TEMPERATURE EFFECT ON CONCRETE BRIDGE

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ABSTRACTS

This thesis presents a practical method of analysis for temperature dependent response of composite concrete section, in both statically determinate and indeterminate structures.

The evaluation of the temperature effects in a structure involves two aspects of calculations, which are to predict the temperature distributions through the structure and to calculate the strain and stress levels induced by these temperature distributions. The mechanisms of heat transfer occur on the concrete bridges is investigated and a fifth-degree parabola curve is selected to present the temperature distribution along the cross-section in this thesis.

The theory used in the analysis of thermal response is based on the theory proposed by Priestley (1978). It also takes the aging effect into account using Age-Adjusted Effective Modulus Method. The elastic modulus is replaced by age-adjusted effective modulus which is affected by two factors, namely creep coefficient $\phi(t, \tau)$ and aging coefficient $\chi(t, \tau)$. The details of analysis of the thermal stress and strain in statically determinate and indeterminate bridge are discussed and described in several cases. A computer program which used in the thermal analysis is also described.

To predict the time-dependent effects on concrete section, the creep and shrinkage effects are modeled using the aging-adjusted effective modulus method, couple with a relaxation procedure.
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Finally I am indebted to my parents for their love, support and understanding, I dedicate this thesis to them.
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NOTATION

All symbols are defined in the thesis where they first appear. The more frequently symbols used and those that appear throughout the book are listed below.

$A, B, I$ area, first moment of area, and second moment of area respectively, calculated about the top surface of the cross section;

$A_e, B_e, I_e$ the properties $A, B, I$ of the transformed cross section;

$A_s, B_s, I_s$ the properties $A, B, I$ of the age-adjusted transformed cross section;

$A_{q_i}, B_{q_i}, I_{q_i}$ area, first moment of area, and second moment of area of $j$-th concrete element about the top surface;

$A_n, A_p$ areas of the non-prestressed and prestressed steel, respectively;

$d_{ci}$ depth to the centroidal axis of concrete element $i$;

$d_{as}$ depth to the position of the resultant axial force;

$d_s, d_p$ depths to the non-prestressed and prestressed steel, respectively;

$E_c, E_s, E_p$ elastics modulii of concrete, non-prestressed steel, and prestressed steel, respectively;

$E_c, E_s$, $E_p$ effective modulus of concrete (Eqn 3.5) and age-adjusted effective modulus of concrete (Eqn 3.6), respectively;

$h$ black-top thickness of concrete bridge in mm;

$kR$ reduced relaxation in the prestressed steel;

$L$ span of a beam;

$M$ moment;

$N_i, M_i$ resultant axial force and moment about the top surface of cross section;

$N_s, M_s$ sustained axial force and moment applied to section;

$\Delta N_i, \Delta M_i$ restraining actions which develop due relaxation on section;
\( \begin{align*}
  n_c, n_s, n_p & \quad \text{modulus ratio (} E_c / E_{c_2}, E_s / E_{c_2} \text{ and } E_p / E_{c_2}, \text{ respectively);} \\
  \overline{n_c}, \overline{n_s}, \overline{n_p} & \quad \text{age-adjusted modulus ratios (} \overline{E_c} / \overline{E_{c_2}}, \overline{E_s} / \overline{E_{c_2}} \text{ and } \overline{E_p} / \overline{E_{c_2}}, \text{ respectively);} \\
  P & \quad \text{prestressing force;} \\
  R & \quad \text{reaction at the support;} \\
  T & \quad \text{temperature;} \\
  t & \quad \text{time;} \\
  x & \quad \text{direction along a member axis;} \\
  y & \quad \text{distance from the top fibre of a cross section measured} \\
   & \quad \text{perpendicular to the member axis;} \\
  \alpha_i & \quad \text{coefficient of thermal expansion;} \\
  \varepsilon_{c, \Delta K_i} & \quad \text{top fibre strain and curvature induced by temperature gradient;} \\
  \varepsilon_{c, K_i} & \quad \text{top fibre strain and curvature in short time analysis;} \\
  \Delta \varepsilon_{c, \Delta K} & \quad \text{time dependent change in top fibre strain and curvature;} \\
  \varepsilon & \quad \text{strain;} \\
  \varepsilon_c & \quad \text{creep strain;} \\
  \varepsilon_i & \quad \text{instantaneous strain;} \\
  \varepsilon_s, \varepsilon_p & \quad \text{strain in non-prestressed and prestressed steel, respectively;} \\
  \varepsilon_{sh} & \quad \text{shrinkage strain;} \\
  \sigma & \quad \text{stress;} \\
  \sigma_i & \quad \text{initial stress in the concrete;} \\
  \sigma_s, \sigma_p & \quad \text{stress in non-prestressed and prestressed steel, respectively;} \\
  \phi(t, \tau_o) & \quad \text{creep coefficient at time } t \text{ for concrete first loading at } \tau; \\
  \chi(t, \tau_o) & \quad \text{aging coefficient at time } t \text{ for concrete first loading at } \tau. 
\end{align*} \)
CHAPTER 1

INTRODUCTION

1.1 OBJECTIVES

Bridge design requires consideration of the effects produced by temperature range and thermal gradients in the structure. Temperature variations within bridges can cause thermal stresses that are comparable in magnitude to stresses induced by live and dead loading. During the past years, significant progress has been made in the study of temperature distribution and thermal stresses. The thermal response of a bridge involves a combination of local air temperatures, wind movement, cloud cover, bridge location and orientation with respect to the sun, daytime solar radiation, and loss of heat by radiation in a night sky. Some attention (Churchward and Sokal 1981; Emerson 1976; Hirst 1982; Priestley 1978) have been focused upon the empirical relationships between the thermal loads (such as the effective temperature) and the parameters (such as ambient temperature and solar radiation). Mathematical modeling of the temperature distribution problem has also been issued by various researchers (Embady and Ghali 1983; Jones 1976; Will 1975).

The methods now available for calculation of thermal stresses in different types of bridges are very diverse, but they all depend on an accurate prediction of temperature distribution. The basis of transfer is the same for all types of structures, but different methods have been employed to solve the heat transfer equations. Each method has its own set of simplifying assumptions and each has its advantages and disadvantages. In this thesis, a practical and usable treatment will be used in the serviceability analysis of concrete bridge.

If the temperature gradient is applied slowly or is sustained for a period of time, the internal stresses induced by temperature are relieved, to some extent, by creep. A time analysis using the Age-Adjusted Effective Modulus Method (AEMM)
can be used conveniently to determine the variation of cross-sectional behavior with time.

1.2 THESIS OUTLINE

Chapter 2 is divided into two parts. In the first part, an investigation is made into the three principal mechanisms of heat energy exchange on concrete bridge: (1) radiation from the sun and reradiation between the surrounding environment and the structure itself, (2) convection of heat between the surface of the structure and its surrounding environment, and (3) conduction of heat between the surface of the structure and its surrounding environment. In the second part a review and evaluation of existing codes and research findings will be revealed; and based on the information generated, a selection of the temperature gradient adopted in the thesis will be made.

Chapter 3 discusses the theory of stress and strain induced by thermal variation based on the Age-Adjusted Effective Modulus Method. If the temperature change is slow or occurs over a period of time, aging is taken into account in the calculation of elastic modulus, with respect to creep coefficient $\phi(t, \tau_c)$ and aging coefficient $\chi(t, \tau_c)$. The determination of these two factors is also discussed. Finally, the analysis of a cross-section consisting of various reinforced or prestressed concrete elements acting compositely together is also presented. The elastic modulus of one of the concrete elements (say element 1) is selected as the modulus of the transformed section, and the areas of both the bonded steel reinforcement and the other concrete element(s) are transformed into equivalent areas of the concrete element 1. The details of analysis of the thermal stress and strain are represent in case 1 and case 2 which is corresponding to singly reinforced cross-section and composite cross-section.

The theory of stress and strain distributions caused by a non-linear temperature gradient on a cross-section of a statically indeterminate member is discussed and evaluated in Chapter 4. In statically indeterminate structure, the displacements may not be free to occur, and restraining forces may develop at the supports. The stresses and
deformations caused by these restraining forces must be calculated and added to the
temperature induced eigenstresses in order to obtain the total effect of temperature.
Case 3 shows the details of the thermal effects in statically indeterminate bridge. In
case 4 to case 7, four sections are used to investigate the effect of increasing the depth
of cross section on both self-equivalent and continuity stresses.

Age-Adjusted Effective Modulus Method is used in time dependent analyses in
Chapter 5. The creep and shrinkage strain is assumed to be independent and may be
calculated separately, and then summed to obtain the total strain together with
temperature strain and instantaneous strain. The effects of shrinkage on the uncracked
cross section and of various shrinkage are discussed in case 8 to case 14. The results of
analyses in case 15 and case 16 show the effects of varying non-prestressed
reinforcements (both tensile zone and compression zone) on concrete section.

An outline of the case-study conducted in this thesis is presented in Table 1-1.

The summary and conclusion of this study are presented in Chapter 6.
<table>
<thead>
<tr>
<th>CROSS-SECTION</th>
<th>ELEVATION</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td><img src="image1" alt="Image" /></td>
<td>This case is used to investigate the thermal effect on singly reinforced concrete simple support bridge</td>
</tr>
<tr>
<td>Case 2</td>
<td><img src="image2" alt="Image" /></td>
<td>This case is used to investigate the thermal effect on composite concrete simple support bridge</td>
</tr>
<tr>
<td>Case 3</td>
<td><img src="image3" alt="Image" /></td>
<td>This case is used to investigate the thermal effect on a composite continuous bridge</td>
</tr>
<tr>
<td>Case 4</td>
<td><img src="image4" alt="Image" /></td>
<td>These cases are used to investigate the effect of the depth of cross section on thermal response</td>
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<tr>
<td>Case 7</td>
<td><img src="image5" alt="Image" /></td>
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<tr>
<td>Case 8</td>
<td>Case 10</td>
<td>These cases are used to investigate the effect of shrinkage on an uncracked cross-section</td>
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<td>--------</td>
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</tr>
<tr>
<td>Case 11</td>
<td>Case 14</td>
<td>These cases are used to investigate the effect of varying shrinkage on an uncracked cross-section</td>
</tr>
<tr>
<td>Case 15</td>
<td>Case 16</td>
<td>These cases are used to investigate the effect of varying non prestressed reinforcement on thermal responses</td>
</tr>
<tr>
<td>Case 17</td>
<td></td>
<td>Both immediate and final long term responses of concrete bridge</td>
</tr>
</tbody>
</table>

Table 1-1  List of cases studied in the thesis
CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

Observations have indicated that stresses due to temperature can seriously affect the serviceability and the structural integrity of bridge structures. In a statically determinate bridge, nonlinear temperature variations produce stresses in the longitudinal direction of the span. These stresses are self-equilibrating since their resultants are equal to zero and no change in reactions occurs. In a statically indeterminate bridge, additional continuity stresses develop in the longitudinal direction. A bridge structure under the effect of dead load may have cracks or may be free of cracking depending on the design, particularly on the amount of prestressing. Tensile stresses due to temperature can be high enough to cause further cracking.

Elbadry and Ghali (1986) described the damage in a box-girder bridge shown in Figure 2.1. Temperature gradients may produce tensile stresses at the bottom fibres and cause vertical cracks. Cracks parallel to the bridge axis may occur at the bottom surface of the deck slab due to thermal gradients through the thickness of the deck. Horizontal cracks may also occur in the web when significant horizontal thermal gradient exists through the web thickness.

Leonhardt, Kolbe, and Peter (1967) reported a case of damage to a prestressed concrete two-span continuous box-girder bridge. Within five years of completion, large cracks were observed along one of the webs of the shorter span. Crack widths greater than 3/16 in. (5 mm) were measured. The damage was attributed, among other factors, to non-uniform temperature variations. In each web, insufficient shear reinforcement (approximately 0.12 percent) had been provided. Because of this small amount of reinforcement, the cracks were able to extend horizontally over a considerable length.
Figure 2.1  Cracking due to temperature variations in different parts of a continuous box-girder bridge (Elbadry and Ghali, 1986)
2.2 MECHANISMS OF HEAT TRANSFER

A bridge deck continuously gains and loses heat from solar radiation, re-radiation to the sky, and convection to or from the surrounding atmosphere (Figure 2.2). Temperature variations induced by these sources depend on geometry, location, and orientation of the bridge, on climatological conditions, and on thermal properties of the material and exposed surfaces. There are three principal mechanisms of heat transfer: (1) radiation from the sun and reradiation between the surrounding environment and the structure itself, (2) convection of heat between the surface of the structure and its surrounding environment, and (3) conduction of heat between the surface of the structure and its surrounding environment.

![Diagram of heat transfer mechanisms]

Figure 2.2 Factors affecting thermal response (Priestley, 1978)

2.2.1 Heat Transfer by Radiation

Heat transfer by radiation is generally considered to be the most important of the three mechanisms. In the daytime, especially during the warm summer months, heat gain is greater than heat loss, primarily as a result of the solar radiation impinging on the surface
of the structures. At night, the reverse is true and the temperature of the structure drops. The intensity of the solar radiation reaching the surface of the earth is dependent on the angle at which the radiation passes through the atmosphere and the length of daylight time. This intensity is dependent on latitude and has an annual variation. In addition, the intensity of the solar radiation reaching the surface of a bridge is dependent on several other factors, each pertaining to the condition of the earth's atmosphere. These factors are shown diagrammatically in Figure 2.3. The intensity of solar radiation varies daily, and, moreover, because of the poor thermal conductivity of concrete, these diurnal variations result in temperature gradients within bridge.

As shown in Figure 2.3, the radiation which penetrates the atmosphere and reaches the surface of the bridge deck has two primary effects. It may be reflected or it may penetrate the surface, be absorbed and converted to the heat. The amount of absorbed radiation in a bridge structure is dependent on the type of surfacing. Various media absorb different quantities of radiation. Colored bodies are distinguished by their selective absorption of different wavelengths of light. A dark, rough surface absorbs a greater amount of solar radiation than does a light, smooth surface. It was found by Priestley (1977) that the temperatures yielded by the black surfacing were about 10 percent lower than those occurring in the unsurfaced concrete.

2.2.2 Heat Transfer by Conduction and Convection

In addition to the heat transfer by radiation, heat transfer by conduction and convection also take place at the structure surface. It is normally considered that the conduction and convection are dependent upon wind velocity, humidity, and the difference in temperature between the air and surface.

Because these boundary conditions vary continuously with time and the conductivity of concrete is relatively low, variation of temperature through a bridge cross section is nonlinear. With nonlinear temperature distribution, stresses are induced even in statically determinate bridges, and also obviously as in a statically indeterminate bridge.
Figure 2.3  Solar radiation reaching the surface of a bridge
(Imbsen, Vandershaf, Schamber and Nutt, 1985)

From the early 19th century, attempts were made to investigate the temperature distributions and their effect in concrete structures. In the following part, a review of previous research on this subject is made.
2.3 LITERATURE REVIEW OF PREVIOUS RESEARCH ON TEMPERATURE DISTRIBUTIONS

2.3.1 Barber (1957)

In 1957, Barber presented a formula to predict the maximum pavement temperature. He pointed out that the pavement temperature was of interest in connection with stabilization of bituminous surfaces, curing of portland cement concrete, and moisture movements in any type of pavement. There was a relation between pavement temperatures and wind, precipitation, air temperature, and solar radiation as controlled by the thermal properties of the pavement. Finally, he compared the calculated values with those observed temperatures for different cases. The calculations indicated the possibility of roughly correlating surface temperatures with values reported by the Weather Bureau so that means were available to extrapolate field observed temperatures to other times and places. To calculate exact temperatures for a given structure, exact values of its thermal properties and the ambient conditions must be known.

