) (cm/s) at the jet’s impact point is proportional to the rotation rate \( \Omega \) (revolution/sec) such that:

\[
V_b = R \omega = 2\pi (25.8) \Omega.
\]

(4.2)

Henceforth, the flow rate \( Q \) and jet speed \( V_j \) will be used interchangeably, and similarly for the bath velocity \( V_b \) and rotation rate \( \Omega \). This convention is for convenient purpose, as the data directly recorded were the \( Q \) and \( \Omega \) values but the parameters of interest are \( V_j \) and \( V_b \). When necessary, the \( Q \) and \( \Omega \) values can be converted to corresponding \( V_j \) and \( V_b \) values via equations (4.1) and (4.2).
4.2 Stationary bath (Ω = 0)

4.2.1 “Critical entrainment speed $V_c$” & merge-state stability

It was learned from previous studies that, for a viscous liquid jet falling vertically into a stationary bath of the same liquid, there was a critical jet speed at which air entrainment would occur – that is, the jet would spontaneously change from the “merge” state (Figure 4.1a) to the “plunge” state (Figure 4.1b) when the jet speed was above a threshold. Since the previous studies did not mention that it would take some time, after the critical jet speed had been reached, for the jet to make the transition, one naturally assumes that the transition will be an instantaneous one once the critical jet speed has been reached, and below the critical jet speed no transition will be possible. But this is not what was observed in the present study. What was observed was that there was not a sharp transition in the jet speed from the merge to plunge state; rather the stability time of the merge state decreased exponentially as the jet speed increased towards the instantaneous transition threshold – as will be shown below.

![Fig. 4.1. Instability of a merge state. (a) The “merge” state and (b) the “plunge” state for Ω = 0. For this particular figure shown, Q = 51 cm³/min and the merge state lasted for 69.88 s before spontaneously transited to the plunge state. These images were recorded by the horizontal CCD camera.](image-url)
Preparation of a merge state. Images are ordered left to right, only every fifth frame is shown, i.e., each frame is 0.2 s apart. $Q = 51 \text{ cm}^3/\text{min}$, $\Omega = 0$. These images were recorded by the vertical CCD camera, thus they are the mirror images of the actual observation.

To see if a particular jet speed was the “critical entrainment velocity $V_c$”, the flow rate $Q$ and release height $H$ of the jet were set at the appropriate values while the jet was slowed down by the thin metal wire such as shown in the first image of Figure 4.2. It was then made sure that the system was free of air bubbles and the jet speed and bath oil level had reached the appropriate steady values. Then the metal wire was slowly and gently moved away to create a merge state (Figure 4.2). The whole process was recorded on video and so the stability time of the merge state could be obtained from analyzing the video file. If at a particular jet speed, the jet plunged as soon as the metal wire was moved away, that speed could be regarded as the “critical entrainment velocity $V_c$”; since the CCD camera had a time resolution of 25 frame s$^{-1}$ only, the jet was considered as plunging “instantaneously” if the merge state did not last for longer than 0.04 sec (1 frame).

With this method, the whole process was repeated many times or at least several times at a particular jet speed during a particular run of the experiment, where the oil jet temperature was approximately constant (no greater than 0.1 °C variation)
to allow several different jet speeds to be tested in the same run. Then many runs –
each having a slight different oil jet temperature – were carried out. In each run at
each jet speed, the stability time of the merge state fluctuated each time around an
average value, which could be used to represent the merge-state stability at that jet
speed and oil jet temperature.

Figure 4.3 gives the results for seven runs done in different times over two
days. The results show that for jet speed $\geq 55$ (cm$^3$/min), the jet plunges
“instantaneously” almost every time, so it can be regarded as the “critical
entrainment velocity $V_c$” (the deviation in couple runs will be discussed shortly after);
and for jet speeds between 53 and 55 (cc/min), the average stability time of the
merge state always increases when the jet speed is decreased.
Fig. 4.3. Merge-state stability for $53 \leq Q \leq 57$ (cm$^3$/min) and $\Omega = 0$. Different symbols indicate different runs of the experiment at different oil jet temperatures. The “average stable time” is the average time the merge state is stable at a jet speed before it spontaneously becomes a plunge state. In each graph, hotter jet temperature indicates the run was done later in time. The first graph is for runs done in the morning of day 1; second graph for runs in the afternoon of day 1; third graph for one run on day 2. (1st graph): Diamond: 22.5 °C; Square: 22.6 °C (deviation from the other runs will be discussed later); Triangle: 22.7 °C. (2nd graph): Diamond: 22.2 °C; Square: 22.4 °C (the deviation from the other runs will be discussed later); Triangle: 22.7 °C. (3rd graph): Diamond: 23.0 °C.

The question that comes to mind after knowing these results is: What will the result be for jet speeds below 53 (cc/min)? So, the same method and procedures as described above were applied to jet speeds between 47 and 52 (cc/min). But here the
oil jet temperature for each run – which consists of testing of several jet speeds – could not be kept constant to within 0.1 °C because the experiment time for each run was much longer than before, so the increase of the oil jet temperature due to the continuous running of the pump could not be avoided. The runs now took longer because for jet speeds \( \leq 49 \) (cc/min), the merge state was very stable; the merge state was stable each time for as long as the process was not terminated manually – the longest time waited was one and half hours. Thus, in each run at those jet speeds, the process was only carried once instead of many times, but the run was repeated many times.