2.3.2 Zuk (1965)

Equations for both longitudinal and transverse stresses in composite beams under various conditions of temperature were developed by Zuk in 1960. Later, in his other paper published in 1965, he described some thermal behavior of both an insulated and uninsulated composite bridge. One was a normal concrete deck in the Middle Atlantic States of U.S.A., the other was a bitumen covered bridge. First of all, based on the theory developed by E. S. Barber (1957), Zuk predicted the maximum surface temperature of a bridge deck. He found that the effect of black topping could easily raise the surface temperature of decks by $15^\circ F$ ($8.4^\circ C$) or more above that of bare concrete decks, it was significant in relation to differential temperatures between the top and bottom of bridges, which in turn affect the induced internal thermal stresses. Zuk also gave a simplified equation for predicting the maximum temperature differential between the top and bottom
of a normal steel-concrete composite highway bridge, as well as the equation for predicting the temperature distribution through the slab depth.

A group of field tests data which were collected for the period from September 1961 to August 1962 were compared with the theoretical results. It was shown that these experimental data correlate well with the theory, the maximum surface differential temperature was $102^\circ F$ ($39^\circ C$), compared to the true measured value of $98^\circ F$ ($36.7^\circ C$). Similarly, the theoretical maximum differential temperature was $24^\circ F$ ($13.4^\circ C$) and the true measured temperature was $23^\circ F$ ($12.8^\circ C$). Around 10 percent agreement was found for the steep thermal gradients in the slab near the upper surface between the computed and practical. He pointed out that the most significant source of error lies in the theoretical assumption that the boundary temperature varied as a sine wave.

2.3.3 Maher (1970)

Experience on continuous structures in England and Australia was investigated. Maher showed that the effect of a temperature gradient through the depth of the structure caused by the top slab heating up was faster than the soffit. The magnitude of these effects on a structure will be dependent on many variables, including width and thickness of top slab, depth of section, angle of incidence of the solar rays, latitude in which the structure is situated, season of the year and time of the day. Maher found that the daily maximum vertical temperature gradient usually occurs in early afternoon. For a solid rectangular section, an equivalent linear temperature distribution was assumed to be used to calculate the induced strains and deflections. For a hollow box section, the thermal variation is linear and is significant through the top slab only.

2.3.4 M.O.W. Head Office (1970)

A simplified design method (Figure 2.4) has been issued in 1970 by the civil engineering division of M.O.W. Head Office. This method idealized the vertical temperature distribution in a box-girder by a uniform temperature increase over the
thickness of the deck slab, with zero temperature rise elsewhere. This method filled a very important gap in design procedure at that time.

Figure 2.4  Temperature Gradient issued by the Civil Engineering Division of M.O.W. Head Office (1970)

2.3.5 Hunt and Cooke (1974)

Hunt and Cooke used a finite difference method to solve the one-dimensional heat flow equation for a concrete box-girder bridge. In this model, the bridge was considered as two layers with different thermal properties, while only the homogeneous bridge decks were considered in Emerson's model. Based on Sokolnikoff's theory, the three-dimensional thermal stress distribution in a prismatic bridge was reduced to a two-dimensional plain-strain problem in thermoelasticity.

2.3.6 Priestley (1972-1976)

M. J. N. Priestley (1972) compared the stress distribution under a different assumed temperature gradient which came from M. O. W. design method. He provided a reasonable approximation for sections of about 4 (1200mm) to 5 ft (1500mm) depth by using a sixth power parabola \( t_p = T y^6 / d^6 \) varying from a maximum at the top of the decks to zero at the soffit. The maximum temperature \( T \) is 60°C (15.6°F) used in his paper, but it may change, depending on the local weather condition.

In his further study, Priestley provided an increased temperature gradient, a fifth-power parabola decreasing to zero at a depth of 1200 mm, of the form
\[ t_r = 22(\gamma / 1200)^5 \, ^\circ C \]  \hspace{1cm} (2.1)

which was in very close agreement with the results from the heat-flow analyses. But there are still two areas where the curve dose not adequately define the behavior: (1) at the soffit of each section, the temperature consistently exceeds the fifth-power curve by about 1.5\(^\circ\)C. (2) the temperature through deck slabs above enclosed air-cells exceeds the fifth-power curve, with the error increasing with depth. A straight line plotted with the same maximum temperature as the fifth-power curve gives good agreement. After a study of the sensitivity analysis, among the location, site wind speed, daily ambient temperature range, block-top thickness and surface solar absorptivity, Priestley took the black-top thickness into account of the concrete surface temperature because it was found that the unsurfaced bridges could be subjected to a maximum stress 50 \% higher than those for corresponding bridges surfaced with a 50 mm black-top layer. A reasonable design gradient was proposed (Figure 2.5), which consisted of three separate sections:

(1) A fifth-power parabola for a depth of 1200 mm with the concrete surface temperature related to the black-top thickness by the expression

\[ T = 32 - 0.2h \, ^\circ C \]  \hspace{1cm} (2.2)

where \(h\) = black-top thickness in mm.

This equation is appropriate for slab, T-beams, and cantilever and web portions of box-section.

(2) A linear thermal gradient applicable to deck slabs above air-cells of

\[(5 - 0.05h) \, ^\circ C / 100 \, \text{mm}\]  \hspace{1cm} (2.3)

with the top surface temperature given in (1)

(3) A linear temperature increase from 0 to 1.5 \(^\circ\)C over the bottom 200 mm of the section. Where the section depth is less than 1400 mm, this is superimposed on the fifth-power curve. Note, that although this temperature increase of the soffit is small, it has a significant and beneficial influence on the soffit tension stress, reducing it by 0.5-0.8 MPa depending on the section shape.
2.3.7 Churchward and Sokal (1980)

Churchward and Sokal presented the results of long term recording programs of monitoring temperature within two concrete box girder bridge in Brisbane, Queensland. They defined the design variables of differential temperature, average bridge temperature and the base temperature, and presented the results from the bridges monitored for these variables. Furthermore, they proposed a method to determine the average bridge temperature from the design variables of base temperature and differential temperature, utilizing standard differential temperature profiles. From temperature measurements on the two bridges instruments, they found that the maximum average bridge temperature normally occurred between 1700-1800 hours in summer and 1600-1700 hours in winter, while the minimum average bridge temperature occurred between 0700-0900 hours both in
winter and in summer. Finally, they compared the differential temperature and its profiles with a number of profiles presently being used by various design authorities. Of those compared, the profiles given by Priestley (1976) (Figure 2.5), New Zealand Specification (1974) and NAASRA (1976) reasonably estimate the upper (95 percentile) limit of internal stresses and curvature. The extreme value of maximum differential temperature as recorded during the three year period was 20°C compared with the value of 24°C as given by NAASRA (1976). Average bridge temperature has been determined for the cross section of Elizabeth Street Ramp and the extreme values measured were 14°C and 40°C. This compared with the range given by NAASRA (1976) of −5°C to 50°C for the Brisbane area.

2.3.8  Hirst (1982)

In his paper of 1982, Hirst presented new designed information for the evaluation of differential temperature effects when a bridge was heated by solar radiation. All results have been obtained using a calibrated computer model which idealizes the bridge as a one-dimensional problem in heat transfer. Thermal loading parameters are computed from bridge characteristics and the standard records of the Bureau of Meteorology. A design method is outlined which used an effective thickness concept to display the effects of surfacing in chart form for particular bridges and sites. The properties of the wearing course affect both the amount of heat energy absorbed at the top surface of the bridge and its transmission to the underlying structures. The amount of radiant heat energy absorbed by a bridge is a function of the absorptivity to solar radiation of the top surface. It was found that a single linear relationship between surface absorptivity and thermal moment is a satisfactory design approximation. The principal climate variable influencing thermal moment is the solar radiation total for the day at the bridge site. Being dependent on climate thermal loading is extremely variable and histograms have been presented which show the probability of occurrence of thermal moment for bridge located in the major Australian population centers.
2.3.9 Hoffman, McClure and West (1983)

The authors presented the findings from field temperature measurements of an experimental segmental bridge. The field observation appears to indicate that the vertical temperature distribution that causes maximum upward bowing can be approximated by a fifth-power polynomial. In addition, the slab above the box cell showed a linear temperature distribution. There is a strong agreement between the observation and New Zealand Specification. The New Zealand Specification is more sophisticated than other temperature distribution methods (such as PCI-PTC temperature distribution); however, a question of accuracy versus complexity still occurs.

2.3.10 Ho and Liu (1989)

 Thermal loading acting on highway bridges are treated as random variables. Values of such loading for a 50-year return period are determined based on an analysis of the statistics of extremes. A one-dimensional finite element heat flow model was first established in this paper. Two sets of analyses have been carried out, one for the summer (July and August) and another for winter (January and February) because the maximum of the thermal loading are found to occur in summer and the minimum in winter. Accuracy of the mathematical model is checked by comparison with field measurements. Evans's method, which is used to calculate the first four moments of a random function, is described. Emphasis is placed on the statistical aspect of the thermal loadings rather than on the numerical modeling of the heat-flow problem.

2.3.11 Branco and Mendes (1993)

To study the temperature distribution in bridges, a numerical method was developed to obtain the temperature evolution, in a defined structure, during a day with specified environment characteristics. A simplification was made and it led to a two-dimensional heat transfer problem in the cross section, which can be expressed by the well-known Fourier equation. With this method the design temperature gradients, for small and
medium span concrete bridges, can be defined, considering a parametric study in which the environment and geometrical parameters are considered.

The parameters to be considered in the definition of concrete bridge design temperatures are identified as belonging to the following groups:

* Climatic: solar radiation, air temperature, wind speed and nebulosity, etc.
* Geographical: altitude, latitude, etc.
* Time: hour of the day, day of the year, etc.
* Geometric: geometry of the cross section, bridge orientation, asphalt thickness, etc.

* Materials: thermal conductivity, density, specific heat, color, etc.

The first three categories are essentially related with the environment characteristics of the bridge location. The final two parameters are specifically related to the bridge characteristics.

For important bridges or special studies, in which an accurate estimation of thermal effects is necessary, the numerical technique can also be applied to define the design values, based on the geometry and materials of the structure and considering the local environment condition.

2.4 LITERATURE REVIEW OF PREVIOUS RESEARCH ON TEMPERATURE RESPONSES

In general heat flow equation which governs the transient heat flow within the boundaries of the bridge superstructure is expressed as

\[
\frac{\partial T}{\partial t} = k \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right]
\]  

(2.4)

where

\begin{align*}
T & \quad = \text{temperature of the mass;} \\
t & \quad = \text{time;} \\
x, y, z & \quad = \text{direction in the cartesian coordinates;}
\end{align*}
\[ k \quad = \quad \text{thermal conductivity}; \]
\[ \rho \quad = \quad \text{density}; \text{ and} \]
\[ c \quad = \quad \text{specific heat}. \]

In the following, reviews on the thermal response are made. Several methods which were used to solve Eqn (2.4) by various researcher are shown.

2.4.1 Zuk (1961)

Before William Zuk's, there was almost no study on thermally induced stresses in composite beams. In 1961, Zuk proposed an analytical approach for computing thermal stresses and deflections in statically determinate composite steel beams. The composite member was considered as a full length beam which was first separated into a slab and a beam, and then reuniting them by the interface shears. From elastic theory, the general expression of the thermal stress is easily made. To maintain the horizontal compatibility, some sort of horizontal shear force at the interface is needed, with the assumption that the entire shear force is concentrated near the ends of the slab and beam. To satisfy the condition of vertical compatibility, a couple was introduced at the ends of the beam and slab as shown in Figure 2.6. Because all the interface force are concentrated at the ends, the slab and beam each deforms in the same arc of a circle geometrically verifying the conditions of plane section remaining plane, as would be the cause for true composite action with no interface slip.

An actual simply supported steel and concrete composite highway bridge was investigated. The examination of the results disclosed that the induced stresses and deflections may indeed be of a large order of magnitude, appearing too large to be ignored. The author also commented that transverse tensile stresses in the concrete slab of the order of magnitude of 1000psi (6.9 MPa) is significant, which may in some cases overstress the transverse steel in the slab.
2.4.2 Liu and Zuk (1963)

There were four characteristic types of prestressed members which were studied by Liu and Zuk: (1) Prestressed beam with straight tendons; (2) Prestressed beam with draped tendons; (3) Prestressed beam with straight tendons, composite with a concrete slab; (4) Prestressed beam with straight tendons, composite with a concrete slab. In brief, the theoretical operations involved the free separation of the flexural members into their particular temperature environment. Appropriate restraining forces were then applied to these elements such as to make all strains compatible with actual combined conditions. This method was similar to those of Zuk's early work, except that the additional total prestress in the tendon $F_i$ was introduced into the analysis.

Some typical simply supported prestressed beams which were examined used the theoretical equations derived in the paper. The results of the investigation indicated that the stresses in the concrete did not normally exceed the magnitude of about 200psi (1.38 MPa) in the compression and about 100psi (0.69 MPa) in tension. The induced stress effected in the prestressing tendons range approximately from a 5000psi (34.5 MPa) lost of stress which was 3% of initial prestress to about 8000psi (55.2 MPa) gain in stress which was 5% of initial prestress. The deflections generally lie below 0.04% of the span.
length. Finally, authors pointed out that attention should also be paid to two special features in regard to composite construction: (1) concerning interface forces and (2) transverse stresses.

2.4.3 Priestley (1972 to 1978)

The theory developed by Priestley in 1972 was very simple to apply. This theory could be used to predict an arbitrary section shape subjected to an arbitrary vertical temperature distribution based on the following equilibrium requirements: (1) The total axial force induced by temperature variations must be zero. (2) The net moment on the section of any temperature stresses must be zero.

A box-girder bridge, given as an example in a simplified design method in common usage in New Zealand, was analyzed under a number of different temperature distributions, some of which are felt to be more realistic than the constant temperature block used at that time. Results showed that local stresses were very sensitive to the type of temperature distribution assumed. Curvatures, and therefore continuity effects, are less sensitive, but still exhibit significant variation. Priestley also presented the design techniques for estimating transverse stresses, and some secondary effects, including bearing loads and horizontal temperature gradients.

In his further study in 1976, Priestley assumed that the thermal behavior of even a complex section can be adequately represented by a linear heat-flow analyses. The results of the analyses based on this assumption were compared with finite element results by Lanigan and experimental results. It was seen that the simpler linear heat-flow model produces almost identical results to the finite element model, and that agrees with measured temperature. Furthermore, the results under the "real day" analyzed for several kinds of sections were compared with the M.W.D. design gradient. It was found that the critical temperature profiles differed from the design gradient in two major respects: (1) The maximum concrete surface temperature rise is only 70% of the design gradient value, and (2) there was small but significant increase in the soffit surface temperature.
In the sensitivity analysis, the following factors were considered: radiation values, solar radiation intensities, wind speed, ambient temperature range, black-top thickness and section with or without black-top. He concluded that the thermal response was reduced by increasing average wind speed, and an increase in the ambient temperature range during a critical day decreased the maximum soffit tension stress in continuous bridge.

In 1978, Priestley presented another paper on this subject. A general analytical method for predicting the vertical distribution of thermal induced stress was developed. Good agreement between theoretical and experimental stresses has been obtained. Priestley pointed out that beside the longitudinal flexural stresses induced by restraint of vertical temperature gradient, a number of other aspects are important, and should be considered in design. Restraint of thermal hogging curvatures involved a redistribution of support reactions, with increased shear force in the end spans and the possibility of bearing failure at abutments. Transverse stresses of substantial magnitude could be induced, particularly in closed sections, such as box girders.

![Diagram](image)

**Figure 2.7** Influence of thermal load on ultimate capacity (Priestley 1978)

The figure (Figure 2.7) of influence of thermal load on ultimate capacity was presented by Priestley. This figure illustrated the significance of thermal loading at both service and ultimate load levels. It was found that the equivalent thermal load at ultimate
load is less significant than that at service loads, and the only significance of thermal load to ultimate performance was a slight reduction to ductility or redistribution capacity.

2.4.4 Cook, Priestley and Thurston (1984)

A theory was developed for prediction of the influence of cracking on thermal response of partially prestressed concrete bridge. A Comparison of predictions from the theory with the behavior of two approximately one-sixth scale partially prestressed micro-concrete models showed close agreement for thermally induced moments, and adequate agreement for deflection profiles and crack widths.

From the theoretical moment-curvature relationship of a section in a partially prestressed beam subjected to thermal load, it could be seen that a flexural instability after the onset of cracking represented as B to C in Figure 2.8. The magnitude of the drop is largest for I- or box-shaped sections subjected to low axial prestress and high thermal load. Compared with values predicted on the basis of crack-free analysis, the behavior of the model bridge indicated that cracking at critical sections reduces thermal moments by as much as 46 and 27 percent for the box-girder model and T-beam model respectively.

![Figure 2.8 General moment-curvature relationship of a prestressed concrete section under thermal loading (Cooke, Priestley and Thurston 1984)](image)
Finally, the authors recommended that bridges designed for cracking under thermal load combinations, but designed to remain uncracked under less severe load combinations, should be designed for thermal moments calculated using section stiffness and thermal curvatures appropriate for uncracked sections. Thermal stresses resulting from nonlinearity of temperature profile (primary thermal stresses) can conservatively be ignored.

2.4.5 Elbadry and Ghali (1986)

A viable design approach was proposed by Elbadry and Ghali, which was to employ partial prestressing, allowing cracking to reduce stresses due to temperature which controlled cracking by provision of an appropriate amount of nonprestress steel.

For qualitative assessment of the effect of cracking, consider an interior member of a continuous rectangular beam of equal spans of the length L subjected to a temperature rise varying linearly over the depth. The difference in temperature between the top and bottom surface is gradually increased from zero to a specific value $\Delta T$. The continuity moment $M$ increases linearly from zero until occurrence of the first crack at the weakest section. Just before cracking, the statically indeterminate moment $M = M_\sigma$ and the corresponding temperature difference is

$$
\Delta T_\sigma = \frac{h}{\alpha_t} = \frac{M_\sigma}{EI_1}
$$

(2.5)

where $I_1$ is the second moment of area of a transformed noncracked section and $h$ is the total depth of the section.

Shown in Figure 2.9, at the location of the first crack, the flexural rigidity is reduced, causing a drop in $M$. A further increase of $\Delta T$ increases $M$ again until the value $M_\sigma$ is reached, thus producing a new crack. The largest moment that can occur during the development of the crack pattern is equal to $M_\sigma$. The crack pattern becomes fully developed when the temperature difference reaches a particular value $\Delta T_\sigma$. A further increase of $\Delta T$ will not produce new cracks, but the cracks will widen and there will be a ratio $(M / M_\sigma)$ increases following a straight line of slope $\equiv EI_2 \alpha_t / (hM_\sigma)$, where $I_2$ is
the second moment of area of a fully cracked transformed section, with the concrete in tension ignored.

When the $m$th cracks occur, the continuity moment may be expressed as

$$M_m = \frac{\alpha_t \Delta T_m}{h} \left\{ EI_t \left[ 1 + \frac{ms}{l} \left( \frac{I_1}{I_2} - 1 \right) \right] \right\}$$

(2.6)

and the corresponding temperature difference $\Delta T_m$ is

$$\Delta T_m = \frac{M_{cr}}{EI_t} \frac{h}{\alpha_t} \left[ 1 + \frac{s}{l} (m-1)(\frac{I_1}{I_2} - 1) \right]$$

(2.7)

when the stabilized pattern is reached, the temperature difference became

$$\Delta T_m = \frac{M_{cr}}{EI_t} \frac{h}{\alpha_t} \left[ 1 + \frac{s}{l} (m_{\text{max}} - 1)(\frac{I_1}{I_2} - 1) \right]$$

(2.8)

where $m_{\text{max}} \equiv \frac{l}{s}$.

Several parametric studies were also made to investigate the temperature variations and the corresponding stresses induced in concrete bridges. The results of these studies indicated that temperature stresses (both self-equilibrating and continuity) are greatest: on a summer day in the early afternoon; when the daily range of ambient temperature is large; when the wind speed is maximum; and / or when the deck surface has no cover, such as a layer of asphalt.
Figure 2.9  Development of cracks and statically indeterminate bending moments due to temperature gradient in a continuous beam (Elbadry and Ghali, 1986)
CHAPTER 3

THERMAL STRESS ANALYSIS OF
STATICALLY DETERMINATE CONCRETE BRIDGE

3.1 INTRODUCTION AND ASSUMPTION

In practice the distribution of temperature over the cross section of members is generally nonlinear. In a cross section composed of a different material such as concrete and steel, the components tend to contract or expand differently because of shrinkage and creep. However, contraction and expansion cannot occur freely and changes in stresses occur. In the following, we consider the effect of temperature rise varying nonlinear over the cross section of a member of a bridge. The following assumptions are made first in development of thermal stress analyses using the one-dimensional beam theory, and it is also true in the time dependent analysis:

(1) The material is homogeneous and exhibits isotropic behaviors.
(2) Euler-Bernoulli assumption that plane sections remain plane is valid.
(3) Temperature varies with depth, but is constant at all point of equal elevation. That is, no transverse temperature variation exists.
(4) Material properties are independent of temperature.
(5) Thermal stresses can be considered independently of stress or strain imposed by other loading conditions. That is, the principle of superposition holds.

In addition, the following sign convention is adopted:

(1) Compressive forces, stresses and deformations are positive. Shrinkage strain and compressive creep strains are positive, as are the losses of tension in any prestressed steel.
(2) Positive bending moments produce tensile stresses in the bottom fibres of a horizontal beam and the corresponding curvature is also positive.
3.2 AGE-ADJUSTED EFFECTIVE MODULUS METHOD

3.2.1 Formulation

Age-adjusted effective modulus method is the simplest and the most widespread method in the creep analysis of concrete structures. This method was first proposed by Trost (1967), which is on the basis of approximate and mostly intuitive considerations. Later the method was more rigorously formulated by Bazant (1972). He pointed out that the age-adjusted effective modulus was theoretically exact for any creep problem in which strain varies linearly with creep coefficient, instant strain increment at the time of first loading being admissible. The method was also extended for an unbounded final value of creep and for the variation of elastic modulus whose omission is found to be responsible for a significant error, offsetting the gain in theoretical accuracy. This method is sometimes called Trost-Bazant Method.

The capacity of concrete to creep is usually defined in terms of the creep coefficient, $\phi(t, \tau)$. Under a constant sustained stress, $\phi(t, \tau)$ is the ratio of the creep strain at time $t$ to the instantaneous elastic strain which is represented by:

$$\phi(t, \tau) = \frac{\varepsilon_c(t, \tau)}{\varepsilon_e(\tau)} \quad (3.1)$$

Since creep strain depends on the age of the concrete at the time of first loading, so too does the creep coefficient. Both the creep and the instantaneous strain components are proportional to stress, the creep coefficient $\phi(t, \tau)$ of equation (3.1) is a pure time function and is independent of the applied stress. As time approaches infinity, the creep coefficient is assumed to approach a final value $\phi^*(\tau)$, where

$$\phi^*(\tau) = \phi(\infty, \tau) = \frac{\varepsilon_c^*}{\varepsilon_e(\tau)} \quad (3.2)$$

The final creep coefficient is a useful measure of the capacity of concrete creep.
By another frequently used function, specific creep \( C(t, \tau) \) which is the creep strain at time \( t \) produced by a sustained unit stress first applied at age \( \tau \), the creep function \( \Phi(t, \tau) \) was defined:

\[
\Phi(t, \tau) = \frac{1}{E_c(\tau)} \cdot C(t, \tau) = \frac{1}{E_c(\tau)} \left[ 1 + \phi(t, \tau) \right] \tag{3.3}
\]

\( \Phi(t, \tau) \) is the sum of the instantaneous and creep strain at time \( t \) produced by a sustained unit stress applied at \( \tau \).

Consider the two concrete stress histories and the corresponding creep-time curves shown in Figure 3.1. In stress histories (a), \( \sigma_o \) is suddenly applied at time \( \tau_o \) and held constant with time. In stress histories (b), the stress \( \sigma(t) \) is gradually applied, beginning at \( \tau_o \) and reaching a magnitude equal to \( \sigma_o \) at time \( \tau_1 \). The creep strain at any time \( t (> \tau_o) \) produced by gradually applied stress is significantly smaller than that resulting from the suddenly applied stress, as shown. This is due to aging. The earlier a concrete specimen is loaded, the greater is the final creep strain. A reduced creep coefficient can therefore be used to calculate creep strain, if stress is gradually applied. Let this reduced creep coefficient be \( \chi(t, \tau_o) \phi(t, \tau_o) \) in which \( \chi(t, \tau_o) \) is aging coefficient and its magnitude generally falls within the range 0.6 to 0.9.

![Figure 3.1 Creep due to both constant and variable stress histories](image)

The creep strain at time \( t \) due to a stress \( \sigma(t) \), which has been gradually applied over the time interval \( t - \tau_o \), may be expressed as
\[ \varepsilon_c(t) = \frac{\sigma(t)}{E_c(\tau_o)} \chi(t, \tau_o) \phi(t, \tau_o) \]  

(3.3)

Consider the stress history shown in Figure 3.2. An initial stress \( \sigma_o \) applied at time \( \tau_o \) is gradually reduced with time. The change of stress \( \Delta \sigma(t) = \sigma(t) - \sigma_o \) may be due to a change of the external loads, or restraint to creep and shrinkage, or variations of temperature, or combinations of these, and is usually unknown at the beginning of an analysis.

The total strain at time \( t \) may be expressed as the sum of the strains produced by \( \sigma_o \) (instantaneous and creep), the strain produced by the gradually applied stress increment \( \Delta \sigma(t) \) (instantaneous and creep), and the shrinkage strain.

\[ \varepsilon(t) = \frac{\sigma_o}{E_c(\tau_o)} [1 + \phi(t, \tau_o)] + \frac{\Delta \sigma(t)}{E_c(\tau_o)} [1 + \chi(t, \tau_o) \phi(t, \tau_o)] + \varepsilon_{sh}(t) \]

\[ = \frac{\sigma_o}{E_s(t, \tau_o)} + \frac{\Delta \sigma(t)}{E_s(t, \tau_o)} + \varepsilon_{sh}(t) \]  

(3.4)

where \( E_s(t, \tau_o) \) is the effective modulus

\[ E_s(t, \tau) = \frac{E_c(\tau)}{1 + \phi(t, \tau)} \]  

(3.5)

and

\[ \overline{E_s} = \frac{E_c(\tau_o)}{1 + \chi(t, \tau_o) \phi(t, \tau_o)} \]  

(3.6)

is the age-adjusted effective modulus.

![Figure 3.2 A gradually reducing stress history](image-url)
3.2.2 Determination of Creep Coefficient $\chi(t, \tau_c)$ and Age Coefficient $\phi(t, \tau_c)$

There are numerous analytical methods for predicting the final creep coefficient. These predictive methods vary in complexity. Some are simple and easy to use and provide a quick estimate of $\chi(t, \tau_c)$, whilst others are much more complicated, involving the magnitude and the rate of development of creep. The predictions made by some of the more well-known methods differ widely. Hilsdorf and Müller (1979) compared and assessed many of the methods then available, including ACI 209-1978, CEB-FIP, 1970 and 1978, and Bazant and Panula (Version 1), 1978. They demonstrated marked differences in predictions particularly for concrete loaded at early ages, for low relative humidities and for small cross-sectional dimensions. The CEB-FIP, 1970 method was found to best predict the final creep coefficient, although its accuracy was not particularly good.

Gilbert (1988) compared several typical predictions, and found the tremendous differences in predictions which gives such widely varying results. The ACI 209-1978 and the Bazant-Panula (1978) models provide the smallest estimates of creep and the highest estimates of shrinkage. The CEB models gives higher estimates of creep but grossly underestimate shrinkage.

Hilsdorf and Müller (1979) point out that the more complex methods are no more accurate than the simpler methods and that, in general, the final creep coefficient $\phi^*(\tau)$ varies between 2.0 and 4.0 for most practical applications. Bakoss et al. (1982) found that the final creep of some Australian concrete was best predicted by the CEB-FIP models, while the final shrinkage strain was predicted well by the ACI-209 method. Roper and Bott (1979) found that the CEB-FIP 1978 model predicted the creep of some Australian concrete better than the CEB-FIP 1970 model, which at that time formed the basis of the local predictive method.

After the comparison, Gilbert (1988) found that much more experimental work was required before a predictive model was produced which accurately accounts for the many parameters which affect creep and shrinkage. He concluded that numerically accurate estimates of creep and shrinkage and their effects on structural behavior were
not possible. BS 8100 suggests that at the design stage, it may be prudent to consider a range of values for both the creep coefficient and shrinkage. In this way, upper and lower limits for the final deformations may be established. It is advisable to bracket the problem in this way because, in many situations, overestimates of the deformation, as well as underestimates, can lead to serviceability failures. Gilbert suggested the designer should take advantage of any local data or experience. A knowledge of local concrete and conditions may prove to be a for more reliable means for determining material properties than any of the predictive models contained in building codes and others.

Like the creep coefficient, \( \chi(t, \tau_o) \) depends on the age at first loading, the duration of load, the size and shape of the member, and so on. Aging coefficients based on the CEB-FIP, 1978 creep coefficients have been established by Favre et al. and Neville et al.. They showed that these aging coefficients provided the closest agreement with experimental data of any coefficients yet proposed. For this reason, they have been adopted in the thesis.

From Figure 3.5, the final aging coefficient \( \chi(t, \tau_o) \) is approximately 0.8 when the age at first loading exceeds about 5 days and the final creep coefficient lies in the range \( 1.5 < \phi(\tau_o) < 3.5 \). This includes most practical problems in which final deformations are required for design. This approximation appears to be both reasonable and logical, particularly when one considers just how uncertain are the predictions of \( \phi(t, \tau_o) \) and other material properties. Taking

\[
\chi(\infty, \tau_o) = \chi^*(\tau_o) = 0.8
\]

(3.7)

simplifies the age-adjusted effective modulus method and usually leads to good approximations of both material and structural behavior (Gilbert 1988).
Figure 3.3  Relationship between aging coefficient $\chi(t, \tau_o)$ and time since loading for $50 \leq h_o \leq 1600$mm according to CEB-FIP, 1978 (A.M. Neville et al. 1983)
Figure 3.4  Relationship between $\chi(t, \tau_o)$ and $\tau_o$ based on the CEB-FIP, 1978 creep coefficients (A.M. Neville et al. 1983)

Figure 3.5  Relationship between $\chi(t, \tau_o)$ and $\tau_o$ for different CEB-FIP, 1978 final creep coefficients (A.M. Neville et al. 1983)
3.3 DEVELOPMENT OF THE THEORY FOR
STATICALLY DETERMINATE BRIDGES

In statically determinate bridges, no stresses are produced when the temperature variation is linear; in this case the thermal expansion occurs freely, without restraint. This results in changes in length or in curvature of the members, but produces no change in the reactions or in the internal forces. When the temperature variation is nonlinear, each fiber, being attached to adjacent fibers, is not free to undergo the full expansion, and this induces stresses. These stresses must be self-equilibrating in an individual cross section as long as the structure is statically determinate. The self-equilibrating stresses caused by nonlinear temperature variation over the cross section of a statically determinate bridge are sometimes referred to as the eigenstresses.