In Figure 4.4, the results for six runs done on four separate days show basically that the merge state is much more stable at jet speeds below or equal to 49 cm\(^3\)/min than above it (except for couple runs that deviate from the rest – this will be discussed below); if the experiment had not been discontinued manually for jet speeds \( \leq 49 \) (cc/min), the results would have shown that the merge state is even more stable at those jet speeds.
Fig. 4.4. Merge-state stability for $47 \leq Q \leq 52$ (cm$^3$/min) and $\Omega = 0$. Different symbols indicate different runs of the experiment done at different oil jet temperature range. Circled data points indicate that the merge state is stable for the time indicated before being discontinued manually; data that are not circled indicate that the merge state is, on average, stable for the time indicated before it spontaneously becomes a plunge state. The first graph is for runs done on day 1; second graph for runs on day 2 to 4. (1st graph): Diamond: 22.0 – 22.5 °C; square: 22.6 – 23.0 °C. (2nd graph): Triangle: day 2, 21.5 – 22.4 °C; ‘‘×’’: day 2, 22.2 – 22.5 °C; Diamond: day 3, 22.1 – 22.4 °C; square: day 4, 21.9 – 22.2 °C. (Deviations of the triangle and diamond data will be discussed below.)
A careful inspection of the data in Figures 4.3 and 4.4 indicates that the deviations of the runs noted in those figures were unlikely to be caused by the small change in the absolute temperature of the oil jet itself. However, the heating up of the oil jet over time, as the oil was continuously circulated from the tank to the pump then back to the tank via the jet (see Ch.3 for the experimental apparatus), was certainly a suspected cause of the deviations noted in Figure 4.3. Note that the oil is a poor conductor of heat. Thus, for the experimental apparatus used in the present study, it was likely that the top part of the oil in the annular bath would be slightly hotter than the bottom part for a transient period of time; note the transient nature of these systematically deviated runs. The uneven temperature distribution in the annular bath – however minor – might cause some undesired flow in the bath, which could interfere with the flow induced by the merging or plunging jet; and it was learned in Chapter 2 that the flow in the bath was critically important to the formation of cusp on the free surface, and hence to the stability of the merge state.

For the systematically deviated runs noted in the second graph of Figure 4.4, the cause was also unresolved. But it was also suspected that some relevant properties of the oil, such as the viscosity or surface tension, might have been altered temporarily after doing the experiment for a long time; the deviation disappeared after the system had been left alone for several days. Note first that over the course of doing the experiment for long continuous hours each day for several consecutive days, a quantity many times over the volume in the annular bath – which held about 6000 c.c. – had been circulated from the tank through the pump and back to the tank; if the oil had not been heated up as it passed through the pump, this might have been okay, but since it was the opposite that actually happened, this might have something
to do with the temporary change of the relevant oil properties. Secondly, in a couple of times in the long experiment, the jet had been left plunging into the bath for much longer than desired (due to temporary lack of attendance), creating a lot of tiny micron-sized air bubbles that might have dissolved into the oil bath temporarily, especially when the oil was heating up during the process; this too might have caused the relevant oil properties to be changed temporarily.

In spite of all these deviations, a look back at the results in the above two figures makes one wonder: Where should the critical entrainment velocity $V_c$ be? Should it be defined as the jet speed at and above which any merge state will instantaneously become a plunge state spontaneously, that is, there is no merge state possible at all, or should it be defined as the jet speed at and above which the merge state is relatively “unstable”, in contrast to the very stable merge state observed at and below 49 cc/min? Moreover, it should be noted that, based on the observations, the plunge state obtained spontaneously after the merge state had lost its stability (between 50 and 55 cc/min) was stable.

It may be helpful to see all the results in one single plot: the first graph in Figure 4.5 is a semi-log plot of all the above results combined (including the deviated runs). One striking observation is that all the data seem to fall on a negatively sloped line except for those corresponding to jet speeds $\leq 49$ or $\geq 55$ (cc/min) – this observation is made clearer in the second graph of the same figure, in which the deviated runs noted earlier have been dropped. The accuracy of the data for jet speeds $\geq 55$ cc/min is limited by the time resolution of the CCD camera; all the “instantaneous” plunging cases in the result shown are assumed to have a merge state.
stable time of 0.04 second (estimated time lapse between two successive frames of
the camera). For jet speeds $\leq 49$ cc/min, the stability time of the merge state could be
infinite and it is impractical to wait that long, so the experiment was discontinued
manually after the merge state had been stable for significant amount of time; if it
had been known earlier that the stability of the merge state seems to increase
exponentially as the jet speed is decreased, longer waiting time would have been
given to the experiment for jet speeds $\leq 49$ cc/min. (So the moral that data analysis
should be done in parallel of the data collection or soon after the data is collected is
really worth keeping in mind; do not wait until all the planned work has been carried
out to start the data analysis.)