Consider a general section shape subjected to an vertical temperature distribution, as shown in Figure 3.6. For the analyses described here, the top fiber of the section is taken to be the reference level. The assumptions and sign convention are made in section 3.1

![Diagram of cross section, temperature distribution, and strain](image)

(a) Cross Section  (b) Distribution of Temperature  (c) Distribution of Strain

Figure 3.6 Analysis of the effects of nonlinear temperature variation

With the temperature rise (drop), the unrestrained thermal strains are:

\[ \varepsilon_y = \alpha_y T_y \]

(3.8)
where: \( \alpha_i \): the coefficient of the thermal expansion

\( \varepsilon_i \): in (-) ve for temperature rise (expansion)

in (+) ve for temperature drop (contraction)

\( \Delta T_y \): the temperature change at level y below the top surface

With such strains, plane sections do not remain plane. When expansion is artificially prevented, with plane sections held plane, then the accompanied internal stress distribution:

\[
\Delta \sigma_{\text{relax}} = -E \varepsilon_i = -E \alpha_i \Delta T_y
\]  

(3.9)

For the concrete, \( E \) in equation (3.9) is the age-adjusted effective modulus \( \overline{E}_c \) calculated over the period of the temperature change. If the temperature change is introduced suddenly or over a relatively short period (less than 1 hour), \( \overline{E}_c \approx E_c \). If the temperature change occurs more slowly or is sustained over a prolonged period, creep begins to relieve the internal relaxation stresses and \( \overline{E}_c \) may be calculated using equation (3.6)

\[
\overline{E}_c(t, \tau) = \frac{E_c(\tau_0)}{1 + \chi(t, \tau) \phi(t, \tau_0)}
\]

For the steel reinforcement, \( E \) in equation (3.9) is \( E_s \).

The resultant of \( \Delta \sigma_{\text{relax}} \) may be represented by a normal force \( -\Delta N_i \) and a moment \( -\Delta M_i \) about the top level, given by:

\[
-\Delta N_i = \int \Delta \sigma_{\text{relax}} dA = -\overline{E}_c \alpha_i \int \Delta T_y dA
\]  

(3.10)

\[
-\Delta M_i = -\int \Delta \sigma_{\text{relax}} y dA = -\overline{E}_c \alpha_i \int \Delta T_y y dA
\]  

(3.11)

applied \( -\Delta N_i \) and \( -\Delta M_i \) in positive directions, that is \( \Delta N_i \) and \( \Delta M_i \) applied to the section to restore equilibrium. The corresponding strain and stress at any fibre which below the top level, are:

\[
\varepsilon_y = \varepsilon_{ot} - \Delta \kappa_y y
\]  

(3.12)

where: \( \varepsilon_{ot} \): the strain at top surface fiber

\( \Delta \kappa_i \): the curvature

\( \varepsilon_y \): the strain at y below the top surface
\( \Delta \sigma_{\text{restore}} \): the stress at \( y \) below the top surface

the addition of \( \Delta \sigma_{\text{relax}} \) to \( \Delta \sigma_{\text{restore}} \) gives the self-equilibrating stress due to temperature:

\[
\sigma_s = \bar{E}_s (\alpha_i \Delta T_y + \varepsilon_{ot} - \Delta K_i y)
\]  
(3.13)

The stress \( \sigma_s \) must have a zero resultant because that the components \( \Delta \sigma_{\text{relax}} \) and \( \Delta \sigma_{\text{restore}} \) have equal and opposite resultant.

By integrating, we obtain the internal force:

\[-\Delta N_i = \int \Delta \sigma_{\text{restore}} y dA = \bar{E}_s \varepsilon_{ot} \int y dA - \bar{E}_s \Delta K_i \int y dA = \bar{E}_s \varepsilon_{ot} A - \bar{E}_s \Delta K_i B \]

(3.14)

\[-\Delta M_i = -\int \Delta \sigma_{\text{restore}} y dA = -\bar{E}_s \varepsilon_{ot} \int y^2 dA + \bar{E}_s \Delta K_i \int y^2 dA = -\bar{E}_s \varepsilon_{ot} B + \bar{E}_s \Delta K_i I \]

(3.15)

where: \( A(=\int dA) \): the area of the cross section

\( B(=\int ydA) \): the first moment of the area about the top surface of the section

\( I(=\int y^2dA) \): the second moment of the area about the top surface of the section

Rearranging (3.14) and (3.15), gives the following expressions for the initial top fiber strain and curvature in terms of the applied axial force \( \Delta N_i \) and the bending moment applied about the top reference surface \( \Delta M_i \).

\[
\varepsilon_{ot} = \frac{B \Delta M_i + I \Delta N_i}{\bar{E}_s (AI - B^2)}
\]

(3.16)

\[
\Delta K_i = \frac{A \Delta M_i + B \Delta N_i}{\bar{E}_s (AI - B^2)}
\]

(3.17)

The initial concrete stress at any point \( y \) below the top of the section may be obtained from:

\[
\sigma_i = \bar{E}_s \varepsilon_i = \bar{E}_s (\varepsilon_{ot} - y \Delta K_i)
\]

(3.18)

The initial steel stress at the \( kth \) layer of non-prestressed reinforcement is:

\[
\sigma_{stk} = E_s (\varepsilon_{ot} - d_{st} \Delta K_i)
\]

(3.19)
and the stress immediately after transfer in the bonded prestressed tendons at the k-th level of pretension steel is:

$$\sigma_{pk} = \frac{P_k}{A_{pk}} + E_p(\sigma_{ct} - d_p \Delta K_t)$$  \hspace{1cm} (3.20)

the first term in this equation is the stress in the tendon prior to the transfer of prestress. The second term is the change of stress in the tendon due to the elastic deformation (usually compressive) of the concrete at the steel level during the initial loading, i.e. when the prestress is transferred to the concrete and external loads are applied.

3.4 EXAMPLES

Case 1

A simple supported reinforced concrete beam is subject a temperature rise which is constant over the beam length but varies over the depth as fifth parabola:

$$T_y = T_{wp} \left( \frac{1200 - y}{1200} \right)^5$$

$$T_{wp} = 32 - 0.2h$$

$$h = 20$$  \hspace{1cm} (3.21)

$T_{wp}$ is the temperature rise in the top fibre. The beam has a section as shown in Figure 3.7. The section contains a layers of non-prestressed bars, and one layer of prestressed bars, i.e. $A_{s1} = 800mm^2$ at $d_{s1} = 50mm$, $A_{s2} = 1500mm^2$ at $d_{s2} = 1150mm$ and $A_p = 1000mm^2$ at $d_p = 1000mm$, materials properties are as follows:

$$E_c = 30000MPa, E_p = E_p = 2 \times 10^5 MPa$$

$$\phi(t, \tau_o) = 0.1, \chi(t, \tau_o) = 1.0$$

so, the age-adjusted effective modulus $E_e$

$$E_e = \frac{E_c}{1 + \chi \phi} = \frac{30000}{1 + 1.0 \times 0.1} = 27272MPa$$
the coefficient of thermal expansion for both concrete and steel is:

\[ \alpha_c = \alpha_s = 10 \times 10^{-6} \text{ per}^\circ C \]

and we assume that the prestressing is bonded to the surrounding concrete.

Figure 3.7  Cross-section of case 1

If the free temperature strain is prevented, the relaxation stresses in the concrete are

\[ \Delta \sigma_{relax} = -E_c \varepsilon_i = -E_c \alpha_t \varepsilon_y \]

\[ = -E_c \alpha_t \times 28 \left( \frac{1200 - y}{1200} \right)^5 \]

\[ = -27272 \times 10 \times 10^{-6} \times 28 \times \left( \frac{1200 - y}{1200} \right)^5 \]

\[ = -7.636 \times \left( \frac{1200 - y}{1200} \right)^5 \]

and the steel stresses are
\[
(\Delta \sigma_{\text{relax}})_{s1} = -2 \times 10^5 \times 10 \times 10^{-6} \times 28 \times \left( \frac{1200 - 50}{1200} \right)^5 = -45.27 \text{MPa}
\]
\[
(\Delta \sigma_{\text{relax}})_{t1} = -2 \times 10^5 \times 10 \times 10^{-6} \times 28 \times \left( \frac{1200 - 1000}{1200} \right)^5 = -7.202 \times 10^{-3} \text{MPa}
\]
\[
(\Delta \sigma_{\text{relax}})_{s2} = -2 \times 10^5 \times 10 \times 10^{-6} \times 28 \times \left( \frac{1200 - 1150}{1200} \right)^5 = -7.033 \times 10^{-6} \text{MPa}
\]

The internal restraining actions \(-\Delta N_i\) and \(-\Delta M_i\) are

\[
-\Delta N_i = \int_0^{2000} 7.636 \times \left( \frac{1200 - y}{1200} \right)^5 dA + 45.27 \times 800 + 7.202 \times 10^{-3} \times 1000 + 7.033 \times 10^{-6} \times 1500
\]
\[
= 7.636 \int_0^{2000} \left( \frac{1200 - y}{1200} \right)^5 dA + 36223.2
\]
\[
= 7.636 \times 200 \times 200 + 36223.2
\]
\[
= 341663.2 N
\]
\[
= 341.663 KN
\]
\[
-\Delta M_i = \int_0^{2000} 7.636 \times \left( \frac{1200 - y}{1200} \right)^5 x y \times 200 \ dy + (-45.27) \times 50 \times 800 + (-7.202 \times 10^{-3}) \times 1000 \times 1000
\]
\[
+ (-7.033 \times 10^{-6}) \times 1150 \times 1500
\]
\[
= -7.636 \times 200 \times 3.429 \times 10^4 - 1818014.132
\]
\[
= -54185702.13 \text{Nm}
\]
\[
= -54.186 KNm
\]

The properties of the age-adjusted transformed section are

\[
\bar{n} = \frac{E_s}{E_s} = \frac{2 \times 10^5}{27272} = 7.334
\]
\[
\bar{A_s} = 200 \times 1200 + (\bar{n} - 1) \times (800 + 1000 + 1500) = 260902.2 \text{mm}^2
\]
\[
\bar{B_s} = 200 \times 1200 \times 600 + (\bar{n} - 1) \times (800 \times 50 + 1000 \times 1000 + 1500 \times 1150) = 161513510 \text{mm}^3
\]
\[
\bar{I_s} = \frac{200 \times 1200^3}{12} + 200 \times 1200 \times 600^2 + (\bar{n} - 1) \times (800 \times 50^2 + 1000 \times 1000^2 + 1500 \times 1150^2)
\]
\[
= 1.3411 \times 10^{11} \text{mm}^4
\]
The top fiber strain and curvature:

\[ \Delta \varepsilon_{ot} = \frac{B_z \Delta M_t + I_z \Delta N_t}{E_s (A_z I_z - B_z^2)} \]

\[ = \frac{161513510 \times 54.186 \times 10^6 + 1.3411 \times 10^{11} \times (-341.663 \times 10^3)}{27272 \times (260902.2 \times 1.3411 \times 10^{11} - 161513510^2)} \]

\[ = -3.707 \times 10^{16} \]

\[ = \frac{2.428 \times 10^{20}}{2.428 \times 10^{20}} \]

\[ = -152.67 \times 10^{-6} \]

\[ \Delta K_i = \frac{A_z \Delta M_t + B_z \Delta N_t}{E_s (A_z I_z - B_z^2)} \]

\[ = \frac{260902.2 \times 54.186 \times 10^6 + 161513510 \times (-341.663 \times 10^3)}{27272(260902.2 \times 1.3411 \times 10^{11} - 161513510^2)} \]

\[ = -4.105 \times 10^{13} \]

\[ = \frac{2.428 \times 10^{20}}{2.428 \times 10^{20}} \]

\[ = -0.169 \times 10^{-6} \text{ mm}^{-1} \]

The change of stress at any point for concrete:

\[ \Delta \sigma = E_s (- \varepsilon_i + \Delta \varepsilon_{ot} - y \Delta K_i) \]

\[ = 27272 \left[ 10 \times 10^{-6} \times 28 \times \left( \frac{1200 - y}{1200} \right)^5 + (-152.67 \times 10^{-6}) - y \times (-0.169 \times 10^{-6}) \right] \]

At

- \( y = 0 \text{ mm} \), \( \Delta \sigma = 3.472 \text{ MPa} \)
- \( y = 50 \text{ mm} \), \( \Delta \sigma = 2.239 \text{ MPa} \)
- \( y = 100 \text{ mm} \), \( \Delta \sigma = 1.239 \text{ MPa} \)
- \( y = 150 \text{ mm} \), \( \Delta \sigma = 0.444 \text{ MPa} \)
- \( y = 200 \text{ mm} \), \( \Delta \sigma = -0.173 \text{ MPa} \)
- \( y = 300 \text{ mm} \), \( \Delta \sigma = -0.969 \text{ MPa} \)
- \( y = 400 \text{ mm} \), \( \Delta \sigma = -1.315 \text{ MPa} \)
- \( y = 500 \text{ mm} \), \( \Delta \sigma = -1.344 \text{ MPa} \)
- \( y = 700 \text{ mm} \), \( \Delta \sigma = -0.842 \text{ MPa} \)
- \( y = 800 \text{ mm} \), \( \Delta \sigma = -0.445 \text{ MPa} \)
\[ y = 900 \text{ mm} \quad \Delta \sigma = -0.0084 \text{ MPa} \]
\[ y = 1000 \text{ mm} \quad \Delta \sigma = 0.446 \text{ MPa} \]
\[ y = 1100 \text{ mm} \quad \Delta \sigma = 0.906 \text{ MPa} \]

in the steel reinforcement

\[ \Delta \sigma_1 = 2 \times 10^5 (226 - 152.67 + 50 \times 0.169) \times 10^{-6} = 13.89 \text{ MPa} \]
\[ \Delta \sigma_2 = 2 \times 10^5 (3.6 \times 10^{-2} - 152.67 + 1000 \times 0.169) \times 10^{-6} = 3.593 \text{ MPa} \]
\[ \Delta \sigma_3 = 2 \times 10^5 (3.517 \times 10^{-5} - 152.67 + 1150 \times 0.169) \times 10^{-6} = 9.10 \text{ MPa} \]
Figure 3.8 The stress and strain distribution in case 1
3.5 THEORY FOR COMPOSITE CROSS SECTION

Figure 3.9 Typical concrete-concrete composite section

Figure 3.10 The transformed composite cross-section
Consider a composite cross section, element (1) is a insitu reinforced concrete slab deck which consists of \(m\) layers of non-prestressed reinforcement; element (2) is precast prestressed concrete girder which consists of \(n\) layers non-prestressed reinforcement and \(j\) layers prestressed steel. These two elements have different elastic modulus, but the same coefficient of expansion.

An uncracked cross section is shown in Figure 3.9, the transformed section is also shown in Figure 3.10. We use concrete of element 1 as base, for concrete of element 2, the age-adjusted effective modular ratio is \(\bar{n}_{c2} = \frac{E_{c2}}{E_{c1}}\), if \(E_{c1} \neq E_{c2}\), the value of \(\bar{n}_{c2}\) is not equal to 1. If the temperature rise or drop occurs suddenly or just over a relatively short period (less than one hour), \(\bar{E}_{c1} \approx E_{c1}, \bar{E}_{c2} \approx E_{c2}\), the modular ratio for element 1 is equal to \(n_{c1} = \frac{E_{c1}}{E_{c1}}\). If the steel is bonded to the concrete, the area of steel at each reinforcement level is transformed into an equivalent area of concrete (element 1), i.e. an area of concrete equal to \(\bar{n}_s A_{sk}\) replaces the area of the bonded steel \(A_{sk}\) at the \(k\)-th level of non-prestressed reinforcement and an area of concrete equal to \(\bar{n}_p A_{pk}\) replaces the area of the bonded steel \(A_{pk}\) at the \(k\)-th level of prestressed reinforcement, as shown. The age-adjusted effective modular ratio \(\bar{n}_s\) and \(\bar{n}_p\) are given by \(\bar{n}_s = \frac{E_s}{E_{c1}}\) and \(\bar{n}_p = \frac{E_p}{E_{c1}}\) respectively. For a short time period, \(\bar{n}_s\) and \(\bar{n}_p\) are replaced by \(n_s = \frac{E_s}{E_{c1}}\) and \(n_p = \frac{E_p}{E_{c1}}\) which need not to be considered the aging effect.

Therefore, for a post-tensioned cross-section, we will at this stage consider the steel is not bonded to the concrete, the steel area does not form part of the transformed section. The cross-sectional area of the hollow duct containing the unbonded tendons should be subtracted from the gross cross-section in the determination of the properties of the transformed section. If after transfer of prestress force the duct is grouted and the tendons are thereby bonded to the surrounding concrete, the properties of the transformed section must be changed for all subsequent analyses by the inclusion of the transformed area of the grouted tendons and the area of the now solid duct.

The properties of transformed section about the reference level are:
the area of transformed section:

$$\bar{A}_e = A_{c1} + \bar{n}_{c2}A_{c2} + \sum_{k=1}^{m+n}(\bar{n}_{sk} - 1)A_{sk} + \sum_{k=1}^{l}(\bar{n}_{pk} - 1)A_{pk}$$  \hspace{1cm} (3.22)$$

the first moment of transformed section about top surface:

$$\bar{B}_e = A_{c1}d_{c1} + \bar{n}_{c2}A_{c2}d_{c2} + \sum_{k=1}^{m+n}(\bar{n}_{sk} - 1)A_{sk}d_{sk} + \sum_{k=1}^{l}(\bar{n}_{pk} - 1)A_{pk}d_{pk}$$  \hspace{1cm} (3.23)$$

the second moment of transformed section about top surface:

$$\bar{I}_e = (I_{c1} + A_{c1}d_{c1}^2) + (\bar{n}_{c2}I_{c2} + \bar{n}_{c2}A_{c2}d_{c2}^2) + \sum_{k=1}^{m+n}(\bar{n}_{sk} - 1)A_{sk}d_{sk}^2 + \sum_{k=1}^{l}(\bar{n}_{pk} - 1)A_{pk}d_{pk}^2$$  \hspace{1cm} (3.24)$$

The internal forces are given by following equation similar as (3.14) and (3.15)

$$-\Delta N_i = \bar{E}_c \alpha_i \int T_y dA_{c1} + \bar{E}_c \alpha_i \int T_y dA_{c2} + \sum_{k=1}^{m}(E_{sk} - \bar{E}_c) \alpha_i T_{da} A_{sk}$$

$$+ \sum_{k=m+1}^{n}(E_{sk} - \bar{E}_c) \alpha_i T_{da} A_{sk} + \sum_{k=1}^{l}(E_{pk} - \bar{E}_c) \alpha_i T_{da} A_{pk}$$  \hspace{1cm} (3.25)$$

$$-\Delta M_i = -(\bar{E}_c \alpha_i \int T_y dA_{c1} + \bar{E}_c \alpha_i \int T_y dA_{c2} + \sum_{k=1}^{m}(E_{sk} - \bar{E}_c) \alpha_i T_{da} A_{sk} d_{sk}$$

$$+ \sum_{k=m+1}^{n}(E_{sk} - \bar{E}_c) \alpha_i T_{da} A_{sk} d_{sk} + \sum_{k=1}^{l}(E_{pk} - \bar{E}_c) \alpha_i T_{da} A_{pk} d_{pk})$$  \hspace{1cm} (3.26)$$

The top surface strain can be obtained:

$$\varepsilon_{ot} = \frac{\bar{B}_e \Delta M_i + \bar{I}_e \Delta N_i}{\bar{E}_c (\bar{A}_e \bar{I}_e - \bar{B}_e^2)}$$  \hspace{1cm} (3.27)$$

and the curvature

$$\Delta K_i = \frac{\bar{A}_e \Delta M_i + \bar{B}_e \Delta N_i}{\bar{E}_c (\bar{A}_e \bar{I}_e - \bar{B}_e^2)}$$  \hspace{1cm} (3.28)$$

The self-equilibrating stress due to temperature is as follows,

for concrete element 1:

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\[
\sigma_{ac1} = \overline{E}_{c1}(\varepsilon_i + \varepsilon_{ot} - y\Delta K_i)
\]

for concrete element 2:

\[
\sigma_{ac2} = \overline{E}_{c2}(\varepsilon_i + \varepsilon_{ot} - y\Delta K_i)
\]

for the k-th layers of non-prestressed steel:

\[
\sigma_{sk} = E_{sk}(\varepsilon_i + \varepsilon_{ot} - d_{sk}\Delta K_i)
\]

for the stress in the k-th layers of prestressed steel:

\[
\sigma_{pk} = E_{pk}(\varepsilon_i + \varepsilon_{ot} - d_{pk}\Delta K_i)
\]
3.6 EXAMPLES

Case 2

Consider a simple support bridge. The composite cross section shown in Figure 3.11 forms parts of bridge deck and consists of an in-situ reinforced concrete deck and precast I-section (with bonded tendons)

![Figure 3.11 Detail of the cross-section in Case 2](image1)

![Figure 3.12 Transformed section in Case 2](image2)
For the concrete deck:

\[ E_{c1}(\tau_o) = 31000\,Mpa \]
\[ \phi_1(\infty, \tau_o) = 2.0 \]
\[ \chi_1(\infty, \tau_o) = 0.8 \]

For the precast I-section:

\[ E_{c2}(\tau_o) = 42000\,MPa \]
\[ \phi_2(\infty, \tau_o) = 1.2 \]
\[ \chi_2(\infty, \tau_o) = 0.8 \]

For the steel reinforcement:

\[ E_s = 200000\,MPa \]
\[ E_p = 200000\,MPa \]

the coefficient of thermal expansion for both concrete and steel:

\[ \alpha_c = \alpha_s = 11 \times 10^{-6}\,\text{per}^\circ C \]

Consider the temperature change occurs over a long period, the age-adjusted effective modulus:

\[ \overline{E}_{c1} = \frac{E_{c1}}{1 + \chi_1\phi_1} = \frac{31000}{1 + 2 \times 0.8} = 11923.1\,MPa \]

\[ \overline{E}_{c2} = \frac{E_{c2}}{1 + \chi_2\phi_2} = \frac{42000}{1 + 1.2 \times 0.8} = 21428.6\,MPa \]

Under elastic analysis, and using element 2 as basis for determining modular ratios:

\[ \overline{n}_{c1} = \frac{\overline{E}_{c1}}{E_{c1}} = 1 \]

\[ \overline{n}_{c2} = \frac{\overline{E}_{c2}}{E_{c1}} = \frac{21428.6}{11923.1} = 1.797 \]

\[ \overline{n}_{sk} = \frac{n_{sk}}{E_{c1}} = \frac{200000}{11923.1} = 16.77 \]

The non-prestressed steel is replaced by

\[ (\overline{n}_{s1} - 1)A_{s1} = (16.77 - 1) \times 2000 = 31540\,\text{mm}^2 \]
\[ (\overline{n}_{s2} - 1)A_{s2} = (16.77 - 1) \times 900 = 14193\,\text{mm}^2 \]
\[ (\overline{n}_{s3} - 1)A_{s3} = (16.77 - 1) \times 1800 = 28386\,\text{mm}^2 \]
The prestressed steel is replaced by
\[
(n_p - 1)A_p = (16.77 - 1) \times 800 = 12616 \quad \text{mm}^2
\]
\[
(n_p^2 - 1)A_p^2 = (16.77 - 1) \times 800 = 12616 \quad \text{mm}^2
\]
The temperature change is a fifth-parabola curve, and a linear temperature increase from 0 to 1.5°C over the bottom 200mm of the section
\[
T_y = T_{wp} \left( \frac{1200 - y}{1200} \right)^5
\]
\[
T_{wp} = 32 - 0.2h
\]
\[
h = 20 \text{mm}
\]
Taking the top surface as reference level, the properties of the transformed section are:
the area of the slab \[A_{c1} = 160 \times 1800 = 288000 \text{mm}^2\]
the area of concrete girder \[A_{c2} = (150 + 130) \times 450 - \frac{1}{2} (450 - 150) + 150 \times 545 \]
\[+ 500 \times (150 + 175) - \frac{1}{2} (500 - 150) \times 175\]
\[= 317125 \text{mm}^2\]
the depth of central axis for the concrete girder \[
H_{c2} = \frac{[126000 \times 300 - 22500 \times (160 + 130 + \frac{2}{3} \times 150) + 150 \times 545 \times (440 + \frac{545}{2})]}{317125}
\]
\[= \frac{241788541.7}{317125}\]
\[= 762.4 \text{mm}\]
the second moment of transformed section about the top surface \[
I_{c2} = \frac{1}{12} (450 \times 280^3 + 150 \times 325^3) - \frac{1}{36} (300 \times 150^3 + 350 \times 175^3)
\]
\[+ 12600 \times 300^2 - 22500 \times (160 + 130 + \frac{2}{3} \times 150)^2\]
\[ +150 \times 545 \times (440 + \frac{545}{2})^2 + 162500 \times (1147.5)^2 \]
\[ -30625 \times (985 + \frac{1}{3} \times 175)^2 \]
\[ = 2.343 \times 10^{11} \text{ mm}^2 \]

So the area of transformed section is obtained from Eqn (3.22)

\[ \bar{A}_x = A_{c2} + n_{c1} A_{c1} + \sum_{k=1}^{m+n} (\bar{n}_{sk} - 1) A_{sk} + \sum_{k=1}^{j} (\bar{n}_{pk} - 1) A_{pk} \]
\[ = 288000 + 1.797 \times 317125 + 31540 + 14193 + 28386 + 12616 \times 2 \]
\[ = 957224.625 \text{ mm}^2 \]

the first moment of transformed section about top surface

\[ \bar{B}_x = A_{c2} d_{c2} + n_{c1} A_{c1} d_{c1} + \sum_{k=1}^{m+n} (\bar{n}_{sk} - 1) A_{sk} d_{sk} + \sum_{k=1}^{j} (\bar{n}_{pk} - 1) A_{pk} d_{pk} \]
\[ = 288000 \times 80 + 1.797 \times 317125 \times 762.4 + 31540 \times 80 + 14193 \times 230 \]
\[ + 28386 \times 1240 + 12616 \times 1030 + 12616 \times 1160 \]
\[ = 526.12 \times 10^6 \text{ mm}^3 \]

the second moment of transformed section about the top surface

\[ \bar{I}_x = (I_{c2} + A_{c2} d_{c2}^2) + (n_{c1} I_{c1} + n_{c1} A_{c1} d_{c1}^2) \]
\[ + \sum_{k=1}^{m+n} (n_{sk} - 1) A_{sk} d_{sk}^2 + \sum_{k=1}^{j} (n_{pk} - 1) A_{pk} d_{pk}^2 \]
\[ = 614.4 \times 10^6 + 288000 \times 80^2 + 1.797 \times 2.343 \times 10^{11} + 31540 \times 80^2 \]
\[ + 14193 \times 230^2 + 28386 \times 1240^2 + 12616 \times 1030^2 + 12616 \times 1160^2 \]
\[ = 498.45 \times 10^8 \text{ mm}^4 \]

The change of force due to temperature increase is

\[ - \Delta N_i = \bar{E}_{c2} \alpha_i \int T_j dA_{c1} + \bar{E}_{c2} \alpha_i \int T_j dA_{c2} + \sum_{k=1}^{m} (E_{sk} - \bar{E}_{c2}) \alpha_i T_{sk} A_{sk} \]
\[ + \sum_{k=m+1}^{n} (E_{sk} - \bar{E}_{c2}) \alpha_i T_{sk} A_{sk} + \sum_{k=1}^{j} (E_{pk} - \bar{E}_{c2}) \alpha_i T_{pk} A_{pk} \]
\[\begin{align*}
&= 11923.1 \times 11 \times 10^{-6} \int_0^{160} \left( \frac{1200 - y}{1200} \right)^5 \times 1800 \, dy \\
&\quad + 21428.6 \times 11 \times 10^{-6} \left[ \int_{160}^{290} \left( \frac{1200 - y}{1200} \right)^5 \right. \\
&\quad \quad \quad + \int_{290}^{440} \left( \frac{1200 - y}{1200} \right)^5 (1030 - 2y) \, dy \\
&\quad \quad \quad + \int_{440}^{985} \left( \frac{1200 - y}{1200} \right)^5 150 \, dy + \int_{985}^{1160} \left( \frac{1200 - y}{1200} \right)^5 (2y - 1820) \, dy \\
&\quad \quad \quad + \int_{1160}^{1310} (0.0075y - 8.325) 500 \, dy \\
&\quad \quad \quad + (200000 - 11923.1) \times 11 \times 10^{-6} \times 28 \left( \frac{1200 - 80}{1200} \right)^5 \times 2000 \\
&\quad \quad \quad + (200000 - 21428.6) \times 11 \times 10^{-6} \left[ 28 \left( \frac{1200 - 230}{1200} \right)^5 \times 900 \\
&\quad \quad \quad + (0.0075 \times 1240 - 8.325) \times 1800 \right] \\
&\quad \quad \quad + (200000 - 21428.6) \times 11 \times 10^{-6} \times [28 \left( \frac{1200 - 1030}{1200} \right)^5 \times 800 \\
&\quad \quad \quad + 28 \left( \frac{1200 - 1160}{1200} \right)^5 \times 800] \\
&= 761874.21 + 221507.88 + 82054.06 + 20530.06 + 2.51 \\
&= 1085968.7 \, Nm \\
\end{align*}\]

\[-\Delta M_i = -\left( \overline{E} \sigma \alpha \int T_y \, ydA_{c1} + \overline{E} \sigma \alpha \int T_y \, ydA_{c2} + \sum_{k=1}^{m} (E_{sk} - \overline{E} \sigma) \alpha_i T_{sk} A_{sk} d_{sk} \right. \\
\quad + \sum_{k=1}^{n} (E_{sk} - \overline{E} \sigma) \alpha_i T_{sk} A_{sk} d_{sk} + \sum_{k=1}^{l} (E_{pk} - \overline{E} \sigma) \alpha_i T_{pk} A_{pk} d_{pk} + \sum_{k=1}^{m} (E_{sk} - \overline{E} \sigma) \alpha_i T_{sk} A_{sk} d_{sk} \right] \\
\quad = -\{11923.1 \times 11 \times 10^{-6} \int_0^{160} \left( \frac{1200 - y}{1200} \right)^5 \times y \times 1800 \, dy \}
\]
+21428.6 \times 11 \times 10^{-6} \int_{160}^{250} 28 \left( \frac{1200 - y}{1200} \right)^3 \times 450 \times y \, dy \\
+ \int_{440}^{480} 28 \left( \frac{1200 - y}{1200} \right)^3 (1030 - 2y) \, y \, dy + \int_{985}^{1160} 28 \left( \frac{1200 - y}{1200} \right)^3 \times 150 \times y \, dy \\
+ \int_{1160}^{1310} 28 \left( \frac{1200 - y}{1200} \right)^3 (2y - 1820) \, y \, dy + \int_{1160}^{1310} (0.0075 - 8.325) \times 500 \, y \, dy \\
+ (200000 - 11923.1) \times 11 \times 10^{-6} \times 28 \left( \frac{1200 - 80}{1200} \right)^5 \times 2000 \times 80 \\
+ (200000 - 21428.6) \times 11 \times 10^{-6} \times 28 \left( \frac{1200 - 230}{1200} \right)^5 \times 900 \times 230 \\
+ (0.0075 \times 1240 - 8.325) \times 1800 \times 1124 \\
+ (200000 - 21428.6) \times 11 \times 10^{-6} \times 28 \left( \frac{1200 - 1030}{1200} \right)^5 \times 800 \times 1030 \\
+ 28 \left( \frac{1200 - 1160}{1200} \right)^5 \times 800 \times 1160} \\
= -(53773181 + 55534359.79 + 20726384.78 + 6564324.48 \\
+ 7803819.22 + 2588.1) \\
= -144404657.3 \text{mm}^2 \\