But if it turns out that the merge state at jet speeds $\leq 49$ cc/min is indeed
orders of magnitude more stable than the merge state $> 49$ cc/min, then it will imply
that there is a meta-stable or unstable range of the merge state between 49 and 55
cc/min (for $\Omega = 0$).
Fig. 4.5. Semi-log plots of merge-state stability for $47 \leq Q \leq 57$ (cm$^3$/min) and $\Omega = 0$. Circled data indicate that a merge state is stable for the time shown before the experiment is discontinued manually; data that are not circled indicate that the merge state is, on average, stable for the time shown before spontaneously becoming a plunge state. (1$^{st}$ graph): All the runs (in Figures 4.3 and 4.4) are included; the deviated runs/data are boxed by rectangles. (2$^{nd}$ graph): All the runs excluding the deviated ones are plotted.
It is seen in the second graph of Figure 4.5 that eliminating the deviated runs has really reduced the scattering of the data. It is certainly possible that the scattering can be further reduced if the experiment is carried out in a more systematic way to give better statistics of the merge-state stability at each jet speed. (The reason this has not been done so is that at the time of the experiment it had not been expected that the average stability time of the merge state could be something of interest to look into, as the focus was to find the “critical entrainment velocity $V_c$.”)
Fig. 4.6. **Stability of the plunge state for \( Q \leq 49 \text{ (cm}^3/\text{min)} \) and \( \Omega = 0 \).** The sequence of frames at each jet speed is ordered from left to right, then from top to bottom. The first image in each sequence shows the jet right before being perturbed. The perturbation of the jet by the wire causes oil drops to be formed at the end of the oil jet; for \( Q \geq 42 \) these drops fall through the bath surface, temporarily prevented from coalescing with the bath by a thin air film which will eventually puncture. (a) \( Q = 41 \): no plunge state is possible; 0.08 s between frames. (b) \( Q = 42 \): plunge state is unstable; 0.08 s between frames. (c) \( Q = 47 \): plunge state is unstable; 0.04 s between frames (unless indicated otherwise). (d) \( Q = 48 \): plunge state is stable, air film is short and steady; 0.08 s between frames. (e) \( Q = 49 \): plunge state is stable, air film is long and air bubbles are forming at its lower end; 0.04 s between frames.
4.2.2 Stability of the plunge state

It was mentioned above that the plunge state was stable for jet speeds between 50 and 55 (cc/min). The plunge state at these jet speeds can be obtained spontaneously from an “unstable” or weakly stable merge state. But for jet speeds ≤ 49 cc/min, as the merge state is very stable, the plunge state cannot be obtained spontaneously from a merge state. One way to obtain a plunge state at these jet speeds is by perturbing the jet with the metal wire (see Experimental apparatus in Chapter 3); it should be remarked that for jet speeds above 49 (cc/min), the plunge state can also be obtained by perturbing the oil jet, and that the stability and characteristics of the plunge state obtained this way are no different from those of a plunge state obtained spontaneously. Figure 4.6 shows the behavior of the plunge state obtained by perturbing the jet at five chosen jet speeds ≤ 49 (cc/min). The observed results could be summarized as follows:

<table>
<thead>
<tr>
<th>Jet speed or Q (cc/min)</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>41</td>
<td>No plunge state or air entrainment is possible.</td>
</tr>
<tr>
<td>42</td>
<td>Plunge state is unstable (or air entrainment is not sustainable); usually lasts for less than ~ 1 s.</td>
</tr>
<tr>
<td>43 – 46</td>
<td>Plunge state is unstable; usually lasts for only ~ 1 s.</td>
</tr>
<tr>
<td>47</td>
<td>Plunge state is unstable; sometimes the behavior is more like that in jet speeds 43 – 46 but other times it is more like that at jet speed 48 except that the plunge state is only stable for ~ 10 s.</td>
</tr>
<tr>
<td>48</td>
<td>Plunge state is stable with steady short air film.</td>
</tr>
<tr>
<td>49</td>
<td>Plunge state is stable with long air film; air bubbles are constantly formed at the lower end of the air film.</td>
</tr>
</tbody>
</table>

N.B.: The length of the air film stays about the same for jet speeds above 49 (cc/min).
These observations have been observed repeatedly over time. Therefore, even though the stability of the plunge state has not been studied in detail (like the stability of the merge state), from repeated observations spanning a long period of time, it can be concluded:

- The plunge state is not possible for jet speed \( \leq 41 \) (cc/min);
- It is unstable for jet speeds between 42 and 47 (cc/min) inclusively;
- It is stable for jet speeds \( \geq 48 \) (cc/min);
- The three results stated just above are highly reproducible and insensitive to the small variation of the oil temperature.

In order to study the stability of the plunge state in detail or systematically, the problem created by the constant production of air bubbles for jet speeds \( \geq 49 \) cc/min needs to be overcome first.

Finally, it is interesting to note that the jet speed at and above which the plunge state is stable, namely 48 cc/min, is just below the jet speed at and below which the merge state was observed to be very stable, namely, 49 cc/min.

4.3 Non-stationary bath \((\Omega > 0)\)

4.3.1 Merge-state stability

It has been shown above how the stability of the merge state depends on the jet speed for a stationary bath (Figures 4.3 and 4.4). When the oil bath is no longer stationary but moves at a constant rotation rate \( \Omega \), the stability of the merge state turns out to depend on both the jet speed and bath speed in a rather complicated way.
Here, only the jet speeds where the merge state changed from being very stable to weakly stable or “unstable” were studied; whether there is still a critical entrainment velocity $V_c$ for $\Omega > 0$ is yet to be looked into. Similar experimental method to that used when the bath was stationary has been employed, but now the oil bath needed to be set to rotate at a constant $\Omega$ value first before the experiment could be started (the issue whether the oil in the bath has reached a solid body movement with the tank was explained in Chapter 3).