The top surface strain and curvature can be obtained from (3.27) and (3.28) \\

\[
\varepsilon_{oe} = \frac{B_0 \Delta M_0 + I_0 \Delta N_t}{E_0 (A_0 I_e - B_0^2)} \\
\]

\[
= \frac{526.12 \times 10^6 \times 144404657.3 + 498.45 \times 10^3 \times (-1085968.7)}{11923.1 \times (957224.625 \times 498.45 \times 10^3 - (526.12 \times 10^6)^2)} \\
= \frac{-4.653 \times 10^{-7}}{2.3885 \times 10^{-11}} \\
= -1.948 \times 10^{-4} 
\]

53
\[ \Delta K_i = \frac{\bar{A}_s \Delta M_i + \bar{B}_s \Delta N_i}{E_0 I (\bar{A}_s I - \bar{B}_s)} \]

\[ = \frac{957224.625 \times 144404657.3 + 526.12 \times 10^4 \times (-1085968.7)}{11923.1 \times (957224.625 \times 498.45 \times 10^9 - (526.12 \times 10^4)^3)} \]

\[ = \frac{-4.3312 \times 10^4}{2.3885 \times 10^{21}} \]

\[ = -1.813 \times 10^{-7} \text{ mm}^{-1} \]

The self-equilibrating stress due to temperature change along the depth of the section for concrete element 1 can be obtained:

\[ \sigma_{sc1} = \frac{E_0 I (-\varepsilon_t + \varepsilon_{ct} - y \Delta K_i)}{11923.1 (11 \times 10^{-4} \times T_y - 1.948 \times 10^{-4} + 1.813 \times 10^{-7} y)} \]

and for concrete element 2

\[ \sigma_{sc2} = \frac{E_0 I (-\varepsilon_t + \varepsilon_{ct} - y \Delta K_i)}{21428.6 (11 \times 10^{-4} \times T_y - 1.948 \times 10^{-4} + 1.813 \times 10^{-7} y)} \]

for the k-th layers of non-prestressed reinforcement

\[ s_{nl} = 2 \times 10^5 (11 \times 10^{-4} \times 28 \left( \frac{1200 - 80}{1200} \right)^5 - 1.948 \times 10^{-4} + 80 \times 1.813 \times 10^{-7}) \]

\[ = 7.569 \text{ MPa} \]

\[ s_{nl} = 2 \times 10^5 (11 \times 10^{-4} \times 28 \left( \frac{1200 - 230}{1200} \right)^5 - 1.948 \times 10^{-4} + 230 \times 1.813 \times 10^{-7}) \]

\[ = -9.362 \text{ MPa} \]

\[ s_{nl} = 2 \times 10^5 (11 \times 10^{-4} \times (0.0075 \times 1240 - 8.325) - 1.948 \times 10^{-4} \]

\[ + 1240 \times 1.813 \times 10^{-7}) \]

\[ = 8.147 \text{ MPa} \]

for the k-th layer of prestressed steel

\[ s_{pl} = 2 \times 10^5 (11 \times 10^{-4} \times 28 \left( \frac{1200 - 1030}{1200} \right)^5 - 1.948 \times 10^{-4} \]

\[ + 1030 \times 1.813 \times 10^{-7}) \]
\[ s_{\text{p2}} = 2 \times 10^4 \left( 11 \times 10^{-4} \times 28 \left( \frac{1200 - 1160}{1200} \right)^3 - 1.948 \times 10^{-4} + 1160 \times 1.813 \times 10^{-7} \right) \]
\[ = 3.102 \text{MPa} \]

Figure 3.13 Stress distributions for Case 2
CHAPTER 4

THERMAL STRESS ANALYSIS OF
STATICALLY INDETERMINATE CONCRETE BRIDGES

4.1 THEORY FOR STATICALLY INDETERMINATE BRIDGES

In statically indeterminate structures, the stresses due to a thermal gradient consist of two components, one is primary stress and the other is secondary thermal stress. If the temperature is constant along the length of the structure but varies with the depth of cross-section, the non-linear thermal distribution induces the primary stress across the section. The primary stresses are self-equilibrating and only depend on the properties of cross-section, in this definition it is not important whether the structure is statically determinate or statically indeterminate. The primary stress is also called self-equilibrating stress.

The deformation due to primary thermal response in a statically indeterminate structure produces reactions, internal forces and stresses because of the restraint imposed by supports. The stresses due to the indeterminate forces are referred to as secondary thermal stresses or continuity stresses. Several methods are used to determinate the continuity stress. Here we apply the five steps of the force method of analysis to determine the continuity stress, as mentioned in the following case study (Case 3). The resulting stresses, referred to as continuity stresses, are produced whether the temperature distribution is linear or nonlinear and must be added to the self-equilibrating stresses to obtained the total thermal stresses.
Figure 4.1 Release of temperature moments in continuous deck

4.2 EXAMPLES

Case 3

A three span composite concrete bridge is considered, which has the same section properties and temperature gradient as Case 2.

The stress distribution in case 2 is the stress so called self-equilibrating stresses. The continuity stresses can be obtained based on the following steps.
Step 1

A released structure and a coordinate system are selected in Figure 4.2(b). Taking advantage of symmetry, the required action is $A = \sigma$, the stress at any fiber of the cross section at B.

Step 2

The displacement of the released structure is the relative rotation of the beams ends at B (or at C).

![Diagram](a) Beam elevation

![Diagram](b) Released structure and coordinate system

\[ D_1 = \frac{\Delta K_{AB} L_{AB}}{2} + \frac{\Delta K_{BC} L_{BC}}{2} \]

\[ = -\frac{1.827 \times 10^{-7}}{2} (1800 + 2000) \]

\[ = -3.471 \times 10^{-4} \]

The value of the required action in the released structure is the self equilibrating stress, hence

\[ A_z = \sigma_z \]

Step 3

Applying $F_1 = 1$ at the coordinate in Figure 4.2(b) gives the flexibility
Figure 4.3  Cross-section of continuous beam, Case 3

Centroidal Axes of Concrete Deck (Element 1)

Centroidal Axes of Concrete Grate (Element 2)

Centroidal Axis of Cross Section

2,150 mm²

12616 mm²

28365 mm²

1030

1240

407.5 mm

Rₐ = 762.4 mm

14193 mm²

150

545

175

80

180

1190

1310

460

1800

590
\[ f_{ii} = \left( \frac{L}{3EI} \right)_{AB} + \left( \frac{L}{2EI} \right)_{BC} = \frac{1600}{E_c I_e} \]

In this case, \( A_u \) is the stress at any fibre of the cross section at B due to \( F_1 = 1 \), that is the stress due to unit bending moment:

\[
d_e = \frac{A_{c1} H_{c1} + A_{c2} H_{c2}}{A_{c1} + A_{c2}}
\]

\[
= \frac{288000 \times 80 + 317125 \times 762.4}{288000 + 317125}
\]

\[
= 437.6 \text{mm}
\]

\[
A_u = \frac{d_e - y}{I_e} = \frac{437.6 - y}{I_e}
\]

where:
- \( d_e \) is the distant of central axis of the transformed section from top level
- \( y \) is the distant measured from the top surface

**Step 4**

The redundant (the connecting moment at B or C) is given by

\[
F_i = -f_{ii}^{-1} D_i = -\left( \frac{1600}{E_c I_e} \right)^{-1} \times (-3.471 \times 10^{-4}) = 2.17 \times 10^{-7} E_c I_e
\]

**Step 5**

The continuity stress from \( F_i \) is

\[
\Delta \sigma = A_u F_i = \frac{F_i (d_e - y)}{I_e}
\]

The continuity stresses for concrete, steel and non-prestressed steel

\[
\Delta \sigma_{cl} = n_{cl} \Delta \sigma,
\]

\[
\Delta \sigma_s = n_s \Delta \sigma,
\]

\[
\Delta \sigma_p = n_p \Delta \sigma
\]

The stress at any fiber is obtained by the superposition equation:

\[
A = A_s + A_u F_i
\]

Substitution gives
\[ \sigma = \sigma_s + (437.6 - \eta) \times 2.17 \times 10^{-7} E_e \]

So the stress in the non-prestressed and prestressed steel
\[ \sigma_{nj} = \sigma_{nj} + (437.6 - d_{nj}) \times 2.17 \times 10^{-7} E_e \]
\[ \sigma_{pj} = \sigma_{pj} + (437.6 - d_{pj}) \times 2.17 \times 10^{-7} E_p \]
Hence, the stress at top and bottom surface
\[ \sigma_{top} = 1.334 + 437.6 \times 2.17 \times 10^{-7} \times 11923.1 \]
\[ = 1.334 + 1.132 \]
\[ = 2.466 \text{MPa} \]
\[ \sigma_{bot} = 1.284 + (437.6 - 1310) \times 2.17 \times 10^{-7} \times 21428.6 \]
\[ = 1.284 - 4.057 \]
\[ = -2.773 \text{MPa} \]

the stress in steel
\[ \sigma_{s1} = 8.365 + (437.6 - 80) \times 2.17 \times 10^{-7} \times 2 \times 10^5 \]
\[ = 8.368 + 15.52 \]
\[ = 23.88 \text{MPa} \]
\[ \sigma_{s2} = -8.76 + (437.6 - 230) \times 2.17 \times 10^{-7} \times 2 \times 10^5 \]
\[ = -8.76 + 9.01 \]
\[ = 0.25 \text{MPa} \]
\[ \sigma_{s3} = 7.435 + (437.6 - 1240) \times 2.17 \times 10^{-7} \times 2 \times 10^5 \]
\[ = 7.435 - 34.82 \]
\[ = -27.39 \text{MPa} \]
\[ \sigma_{p1} = -2.048 + (437.6 - 1030) \times 2.17 \times 10^{-7} \times 2 \times 10^5 \]
\[ = -2.048 - 25.71 \]
\[ = -27.76 \text{MPa} \]
\[ \sigma_{p2} = 3.319 + (437.6 - 1160) \times 2.17 \times 10^{-7} \times 2 \times 10^5 \]
\[ = 3.319 - 31.35 \]
\[ = -28.03 \text{MPa} \]

The stress distributions are shown in figure 4.4. From the result of the calculation, the continuity stress for top surface is 1.132 MPa, 15.1% less than self-
equilibrium stress 1.334 MPa. The stress of the bottom fiber has changed from compression (1.284 MPa) to tension (-2.773 MPa) due to the continuity effect. The continuity stresses are proportional to the statically determinate curvature $\Delta K_i$, and flexural rigidity EI. Negative curvature $\Delta K_i$ results in positive continuity stresses producing compression at the top fiber.

4.3  EFFECT OF CROSS SECTION DEPTH ON THERMAL RESPONSE

Case 4-Case 7

In table 4-1, the result of analysis of case 4-case 7 are presented. Four different depths of cross section are considered. The width of cross section, areas of prestressed and non-prestressed steels are same in every section. $A_{n1} = 2000mm^2$, $A_{n2} = 900mm^2$, $A_{n3} = 1800mm^2$, $A_{p1} = 800mm^2$, $A_{p2} = 800mm^2$.

The effect of increasing the depth of cross section is to increase the self-equilibrating stresses on the top surface, and so too does the maximum negative stresses in the statically determinate structures. The difference between the tension and compression stresses are large in deep section, while the continuity stresses are large in shallow section.
Figure 4.4  Stress distributions in Case 3

(a) Total stress

(b) Continuity stress

(a) Thermal stress
Figure 4.5  Detail of cross-section in Case 4

Figure 4.6  Detail of cross-section in Case 5
Figure 4.7  Detail of cross-section in Case 6

Figure 4.8  Detail of cross-section in Case 7
<table>
<thead>
<tr>
<th>Case</th>
<th>Statically Determinate Curvature ( \Delta K_i )</th>
<th>Self-equilibrating Stress on the Top Surface (MPa)</th>
<th>Self-equilibrating Stress on the Bottom (MPa)</th>
<th>Maximum Negative Stress (MPa)</th>
<th>Continuating Stress on the Top Surface (MPa)</th>
<th>Continuating Stress on the Bottom (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>(-2.022 \times 10^{-7})</td>
<td>1.250</td>
<td>1.371</td>
<td>-1.780</td>
<td>1.193</td>
<td>-4.233</td>
</tr>
<tr>
<td>5</td>
<td>(-1.827 \times 10^{-7})</td>
<td>1.335</td>
<td>1.282</td>
<td>-1.811</td>
<td>1.132</td>
<td>-4.057</td>
</tr>
<tr>
<td>6</td>
<td>(-1.694 \times 10^{-7})</td>
<td>1.383</td>
<td>1.249</td>
<td>-1.856</td>
<td>1.114</td>
<td>-3.947</td>
</tr>
<tr>
<td>7</td>
<td>(-1.563 \times 10^{-7})</td>
<td>1.456</td>
<td>1.260</td>
<td>-1.865</td>
<td>1.081</td>
<td>-3.860</td>
</tr>
</tbody>
</table>

Table 4-1  Effect of the depth of cross-section to both self-equilibrating and continuing stresses (Case 4-7)
Figure 4.9  Stress distributions in Case 4
Figure 4.10 Stress distributions in Case 5
Figure 4.11  Stress distributions in Case 6

(a) Thermal stress

(b) Continuity stress

(c) Total stress
(a) Thermal stress

(b) Continuity stress

(c) Total stress

Figure 4.12  Stress distributions in Case 7
CHAPTER 5

TIME ANALYSES OF CONCRETE BRIDGES

5.1 INTRODUCTION

When a concrete specimen is subject to load, its response is both immediate and time-dependent. Under sustained load, the deformation of a specimen gradually increases with time and eventually may be many times greater than its instantaneous value. At any time t, the total concrete strain \( \varepsilon(t) \) in a uniaxially loaded specimen consists of a number of components, which include the instantaneous strain \( \varepsilon_i(t) \), the creep strain \( \varepsilon_c(t) \), the shrinkage strain \( \varepsilon_{sh}(t) \) and the temperature strain \( \varepsilon_T(t) \). Although not strictly correct, it is usual to assume that all four components are independent and may be calculated separately and summed to obtained the total strain

\[
\varepsilon(t) = \varepsilon_i(t) + \varepsilon_c(t) + \varepsilon_{sh}(t) + \varepsilon_T(t) \tag{5.1}
\]

From the previous research results, the creep and shrinkage characteristics of concrete are highly variable and are difficult to exactly know. In addition, methods for the time analysis of concrete structures are plagued by simplifying assumptions and approximations. Each method has its advantages and disadvantages.

In the following analyses the age-adjusted effective modulus method is used, assumption and convection is already made in section 3.1.

5.2 THEORY

The cross section of composite beam which consisting of various reinforced or prestressed concrete elements acting compositely together and subjected to a sustained bending and axial force is shown in Figure 5.1. The short-term analysis is undertaken first, followed by a time analysis based on the relaxation procedure described by Gilbert (1988) for composite concrete-concrete cross section.
5.2.1 Development of Theory for Short-Term Analyses

For the composite cross-section shown in Figure 5.1, the elastic modules of one of the concrete elements is selected as the modules of the transformed section, say $E_{el}$ of Element 1 (the slab deck). The areas of both the bonded steel reinforcement and the other concrete element(s) are transformed into equivalent areas of the concrete of Element 1, as shown in Figure 5.2.

The properties of this transformed section about the top reference level are
Figure 5.2  The transformed composite cross-section

\[ A = \sum_{j=1}^{2} n_{qj} A_{qj} + \sum_{k=1}^{3} (n_{sk} - 1) A_{sk} + \sum_{k=1}^{2} (n_{pk} - 1) A_{pk} \]  \hspace{1cm} (5.2) \]

\[ B = \sum_{j=1}^{2} n_{qj} A_{qj} d_{qj} + \sum_{k=1}^{3} (n_{sk} - 1) A_{sk} d_{sk} + \sum_{k=1}^{2} (n_{pk} - 1) A_{pk} d_{pk} \]  \hspace{1cm} (5.3) \]

\[ I = \sum_{j=1}^{2} (n_{qj} I_{qj} + n_{qj} A_{qj} d_{qj}^2) + \sum_{k=1}^{3} (n_{sk} - 1) A_{sk} d_{sk}^2 + \sum_{k=1}^{2} (n_{pk} - 1) A_{pk} d_{pk}^2 \]  \hspace{1cm} (5.4) \]

where \( n_{qj} = E_{qj} / E_{el} \), \( n_{sk} = E_{sk} / E_{el} \), and \( n_{pk} = E_{pk} / E_{el} \). Of course, if the prestressed steel were post-tensioned and unbonded, it would not form part of the transformed section. The strain at any point on the cross-section may be expressed in term of the top fiber strain and the section curvature:

\[ \varepsilon_i = \varepsilon_{el} - y K_i \]  \hspace{1cm} (5.5) \]

where

\[ \varepsilon_{el} = \frac{BM_i + IN_i}{E_c (AI - B^2)} \]  \hspace{1cm} (5.6) \]

\[ K_i = \frac{AM_i + BN_i}{E_c (AI - B^2)} \]  \hspace{1cm} (5.7) \]
and A, B and I are the properties of the transformed section; $N_i$ is the resultant axial force on the transformed section ($N_i = N_s$), and $M_i$ is the resultant moment about the top of the section ($M_i = M_s - N_s d_{ns}$). $E_e$ is the elastic modules of the concrete of element 1 ($E_{el}$). The changes of the concrete stress in the $j$-th concrete element at $y$ below the top fibre and the changes of stress in the bonded steel reinforcement caused by $N_i$ and $M_i$ may be calculated from

$$\Delta \sigma_i = E_{el}(\varepsilon_{ei} - y K_i)$$  \hspace{1cm} (5.8) \\
$$\Delta \sigma_{sk} = E_{sk}(\varepsilon_{sk} - d_{sk} K_i)$$  \hspace{1cm} (5.9) \\
$$\Delta \sigma_{pk} = E_{pk}(\varepsilon_{pk} - d_{pk} K_i)$$  \hspace{1cm} (5.10)

5.2.2 Development of Theory for Time analysis

An alternative and powerful approach for time analysis of the cross-section considered here is the use of a relaxation procedure proposed by Bresler and Selna which is perhaps the most convenient approach for the time analysis of a complex cross-section.

During any time interval, the strain state is frozen, i.e. the strain distribution is assumed to remain unchanged. If the total strain is held constant and the creep, shrinkage and temperature components change, then the instantaneous component of strain must also change by an equal and opposite amount. As the instantaneous strain changes, so too does the concrete stress. The stress on the cross-section is therefore allowed to vary freely due to relaxation. As a result, the internal actions change and equilibrium is not maintained. To restore equilibrium, an axial force $\Delta N$ and bending moment $\Delta M$ must be applied to the section.

The change of strain due to creep and shrinkage may be considered to be artificially prevented by restraining actions $-\Delta N$ and $-\Delta M$. When $\Delta N$ and $\Delta M$ are applied to the section, the restraining actions are removed and equilibrium is restored.
\[-\Delta N = - \sum_{j=1}^{n} \bar{E}_{ej} [\phi_j (A_{cj} \varepsilon_{oi} - B_{ej} K_i) + \varepsilon_{adj} A_{cj}] + \sum_{k=1}^{m} K_k R_k \]  

(5.11)

\[-\Delta M = - \sum_{j=1}^{n} \bar{E}_{ej} [\phi_j (-B_{ej} \varepsilon_{oi} + I_{ej} K_i) - \varepsilon_{adj} B_{ej}] - \sum_{k=1}^{m} K_k R_k d_{pk} \]  

(5.12)

where \(A_{cj}\) is the area of the \(j\)-th concrete element, \(B_{ej}\) and \(I_{ej}\) are the first and second moments of the area of the \(j\)-th concrete element about the top reference level, respectively.

The age-adjusted effective modulus of one of the concrete elements (say \(\bar{E}_{e1}\)) is selected as the modulus of the age-adjusted transformed section. The concrete in all other element is transformed into equivalent areas of the concrete of Element 1 by multiplying by the age-adjusted modular ratio \(\bar{n}_{ej} = \bar{E}_{ej} / \bar{E}_{e1}\). The bonded steel area is transformed by multiplying by \(\bar{n}_{sk} = E_{sk} / \bar{E}_{e1}\) or \(\bar{n}_{pk} = E_{pk} / \bar{E}_{e1}\).

For cross-section shown in Figure 5.1, the area of the age-adjusted transformed section \(\bar{A}_e\) and the first and second moments of the transformed areas about the top surface \(\bar{B}_e\) and \(\bar{I}_e\), respectively, are

\[\bar{A}_e = \sum_{j=1}^{2} \bar{n}_{ej} A_{cj} + \sum_{k=1}^{3} \bar{n}_{sk} A_{sk} + \sum_{k=1}^{2} \bar{n}_{pk} A_{pk}\]  

(5.15)

\[\bar{B}_e = \sum_{j=1}^{2} \bar{n}_{ej} A_{cj} d_{cj} + \sum_{k=1}^{3} \bar{n}_{sk} A_{sk} d_{sk} + \sum_{k=1}^{2} \bar{n}_{pk} A_{pk} d_{pk}\]  

(5.14)

\[\bar{I}_e = \sum_{j=1}^{2} (\bar{n}_{ej} I_{ej} + \bar{n}_{ej} A_{ej} d_{ej}^2) + \sum_{k=1}^{3} \bar{n}_{sk} A_{sk} d_{sk}^2 + \sum_{k=1}^{2} \bar{n}_{pk} A_{pk} d_{pk}^2\]  

(5.15)

The time-dependent change in top fiber strain and curvature is therefore

\[\Delta e_o = \frac{\bar{B}_e \Delta M + \bar{I}_e \Delta N}{\bar{E}_e (\bar{A}_e \bar{I}_e - \bar{B}_e)}\]  

(5.16)

\[\Delta K = \frac{\bar{A}_e \Delta M + \bar{B}_e \Delta N}{\bar{E}_e (\bar{A}_e \bar{I}_e - \bar{B}_e)^2}\]  

(5.17)
The change of stress at a point in the j-th concrete element at a depth y below the top fibre is equal to the sum of the stress loss due to relaxation of the age-adjusted transformed section, when creep and shrinkage are fully restrained, and the stress which results when $\Delta N$ and $\Delta M$ are applied to the cross-section.

$$\Delta \sigma = -\bar{E}_o [\phi_j (e_{oi} - yK_i) + e_{alj} - (\Delta \varepsilon_o - y\Delta K)]$$  \hspace{1cm} (5.18)

The stress change with time in the k-th layer of non-prestressed reinforcement is

$$\Delta \sigma_{sk} = E_{sk} (\Delta \varepsilon_o - d_{sk} \Delta K)$$  \hspace{1cm} (5.19)

and in the k-th layer of prestressed steel

$$\Delta \sigma_{pk} = E_{pk} (\Delta \varepsilon_o - d_{pk} \Delta K) + \frac{K_k R_k}{A_{pk}}$$  \hspace{1cm} (5.20)

5.2.3 Examples

Case 8 - Case 10

In these cases, the instantaneous and final long-term behavior of the reinforced concrete cross-section shown in Figure 5.3 is to be calculated. The section is reinforced with a single layer of conventional, non-prestressed reinforcement of area $A_s = 1800\text{mm}^2$. The section is first subjected to a constant sustained bending moment $M_s = 40\text{KNm}$ (with and without shrinkage). The analysis is then repeated for $M_s = 0$ to examine the effects of shrinkage on an uncracked cross-section. Material properties are as follows:

$$E_c (\tau_o) = 30000\text{MPa}; \ E_s = 200000\text{MPa};$$

$$\phi^* = \phi(\infty, \tau_o) = 2.5; \ \chi^* = \chi(\infty, \tau_o) = 0.8;$$

$$\varepsilon_{sh} = \varepsilon_{sh} (\infty) = 500 \times 10^{-6} \text{ or 0}$$
A summary of results obtained using the computer program for case 8, 9 and 10 is provided in Table 5-1. The initial and final strain and stress distribution is plotted in Figure 5.4. In case 10, shrinkage is considered individually without any other factors. In the absence of shrinkage, tensile creep cause the tensile steel stress to increase with time and causing the tensile stress in the bottom concrete fibre to decrease. It is because that the resultant internal tensile force is gradually transferred from the concrete to the steel.

**Case 11 - Case 14**

Further more, consider the Case 11 to Case 14, the same cross-section subjected to the a constant bending moment $M_x = 40KNm$, but the shrinkage strain is varying from $300 \times 10^{-6}$ to $600 \times 10^{-6}$. In Table 5-3, we can see that with the increase of $\varepsilon_{sh}$, (Case 11 - Case 14) the restrain internal force $\Delta N$ and $\Delta M$ increase, and so too dose the change of strain $\Delta \varepsilon_o$ and the change of curvature $\Delta k$ with time. Larger $\varepsilon_{sh}$ get the larger stress in the top surface, and cause the bottom fibber from tension to compression (Figure 5.5).
<table>
<thead>
<tr>
<th></th>
<th>Case 8</th>
<th></th>
<th>Case 9</th>
<th></th>
<th>Case 10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Concrete strain $\varepsilon\left(10^{-6}\right)$</td>
<td>Concrete stress $\sigma$(MPa)</td>
<td>Concrete strain $\varepsilon\left(10^{-6}\right)$</td>
<td>Concrete Stress $\sigma$(MPa)</td>
<td>Concrete strain $\varepsilon\left(10^{-6}\right)$</td>
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<td>top</td>
<td>bottom</td>
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<td>bottom</td>
<td>top</td>
</tr>
<tr>
<td>Initial</td>
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<td>2.944</td>
<td>-2.63</td>
<td>98.15</td>
</tr>
<tr>
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<td>-215.3</td>
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<td>913.9</td>
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<td>Change</td>
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<td>-127.35</td>
<td>-0.3854</td>
<td>0.9152</td>
<td>815.8</td>
</tr>
</tbody>
</table>

**Note:**

- Case 8: $M_s = 40$ kNm \& $\varepsilon_{sh} = 0.0$;
- Case 9: $M_s = 40$ kNm \& $\varepsilon_{sh} = 500 \times 10^{-6}$;
- Case 10: $M_s = 0.0$ kNm \& $\varepsilon_{sh} = 500 \times 10^{-6}$.

Table 5-1  The effects of shrinkage on uncracked cross section (Case 8 - Case 10)
(This table is represented in Figure 5.4)
Figure 5.4  Strain and stress distributions for the case of shrinkage effect (Case 8 - Case 10)
<table>
<thead>
<tr>
<th></th>
<th>Case 11</th>
<th>Case 12</th>
<th>Case 13</th>
<th>Case 14</th>
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<tbody>
<tr>
<td></td>
<td>Concrete strain $\varepsilon \left(10^{-6}\right)$</td>
<td>Concrete stress $\sigma$(MPa)</td>
<td>Concrete strain $\varepsilon \left(10^{-6}\right)$</td>
<td>Concrete Stress $\sigma$(MPa)</td>
</tr>
<tr>
<td>top bottom</td>
<td>98.15 -87.65</td>
<td>2.944 -2.63</td>
<td>98.15 -87.65</td>
<td>2.944 -2.63</td>
</tr>
<tr>
<td>Initial</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>top bottom</td>
<td>670.3 -70.47</td>
<td>3.2122 -3.2669</td>
<td>792.1 -22.2</td>
<td>3.43 -3.784</td>
</tr>
<tr>
<td>Final</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change</td>
<td>572.2 17.18</td>
<td>0.2682 -0.6369</td>
<td>693.95 65.45</td>
<td>0.486 -1.154</td>
</tr>
</tbody>
</table>

**Note:**

Case 1: $M_s = 40 \text{ kNm}$ & $\varepsilon_{sh} = 300 \times 10^{-4}$;
Case 2: $M_s = 40 \text{ kNm}$ & $\varepsilon_{sh} = 400 \times 10^{-4}$;
Case 3: $M_s = 40 \text{ kNm}$ & $\varepsilon_{sh} = 500 \times 10^{-4}$;
Case 4: $M_s = 40 \text{ kNm}$ & $\varepsilon_{sh} = 600 \times 10^{-4}$.

**Table 5-2** The effects of varying shrinkage on uncracked cross-section (Case 11 - Case 14)
(This table is represented in Figure 5.5)
Figure 5.5  Stress and strain distributions for the case of varying shrinkage (Case 11 - Case 14)
<table>
<thead>
<tr>
<th>Case</th>
<th>$\Delta N$ (kN)</th>
<th>$\Delta M$ (kNm)</th>
<th>$\Delta \varepsilon_o$ (x10^-6)</th>
<th>$\Delta k$ (x10^-6) (mm^-1)</th>
<th>$\Delta \sigma_o$ (MPa)</th>
<th>$\Delta \sigma_b$ (MPa)</th>
<th>$\Delta \sigma_s$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>428.3</td>
<td>-88.02</td>
<td>572.2</td>
<td>1.009</td>
<td>0.2682</td>
<td>-0.6369</td>
<td>13.53</td>
</tr>
<tr>
<td>12</td>
<td>564.0</td>
<td>-124.9</td>
<td>694.0</td>
<td>1.143</td>
<td>0.4860</td>
<td>-1.154</td>
<td>24.52</td>
</tr>
<tr>
<td>13</td>
<td>699.7</td>
<td>-161.8</td>
<td>815.8</td>
<td>1.276</td>
<td>0.7038</td>
<td>-1.672</td>
<td>35.51</td>
</tr>
<tr>
<td>14</td>
<td>835.4</td>
<td>-198.8</td>
<td>937.5</td>
<td>1.410</td>
<td>0.9217</td>
<td>-2.189</td>
<td>46.52</td>
</tr>
<tr>
<td>8</td>
<td>21.23</td>
<td>22.72</td>
<td>206.8</td>
<td>0.6081</td>
<td>-0.3854</td>
<td>0.9152</td>
<td>-19.44</td>
</tr>
<tr>
<td>10</td>
<td>678.5</td>
<td>-184.6</td>
<td>608.9</td>
<td>0.6684</td>
<td>1.089</td>
<td>-2.587</td>
<td>54.95</td>
</tr>
</tbody>
</table>

Note:

Case 11: $M_s = 40$ kN \& $\varepsilon_{sh} = 300 \times 10^{-6}$ ;
Case 12: $M_s = 40$ kN \& $\varepsilon_{sh} = 400 \times 10^{-6}$ ;
Case 13: $M_s = 40$ kN \& $\varepsilon_{sh} = 500 \times 10^{-6}$ ;
Case 14: $M_s = 40$ kN \& $\varepsilon_{sh} = 600 \times 10^{-6}$ ;
Case 8: $M_s = 40$ kN \& $\varepsilon_{sh} = 0.0$ ;
Case 10: $M_s = 0.0$ \& $\varepsilon_{sh} = 500 \times 10^{-6}$ .