The result for a short range of $\Omega$ values (bath speeds): $0 \leq \Omega \leq 0.05$ (rev/s) is shown in Figure 4.7; the merge-state stability for other $\Omega$ values (bath speeds) has not been studied in detail – only preliminary data were obtained. This result is from repeated data collected over a month’s period of time, and the data were no longer separated into runs by their oil jet temperature but grouped together according to their applicable $Q$ (flow rate) and $\Omega$ (rotation rate) values; the range of oil jet temperature applicable to these data was approximately 22.0 – 23.0 °C. The first graph is a plot similar to those in Figure 4.3 or 4.4 and the series are labeled by the different bath speeds. The second graph is a plot of merge-state stability time against bath speed and the series are labeled by the different jet speeds. The result shows that the effect of a slow bath speed on the stability of the merge state is so different for jet speeds 50 and 51 (cc/min), compared to the other two jet speeds 52 and 53 (cc/min) – of which the stability of the merge state is essentially the same whether the oil bath is moving or not. At the jet speeds 50 and 51 (cc/min), a very slow bath motion can cause the merge state to be a lot more stable than when the bath is stationary, but this effect diminishes once the bath speed becomes faster. Also, this
effect is more gradual at the jet speed 51 (cc/min) than at 50 (cc/min), of which the change in the stability time of the merge state is “sharper”. 

Fig. 4.7. Stability of the merge state for $0 \leq \Omega \leq 0.05$ (rev/s) and $50 \leq Q \leq 53$ (cm$^3$/min). Circed data indicate that a merge state is stable for the time shown before the experiment is discontinued; data that are not circled indicate that the merge state is, on average, stable for the time shown before it spontaneously becomes a plunge state. The data were repeatedly collected in one month’s time and no differentiation in oil jet temperature was made; the applicable oil jet temperature range is 22.0 – 23.0 °C. (1st graph): the series are labeled by the different $\Omega$ (bath speed) values. (2nd graph): the series are labeled by the different $Q$ (jet speed) values.
Fig. 4.8. **The merge state at different jet speeds and bath speeds.** The bath is moving right to left. The numbers on the left indicate flow rate (cc/min) and the numbers at the bottom indicate rotation rate (rev/s). Empty space indicates no data was taken. At three locations in this figure, a plunging state rather than a merge state is shown because no merge state lasting longer than 0.04 s was observed in these cases. These pictures were not all taken at the same camera setting, so deviation of the image from the rest occurs for some of them.
From earlier observation that the impinging jet is asymmetric when the bath is in motion (see Figure 1.3 (e)), and the understanding (from Chapter 2) that the instability of the merge state is intimately related to the sharp curvature of the free surface formed around the impinging jet, one may wonder whether there is a correlation between the peculiar dependence of the merge-stage stability on jet speed and bath speed, and the asymmetric curvature of the free surface formed around an impinging jet when the bath is moving. From the comparison of merge-state photos at the relevant jet speeds and bath speeds shown in Figure 4.8, there is no obvious correlation between the two. In particular, the merge states at the jet speed 53 (cc/min) and relevant Ω values seem to look the same as those at the jet speed 50 (cc/min) and same Ω values, even though the merge-state stability times for those Ω values were so different at the jet speeds 50 and 53 (cc/min). But this may be due to the reason that the level of magnification in these photos may not be great enough to tell if there is indeed any difference in the free surface shapes or profiles of the different merge states shown.

4.3.2 The bounce state region

A viscous jet falling into a moving bath of the same liquid can bounce off the bath surface, demonstrating the bouncing jet phenomenon. It was learned in Chapter 2 that the bouncing jet could only be observed or was only “stable” in a certain region in the control parameter space. This “bouncing jet region” or “bounce state region” was also mapped out in the present study (see Figure 4.9), of which the viscosity parameter was fixed, but the other control parameters were slightly different from before: the present study has fixed the jet’s diameter at the entry point (into the bath), and varied the flow rate Q and nozzle height H accordingly; fixing the
jet’s diameter (at the entry point) helps to keep the effect of the jet’s diameter constant, and to really observe the effect of changing the jet and bath velocity. (The jet’s velocity was changed by varying both the flow rate Q and nozzle height H according to the calibration results for the constant jet diameter; see *Calibration for constant jet diameter* in Chapter 3)

The method used in the present study in obtaining the bounce state region was similar to that employed by Thrasher *et al.*: a bounce state was initiated by passing the metal wire through the jet; the bounce state was considered “stable” if it lasted for more than a few seconds. Usually the difference in the stability of the bounce state across the lower boundary was very obvious, but because of the complication of perturbing the jet to initiate a bounce, the lower threshold of the bath velocity could not be obtained at exactly the same bath speed each time, and has a relatively large uncertainty: about 0.005 rev/s (five times the smallest that can be varied). The upper boundary was formed by the higher thresholds of the bath velocity at which a bouncing jet became a trailing jet; the error in this boundary was mainly due to the human judgment of when a trailing jet had been formed.

Finally, note that the bounce state region was obtained much earlier than the rest of the results in the present study, under a slightly different calibration of the pump, so if it is combined with the other results (as it will be later on), it might be slightly shifted relative to the other results.
The bounce state region (in the control parameter space formed by the jet and bath speed). The liquid is silicone oil of nominal kinematics viscosity 150 cSt, and density 0.86 g/cm$^3$. Below the bounce state region, no bounce state is observed; above the bounce state region, a flat bouncing jet or trailing jet is observed. The diagram could not be completed because Rayleigh instability of the jet makes it impossible to do the experiment at very slow jet speeds.

4.3.3 The half-plunge state region and spontaneous transitions

The half-plunge state or half-entraining jet (as was called by Thrasher et al.) was introduced in Chapters 1 and 2. It is a stable state that cannot be observed when the bath is stationary; it is only stable, and hence observable, in a certain region of the control parameter space excluding zero bath speed.

The half-plunge state is so called because, as can be seen from Figure 1.3 (b), the side of the jet that faces the incoming bath liquid entrains air into the bath and is separated from the bath by a thin layer of air – much like the thin air film wrapping around an air-entraining or a plunging jet when the bath is stationary; but the other
side of the jet, facing away from the incoming bath liquid, does not entrain air and merges smoothly with the bath surface. The shape of the half-plunge state will change as the jet speed or bath speed varies, but the main characteristics that distinguish it from the other states remain the same, or in other words, the topology of the state remains unchanged.

Fig. 4.10. **Stable region of the half-plunge (P/2) state and its spontaneous transitions to other stable states.** The control parameters are the jet speed and bath speed. Different symbols indicate the different spontaneous transitions of the P/2 state to other states. All these data were collected in a period of time covering about several weeks except indicated otherwise. “P/2” stands for half-plunge state, “P” stands for plunge state, “B” stands for bounce state, and “M” stands for merge state.

- **Diamond**: P/2 to B, or also called the “auto-bounce” transition; oil jet temperature = 22.6 ± 0.1 °C.
- **Square**: P/2 to P transition; oil jet temperature = 22.0 ± 0.2 °C (note that the temperature was different from the others).
- **Triangle**: P/2 to M transition; oil jet temperature = 22.6 ± 0.1 °C.
- **×**: P/2 to B (or auto-bounce) transition; oil jet temperature = 22.6 °C. This data set was collected several months earlier than the rest of the data in the figure, but was seen overlapping with a similar set collected later, showing the reliability of the experimental setup and the reproducibility of the result.
The stable region of the half-plunge state in the control parameter space (formed by the jet and bath speed) was mapped out as shown in Figure 4.10; it is the area inside the plotted curves. Each of the curves corresponds to a spontaneous transition of the half-plunge state to other states at those points, as indicated in the figure. (This will become clear when the regions of the different states are combined onto one single plot.) The half-plunge state is very stable (inside its mapped stable region) once it is created, either by perturbing the jet or by the spontaneous decay of the other states (this will also become clear later on); in fact, the half-plunge state (when it is stable) is much more stable than the “stable” bounce state. (However, the bounce state seemed to vary in stability within its mapped region, but no detailed study on this is available yet.)

**Spontaneous transition to the merge state**

Figure 4.11 illustrates how the spontaneous transition of the half-plunge state to the merge state was located on the diagram in Figure 4.10: at a particular jet speed, a half-plunge state was created at different bath speeds by passing the metal wire swiftly through the jet (that is, by perturbing the jet), and then it was left alone to see at which bath speed the half-plunge state could or could not be sustained. The fastest bath speed at which the half-plunge state could *not* be sustained and quickly decayed to the merge state was defined as the “P/2-to-M transition bath speed” (for that particular jet speed). Moreover, it was shown that whether the half-plunge state was created after the transition bath speed had been reached (Figure 4.11 (a)) or before that (Figure 4.11 (b)), the result was the same. Note that the role of perturbing the jet was just to create a half-plunge state; it played no part in determining whether the half-plunge state created was stable or not.
Fig. 4.11. **Method of finding the P/2-to-M transition bath speed.** Parameters for this figure: Q = 49 (cc/min), oil jet temperature = 22.7 °C. The numbers in each row are times in seconds. (a) The P/2 state created at Ω = 0.050 (rev/s) by perturbing the jet could not be sustained, and spontaneously decayed to an M state in a few seconds. (b) A stable P/2 state was first created at Ω = 0.055 (rev/s) by perturbing the jet, but shortly after the bath speed was decreased to 0.050 (rev/s), the P/2 state spontaneously decayed to an M state. Remarks: The bath speeds between 0.050 and 0.055 (rev/s) were not tested, because otherwise the sharp contrast of the stability of the P/2 state would be obscured.

**Spontaneous transition to the plunge state**

Figure 4.12 illustrates the method used to locate the spontaneous transition of the half-plunge state to the plunge state on the diagram in Figure 4.10: at a particular jet speed, a half-plunge state was created at different bath speeds, by perturbing the jet or by spontaneous decay from the other states, and then left to see at which bath speed it could or could not be sustained. The fastest bath speed at which the half-plunge state could not be sustained and quickly decayed to the plunge state was
defined as the “P/2-to-P transition bath speed” (for that particular jet speed). Moreover, it was shown that it did not matter whether the half-plunge state was created by spontaneous decay from the other states (Figure 4.12 (a)) or by perturbing the jet (Figure 4.12 (b)); and it also did not matter whether the half-plunge state was created after the transition bath speed had been reached (Figure 4.12 (a)) or before that (Figure 4.12 (b)).

Fig. 4.12. Method of finding the P/2-to-P transition bath speed. Parameters for this figure: $Q = 51$ (cc/min), oil jet temperature $= 23.0$ °C. The numbers in each row are times in seconds. (a) $\Omega = 0.055$ (rev/s) throughout. A P/2 state appeared spontaneously out of a merge state about 75 seconds after it had been created, and then spontaneously decayed to a plunge state shortly after. (b) A stable P/2 state was first created by perturbing the jet at $\Omega = 0.060$ (rev/s), but shortly after the bath speed was decreased to 0.055 (rev/s), the P/2 state quickly decayed to a plunge state. Remarks: (1) The bath speeds between 0.055 and 0.060 (rev/s) were not tested, because otherwise the sharp contrast of the stability of the P/2 state might be obscured. (2) The oil jet temperature for this figure was higher than that of the P/2-to-P transition in Figure 4.10; hence, the transition bath speed has shifted to 0.055 rev/s (current figure) from 0.060 rev/s (result in Figure 4.10 for the jet speed 51 cc/min).
Spontaneous transition to the bounce state ("auto-bounce" transition)

Figure 4.13 shows in general how the “auto-bounce” transition bath speed for a particular jet speed was obtained: a half-plunge state was first created, by perturbing the jet or by spontaneous decay from the other states, and left alone to see if it was stable or not at the bath speed tested; the slowest bath speed at which the half-plunge state was unstable and spontaneously transited to a bounce state was the auto-bounce transition bath speed. Similar to the other two transitions above, the result was the same whether the half-plunge state was created after the transition bath speed had been reached or before that (not illustrated in the figure).
Fig. 4.13. Method of finding the P/2-to-B or “auto-bounce” transition bath speed. Parameters for this figure: jet speed = 51 (cc/min), bath speed = 0.140 (rev/s), oil jet temperature = 22.2 °C. The numbers below each photo are times in seconds. For Ω ≤ 0.139 (rev/s), the P/2 state created was stable; whereas for Ω ≥ 0.140 (rev/s), the P/2 state was unstable (not shown in this figure). The P/2 state seen here was created by perturbing the jet, but it could also be created by waiting long enough for a merge state to spontaneously decay to a half-plunge state at the particular jet speed and bath speed. Remarks: the “auto-bounce” transition bath speed would increase (decrease) if the oil jet temperature was increased (decreased).

However, unlike the other two transitions, the auto-bounce transition was very sharp and its transition bath speed could be resolved down to the smallest possible with the current setup, which was 0.001 (rev/s). In other words, a bath speed difference of 0.001 (rev/s) would determine whether the jet was a stable half-plunge state or an unstable half-plunge state that would spontaneously transit to a bounce state.
As remarked earlier in Chapter 3, such property of the auto-bounce transition could be utilized to estimate the response time of the oil bath to the change of its speed by changing the tank rotation rate. For example, starting from a bath speed above the auto-bounce transition at which no stable half-plunge state can be sustained, decrease the tank rotation rate to a targeted bath speed at which a half-plunge state is stable while at the same time try to create a half-plunge state by perturbing the jet. This process can be recorded by the CCD camera (and repeated as many times as desired), and then the time it takes for a stable half-plunge to be created, from the time the targeted tank rotation rate has been reached, can be obtained by analyzing the video files – this will give an estimate of the delay or response time of the oil bath to the change of its speed. Using this method and a rate of change of the rotation rate: 0.001 (rev/s²), it was found that the estimated response time of the oil bath was quite short, about 1 second or less, for the oil used in this study.
Temperature Dependence of "Auto-bounce"

![Graph showing temperature dependence of auto-bounce transition.](image)

**Fig. 4.14.** The temperature dependence of the auto-bounce transition. The series are labeled by the oil jet temperature. Solid Diamond: 22.2 °C. Solid Square: 22.4 °C. Solid Triangle: 22.5 °C. “×”: 22.6 °C. Open Diamond: 22.7 °C. Open Triangle: 22.8 °C. “+”: 22.9 °C. The data in each series were not collected in one single continuous experiment; these data were accumulated results over a period of time of about two weeks (as the oil jet temperature could not be controlled).

**Dependence of the spontaneous transitions on temperature**

In the course of trying to obtain the spontaneous transitions of the half-plunge state to the other states, it was found that the transition curves would shift if the oil jet temperature changed; it seemed that the stable half-plunge region would expand both upwards and downwards if the oil jet temperature was increased. In particular, for example, at jet speed 51 (cc/min), the P/2-to-P transition bath speed was shifted from 0.060 to 0.055 (rev/s) when the oil jet temperature increased from 22.0 to 23.0 °C, and this behavior has been observed repeatedly with weeks apart. The dependence of the (spontaneous) transitions on oil jet temperature was more strikingly demonstrated by the auto-bounce transition, as can be seen from Figure...
4.14. (No data on this temperature dependence was collected for the transition from the half-plunge state to the merge state.)

If it is indeed true that the stable half-plunge region expands when the oil (jet) temperature increases, this will be similar to the expanding of the bouncing regime, due to the decrease of the liquid viscosity, observed by Thrasher et al. (see Figure 2.10 in Chapter 2). And certainly this does seem reasonable, because the viscosity of the oil will decrease as its temperature increases.

4.3.4 Multiplicity

To recapitulate the results shown so far (for $\Omega > 0$): First, for slow bath speeds, the result from Sub-section 4.3.1 implied that the very stable and “meta-stable” regions of the merge state could be separated by a curve (for faster bath speeds, only preliminary data was obtained for this curve); second, the region (in the parameter space) in which the bounce state could be observed was mapped out; third, the stable region of the half-plunge state was also mapped.

To complete the picture, preliminary data has also been collected for the stable region of the plunge state (actually, the stable “region” of the plunge state was obtained near the end of Section 4.2, for the case of zero bath speed or stationary bath). Figure 4.15 combines all these results into one single plot.
Fig. 4.15. **The regions of the different states.** Dashed lines indicate that only preliminary data were collected at those points. To the left of the black, closed-bracket-shaped, line (solid or dashed) is the (incomplete and hence not mapped) region of stable merge state; to the right of the blue, open-bracket-shaped, dashed line is the region of stable plunge state in blue vertical lines. The region of the bounce state is in green negatively sloped lines, and the stable region of the half-plunge state is in red positively sloped lines.

It can be seen from Figure 4.15 that there is a small region, in the parameter space, where the three regions corresponding to the bounce, half-plunge and plunge state overlap one another; this is the region where the bounce, half-plunge and plunge states can all be observed. (Whether there are locations in the control parameter space where all four states, including the merge state, can be observed, is something that needs to be looked into more carefully; recall that the bounce state region was obtained much earlier than the rest of the results, under a slightly different pump calibration.)
Just to the right of the black line in the figure, the merge state is “meta-stable” or “unstable” (see the results in Sub-sections 4.2.1 and 4.3.1); hence, any merge state (to the right of the black line) will spontaneously decay to the half-plunge or plunge state. To the left of the black line, the merge state is very stable, and it will not spontaneously decay to a half-plunge state, thus, perturbation of the jet is required to create a half-plunge state to the left of the black line.

It is also clear from Figure 4.15 why the half-plunge state would spontaneously decay or transit to the other states outside its stable region, as described and explained in the last sub-section.

Finally, it is noted that several places in the control parameter space, mostly at the boundaries of the stable half-plunge region, are likely to exhibit hysteresis (in the broad sense explained in Chapter One). For example, at the upper boundary of the stable half-plunge region, the bath speed can be increased continuously past the auto-bounce transition bath speed, and then decreased back to the starting value (in the present study the bath speed, but not the jet speed, can be varied continuously); in this way the “reverse transition” from the bounce state to the half-plunge state is likely to be delayed or lagged behind, due to the fact that both of these states can be observed (at locations in the parameter space) below the auto-bounce transition curve (i.e. the upper boundary of the stable half-plunge region). So the last part of the present study was to try to demonstrate this possible hysteretic behavior.
4.3.5 “Hysteresis” at the auto-bounce transition

To demonstrate this hysteretic behavior, a stable half-plunge state was first created inside the stable half-plunge region (at a particular jet speed), and then the bath speed was increased gradually past the auto-bounce transition bath speed to make the half-plunge state spontaneously transit to a bounce state; after the bounce state had been formed, the bath speed was decreased gradually past the transition bath speed in the opposite direction, until the bounce state spontaneously decayed back to a half-plunge state. Both processes were recorded with the CCD cameras and stored away for later analyses; their corresponding oil jet temperatures were also recorded. And such processes were repeated many times. Note that as there was no real-time measurement of the bath speed – only the period of rotation could be measured – in order to keep track of the bath speed, the rotation rate could only be changed at increment(s) of the period of one rotation of the tank.

It turns out that the “hysteresis” that was to be demonstrated seemed to depend very much on the stability of the bounce state when the bath speed was decreasing; the more stable the bounce state was (when the bath speed was decreasing), the more affirmative the “hysteresis” was. Figures 4.16 to 4.18 give three representative examples of the result, with Example 1 being the typical case and Example 3 the atypical one, and Example 2 was not often but was also not rare.
Fig. 4.16 (a). **Timeline of an auto-bounce transition (Example 1).** $Q = 49$ (cc/min), oil jet temperature = 22.2 ± 0.1 °C. A stable half-plunge state was created at $\Omega = 0.132$ (rev/s), and it spontaneously transited to a bounce state at the transition bath speed $\Omega = 0.136$ (rev/s).

Fig. 4.16 (b). **Timeline of the reverse transition of Example 1.** $Q = 49$ (cc/min), oil jet temperature = 22.2 °C. This was carried out immediately after the process in Figure 4.16 (a). A bounce state was created at $\Omega = 0.137$ (rev/s) above the transition bath speed, which became unstable at $\Omega = 0.134$ (rev/s) below the transition bath speed 0.136 (rev/s), and spontaneously decayed to a stable half-plunge state.
Fig. 4.16 (c). “Hysteresis” at the auto-bounce transition of Example 1. Diamond: corresponds to the process in Figure 4.16 (a). Square: corresponds to the process in Figure 4.16 (b).
Fig. 4.17 (a). **Timeline of an auto-bounce transition (Example 2).** Q = 49 (cc/min), oil jet temperature = 21.7 ± 0.1 °C. A stable half-plunge state was created at Ω = 0.129 (rev/s), and it spontaneously transited to a bounce state at the transition bath speed Ω = 0.133 (rev/s).

Fig. 4.17 (b). **Timeline of the reverse transition of Example 2.** Q = 49 (cc/min), oil jet temperature = 21.7 °C. This was carried out immediately after the process in Figure 4.17 (a). A bounce state was created at Ω = 0.134 (rev/s) above the transition bath speed, which became unstable at Ω = 0.126 (rev/s) below the transition bath speed 0.133 (rev/s) and spontaneously decayed to a stable half-plunge state.
Fig. 4.17 (c). “Hysteresis” at the auto-bounce transition of Example 2. Diamond: corresponds to the process in Figure 4.17 (a). Square: corresponds to the process in Figure 4.17 (b).
Fig. 4.18 (a) **Timeline of an auto-bounce transition (Example 3).** \( Q = 49 \) (cc/min), oil jet temperature = 22.1 °C. A stable half-plunge state was created at \( \Omega = 0.134 \) (rev/s), and it spontaneously became a bounce state at the transition bath speed \( \Omega = 0.135 \) (rev/s).

Fig. 4.18 (b) **Timeline of the reverse transition of Example 3.** \( Q = 49 \) (cc/min), oil jet temperature = 22.1 °C. This was carried out immediately after the process in Figure 4.18 (a). A bounce state was created at \( \Omega = 0.136 \) (rev/s) above the transition bath speed, which became unstable at the transition bath speed \( \Omega = 0.135 \) (rev/s) and spontaneously became a stable half-plunge state.
Fig. 4.18 (c). **Non-hysteretic behavior of the auto-bounce transition of Example 3.** Diamond: corresponds to the process in Figure 4.18 (a). Square: corresponds to the process in Figure 4.18 (b).

It can be seen from the timeline diagrams of Figures 4.16 to 4.18, that the bounce state was not very stable even at bath speeds above the auto-bounce transition; the bounce state would temporarily “die” and become an unstable half-plunge state that quickly changed back to a bounce state, and such process would repeat itself. One possible reason for this may be that there was dust or other impurities present in the oil jet or on the bath surface: the presence of these unwanted impurities would destroy the non-coalescence created by a constant and continuous thin air layer in between the oil jet and bath, which is critical to the stability of the bounce state. Also, it was observed (in a side experiment) that when the patterns of air flow just above the bath surface, near the jet, was disturbed, the frequency that the bounce state “died” but quickly “came back to life” was higher; this indicates that the patterns of air flow above the bath surface – which the present experiment had no control of –
affect the stability of the bounce state. Therefore, it does not seem surprising that the
“hysteresis” (that was to be demonstrated) seems to be probabilistic in nature, as the
observations have suggested. (Whether the bounce state can be made to be more
stable if the control over the entire system is improved is yet to be looked further
into.)

In spite of the probabilistic nature of the “hysteresis”, it does seem probable
that the slowing down of the bath surface may help to make the non-coalescence
between the oil jet and the bath last longer than it would otherwise be, and hence
causes the reverse transition of the auto-bounce to be slightly hysteretic; the slowing
down of the bath surface may help to “trap” air in between the bended oil jet and the
indented bath surface, or to slow down the drainage of the air in between them.

Finally, it is noted that, in some of the auto-bounce transition processes, the
auto-bounce transition did not happen right away after the transition bath speed had
been reached; in other words, the system seemed to take longer than ~ 1 s to make
the transition after the transition bath speed had been reached. This does not seem to
be an indicator that the bath speed was delayed more than it has been estimated (see
the estimate of the response time of the oil bath given right after Figure 4.13), but
rather, that the system itself needed time to make the transition from the half-plunge
state to the bounce state.
Chapter 5: Conclusions

To summarize the main results obtained in the previous chapter, first, it was observed that the spontaneous transition from the merge state to the plunge state (see Figure 1.2), when the bath was stationary, happened instantaneously when the jet velocity (at the entry point) was above a threshold (given by a constant jet diameter flow rate 55 cm$^3$/min), which could be regarded as the critical entrainment velocity, $V_c$, predicted by equation (2.7): $V_c \sim (\gamma/\mu) \ln [\mu/\mu_0]$. However, it was found that the spontaneous transition could also happen when the jet velocity was below the critical threshold $V_c$, except that the transition was not instantaneous – it took a while, and the amount of time it took, on average, depended on the jet velocity; a decrease of the jet velocity by 1%, for example, would increase this (average) time by at least three orders of magnitude. This has not been reported, or even mentioned, in any of the previous studies on this phenomenon of air entrainment by plunging viscous liquid streams.

Second, the “(stable) state diagram” in the parameter space formed by jet velocity and bath velocity, at fixed viscosity and jet diameter (at the entry point into the bath), was obtained (see Figure 4.15 and Section 4.1). It was seen that the system (an oil jet impinging vertically into a bath of the same liquid) behaved in a more complex way when bath motion was introduced into the problem. The complexity was brought in by the states that were only observed when there was motion of the bath: the “bounce” and “half-plunge” state (see Figure 1.3). But in a way, the problem was not that complicated: each of the states (“merge”, “plunge”, “half-plunge” and “bounce”; see Figures 1.2 and 1.3) had its stable or meta-stable region in
the parameter space (formed by the jet velocity and bath velocity), and these regions would overlap one another. In the sub-regions where there was any overlapping, at least two of the states could be observed and sometimes even three.

Third, the spontaneous transitions of the half-plunge state to the other states were obtained and mapped in the parameter space. One of these was the “auto-bounce” transition (spontaneous transition from half-plunge to bounce state); this transition was sharp and highly reproducible, and it traced out a smooth curve in the parameter space, but it remains to be explained theoretically.

Lastly, the present study tried to demonstrate one possible hysteretic behavior of the system, namely, the “hysteresis” of the spontaneous half-plunge to bounce state transition, by varying the bath speed (one of the two control parameters) continuously. In the broad sense that hysteresis means that the reverse transition of the system (bounce state to half-plunge state) is lagged behind, the system did exhibit slight hysteretic behavior (see Sub-section 4.3.5). But the hysteretic behavior was probabilistic in nature, meaning that there was only a percentage of chance that it would occur each time (90 % roughly). This was unresolved but it might be due to some imperfection of the current system such as presence of dust or other impurities in the oil that could destroy an otherwise stable non-coalescence between the oil jet and bath surface, for example, as the bounce state is the predominated state above the auto-bounce transition (it seemed that the bounce state was more stable here than other places in the bounce state or bouncing jet region).
Bibliography

Air entrainment by liquid jets


The bouncing jet


Non-coalescence


Others


General Fluid Physics