Table 5-3 The effect of shrinkage on restrain internal force $\Delta N$ and $\Delta M$ (Case 8,10,11,12,13,14) (This table is represented in Figure 5.4 and 5.5)
Case 15 - Case 16

In the following cases, the effects on time-dependent behaviour of varying the quantities of compressive and tensile non-prestressed steel \( A_{s1} \) and \( A_{s2} \), respectively, and the sustained moment \( M_s \) are presented.

Figure 5.6 shows a partially-prestressed concrete section containing two layers of non-prestressed reinforcement and one post-tensioned cable. The cross-section area of the prestressing steel is \( A_p = 780 \text{mm}^2 \) and the diameter of the duct containing the tendons is 60mm. The cross-section is subjected to a constant sustained external moment \( M_s \). Both the short-term and time-dependent behaviour of the cross-section are to be calculated for several different combinations of non-prestressed reinforcement and for three different magnitudes of \( M_s \). The initial tensile force in the post-tensioned tendon prior to transfer (at time \( \tau_o \)) is \( P=1100 \text{KN} \). The prestressing tendon is not bonded to the concrete at transfer and therefore, for the short-time analysis, it is only the non-prestressed steel areas which are transformed into equivalent concrete areas. Soon after the prestress is transferred to the concrete, the post-tensioned duct is grouted, thereby bonding the tendon to the concrete and ensuring compatibility of concrete and the steel strains throughout the period of the time analysis. The relevant material properties assumed for the time period considered are as follows:

\[
E_s(\tau_o) = 30000 \text{MPa}; \quad E_p = 2 \times 10^5 \text{MPa};
\]

\[
\phi(t, \tau_o) = 1.8; \quad \varepsilon_{sh}(t - \tau_o) = 400 \times 10^{-6}; \quad \chi(t, \tau_o) = 0.8; \quad \text{and kR}=20\text{kN}
\]

In Table 5-4 and 5-5, the results of several time analysis are shown. Three levels of external moment area considered. At \( M_s = 100 \text{kNm} \), the initial concrete stress distribution is approximately triangular (Figure 5.7(a)) with higher compressive stress in the bottom fibres. At \( M_s = 280 \text{kNm} \), the initial concrete stress distribution is approximately uniform (Figure 5.7(b)) over the depth of the section and curvature is small. At \( M_s = 400 \text{kNm} \), the initial stress distribution is again triangular (Figure 5.7(c)) with high compressive stresses in the top fibres.
The effect of increasing the quantity of non-prestressed tensile reinforcement, $A_{x2}$, is to increase the change in positive curvature with time. This increase is most dramatic when the initial concrete compressive stress at the level of the steel is high, i.e. when the sustained moment $M_s$ is low and the section is initial subjected to a negative curvature. In Table 5-4, when $A_{x2} = 2000 \, mm^2$ and $M_s = 100 \, kNm$, $\Delta k$ is positive despite a large negative initial curvature. Table 5-4 indicate that the addition of non-prestressed steel in the tensile zone will reduce the time-dependent camber which often cause problem in precast member subjected to low sustained loads. For sections on which $M_s$ is sufficient to cause an initial positive curvature, an increase in $A_{x2}$ will cause an increase in time-dependent curvature and here an increase in final deflection. The inclusion of non-prestressed steel in the compressive zone, $A_{x1}$, increases the change in negative curvature with time. Reference to Table 5-5, when $M_s = 400 \, kNm$, the sections where the initial curvature $k_i$ is positive, the inclusion of $A_{x1}$ reduce the time-dependent change in curvature (and hence deflection). However, when $k_i$ is negative, $A_{x1}$ cause an increase in negative curvature and hence an increase in the camber of the member with time.
<table>
<thead>
<tr>
<th>$M_z$ (kNm)</th>
<th>$A_{n1}$ (mm$^2$)</th>
<th>$A_{n2}$ (mm$^2$)</th>
<th>$\varepsilon_{ao}$ (x10^{-6})</th>
<th>$k_{i}$ (x10^{-6} (mm$^{-1}$)</th>
<th>$\Delta N$ (kN)</th>
<th>$\Delta M$ (kNm)</th>
<th>$\Delta \varepsilon_{ao}$ (x10^{-6})</th>
<th>$\Delta k_{i}$ (x10^{-6} (mm$^{-1}$)</th>
<th>$\Delta \sigma_{o}$ (MPa)</th>
<th>$\Delta \sigma_{p}$ (MPa)</th>
<th>$\Delta \sigma_{n1}$ (MPa)</th>
<th>$\Delta \sigma_{n2}$ (MPa)</th>
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</table>

Note: The results of several time analyses present the effect of varying non-prestressed steel in tensile zone $A_{n2}$ ($A_{n1} = 0$)

Table 5-4 The effects of varying non-prestressed reinforcements (tensile zone) on cross section (Case 15)
(This table is represented in Figure 5.7)
(a) \( M_s = 100\, kNm, A_{x1} = 0, A_{x2} = 0. \)

(b) \( M_s = 280\, kNm, A_{x1} = 0, A_{x2} = 0. \)

(c) \( M_s = 400\, kNm, A_{x1} = 0, A_{x2} = 0. \)

Strain Distribution \( \times 10^{-6} \)

Stress Distribution (MPa)

Figure 5.7 Stress and Strain distributions for Case 15
| $M_s$ (kNm) | $A_{s1}$ (mm$^2$) | $A_{s2}$ (mm$^2$) | $\epsilon_{\alpha}$ (x10$^{-6}$) | $k_i$ (x10$^{-6}$) (mm$^{-1}$) | $\Delta N$ (kN) | $\Delta M$ (kNm) | $\Delta \epsilon_{\alpha}$ (x10$^{-6}$) | $\Delta k$ (x10$^{-6}$) (mm$^{-1}$) | $\Delta \sigma_{\alpha}$ (MPa) | $\Delta \sigma_{\beta}$ (MPa) | $\Delta \sigma_{\tau}$ (MPa) | $\Delta \sigma_{\tau}$ (MPa) | $\Delta \sigma_{\gamma}$ (MPa) |
|---------|-----------------|-----------------|-----------------|-----------------|----------------|---------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 100     | 0.0000          | 0.0000          | -22.99          | -0.5018         | 1728           | -798.1        | 500.0           | -0.2722         | 1.738           | -4.469          | 0.9516          | 0.3193          | 0.0000          | 0.0000          |
|         | 0.5000          | 0.5000          | -23.10          | -0.5020         | 1726           | -798.0        | 435.8           | -0.3819         | 0.9516          | 4.179           | 90.98           | 81.55           | 147.4           | 163.5           |
|         | 1.0000          | 1.0000          | -23.19          | -0.5021         | 1723           | -797.8        | 384.2           | -0.4702         | 0.3193          | 3.946           | 90.98           | 81.55           | 147.4           | 163.5           |
| 280     | 0.0000          | 0.0000          | 195.2           | 0.0353          | 1750           | -681.7        | 851.1           | 0.5148          | 1.228           | -3.211          | 0.0000          | 0.0000          | 0.0000          | 0.0000          |
|         | 0.5000          | 0.5000          | 185.9           | 0.0186          | 1733           | -680.6        | 733.2           | 0.3134          | -1.512          | -2.769          | 143.5           | 126.4           | 131.5           | 133.6           |
|         | 1.0000          | 1.0000          | 177.2           | 0.0033          | 1718           | -679.6        | 639.6           | 0.1537          | -0.9759         | -2.430          | 126.4           | 104.9           | 131.5           | 133.6           |
| 400     | 0.0000          | 0.0000          | 340.6           | 0.3933          | 1764           | -604.2        | 1085            | 1.039           | 0.8874          | -2.373          | 0.0000          | 0.0000          | 0.0000          | 0.0000          |
|         | 0.5000          | 0.5000          | 325.1           | 0.3656          | 1738           | -602.4        | 931.5           | 0.777           | -0.6596         | -1.829          | 178.5           | 156.3           | 76.53           | 113.6           |
|         | 1.0000          | 1.0000          | 310.8           | 0.3402          | 1714           | -600.8        | 809.9           | 0.5697          | -1.839          | -1.419          | 156.3           | 76.53           | 113.6           | 113.6           |

Note: The results of several time analyses present the effect of varying compression steel $A_{s1}$ ($A_{s2} = 1000 mm^2$)

Table 5-5  The effects of varying non-prestressed reinforcements (compression zone) on cross section (Case 16)
(This table is represented in Figure 5.8)
(a) \( M_s = 100 \text{kNm}, A_{x1} = 500 \text{mm}, A_{x2} = 1000 \text{mm} \).

(b) \( M_s = 280 \text{kNm}, A_{x1} = 500 \text{mm}, A_{x2} = 1000 \text{mm} \).

(c) \( M_s = 400 \text{kNm}, A_{x1} = 500 \text{mm}, A_{x2} = 1000 \text{mm} \).

Figure 5.8 Stress and Strain distributions for Case 16
CHAPTER 6

CONCLUSIONS

The objective of this study has been to investigate the change of stresses and strains under non-linear temperature distribution in both statically determinate and indeterminate concrete bridges, the thermal gradient varying along the depth of section and the parameters affecting the temperature conditions have to be considered first.

In the second chapter, from the analysis of heat transfer and other research results on this topic, it was found that: there were several groups of parameters to be considered in temperature gradient. One is environment characteristics of the bridge location, such as solar radiation, air temperature, wind speed and nebulosity, altitude, latitude, time etc. The characteristics of the bridge, such as geometry of the cross-section, bridge orientation, asphalt thickness, thermal conductivity, density, specific heat, color, etc. are another group of parameters,

The temperature distribution proposed by Priestley (1976) which consisted of three individual parts was used in the studies of this thesis. In the first part, temperature were assumed to decrease non-linearly from a maximum of the top surface of the deck slab to minimum at a depth of 1,200 mm. The nonlinear variation was represented by a fifth-order parabola. The second part of the revised distribution applies only to a deck slab over an enclosed cell of box girder in which case temperature was assumed to decrease linearly. The third and final part of the revised distribution assumes a linear variation of temperatures over the bottom 200 mm of the cross-section. It had been found that this temperature design gradient had a very close agreement compared with the results from the heat-flow analyses.

In the third chapter, thermal response in a statically determinate bridge was discussed, and details of the analyses were given in case 1 and case 2. The procedure for deriving expressions for the self-equilibrating stresses based on the Euler-Bernoulli assumption consisted of the stresses in an artificially restrained structure, plus stresses
resulting from axial loads and bending moments, that would be required to remove the artificial restraints.

Thermal responses in a statically indeterminate bridge were described in chapter 4. Continuity stresses were introduced in thermal analyses due to the compatibility of the structure. To calculate the continuity stresses, a sufficient number of internal redundancies are first removed for the purpose of making the structure statically determinate. The inadmissible deformations induced at the locations of removed redundancies are then eliminated by the application of appropriate forces and moments. The stresses induced by this re-establishment of compatibility are known as continuity stresses.

The magnitude and distribution of the continuity stresses are, of course, dependent on the particular bridge structure and support conditions being analyzed. Nonetheless, the total state of stresses, which is obtained by the principle of superposition, is the sum total of the self-equilibrating and continuity stress sets. The results of four case studies indicated that the self-equilibrating stresses on the top surface were large while the cross-section was deeper, and so too were the maximum negative stresses. The difference between the tension and compression stresses are large in a deep section, while the continuity stresses were large in a shallow section.

In the fifth chapter, time dependent analyses on concrete bridge were investigated using age-adjusted effective modulus method, as well as thermal responses analyses. In the studies of case 10, the effect of creep was considered individually. It was shown that the tensile creep caused the tensile steel stress to increase with time and caused the tensile stress in the bottom concrete fibre to decrease. The increment of shrinkage strain caused the restraint internal force to increase, and then the change of the strain on the top surface and the change of curvature. Larger shrinkage strain can cause the bottom fibre to change from tension to compression. The quantity of both non-prestressed reinforcement $A_{nt}$ in the tension zone and compression zone $A_n$ affect the curvature and structure and stress distribution, and hence the deflection of the member with time.
REFERENCE


Appendix

Computer Program
PROGRAM MAIN

Parameter ( M = 150 )

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DIMENSION WM(M),DM(M),AM,M,Y(M),AREA(M),PHI(M),X(M),SH(M),
1 Ac(M),AcD(M),AcDD(M),AcCc(M),AcDe(M),AcDDe(M),
2 Ad(M),AdD(M),AdDD(M),TAd(M),
3 Apr(M),AprD(M),AprDD(M),AprCc(M),AprDe(M),TApr(M),TAprc(M),
4 Apo(M),ApoD(M),ApoDD(M),ApoCc(M),ApoDe(M),TApo(M),TApec(M),
5 As(M),AsD(M),AsDD(M),AsCc(M),AsDe(M),AsDDe(M),TAs(M),TAsc(M),
6 Hc(M),Hp(M),Hd(M),Hpo(M),Hpr(M),
7 Ec(M),Ee(M),Es(M),Epo(M),Epr(M),
8 RNc(M),RNs(M),RNP(M),RNpo(M),
9 RNec(M),RNes(M),RNep(M),RNepo(M),
1 Ppr(M),Ppo(M),
2 TAc(M),TBco(M),TLco(M),
3 ClO(M),Cl(M),
4 SIGMAY1(M),SIGMAY2(M),SIGMAS(M),SIGMAPR(M),SIGMAPO(M),
5 Rpr(M),Rpo(M),
6 dSTRESS(M),dSTRESSpr(M),dSTRESSpo(M),
7 Dy(M),DH(M),
8 deSic(M),deSigs(M),deSigpr(M),deSigpo(M),
9 Yi(M),Bi(M),Ai(M),Et(M),dNTc(M),dNTc(M),
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2 Sigco(M),Sigst(M),Siggr(M),Sigpo(M)

COMMON /INPUT/NNI,N,NJ,NK,NL,exNS,exMs,
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2 Ec,Ee,Ep,Epr,Ppr,Ppo,HH,Alphat,NS1,H,S2,Dns,nfo,Fileout

COMMON /ALL/SUMD,SA,SB,SI,TA,TB,TC,TAC,TACe,
1 Prk,PrkH,Pok,PokH,SuPpr,SuPprH,SuPpo,SuPpoH,
2 axSi,axMi,EPSIL,EPISLB,CURVA,
3 dSTRES1,dSTRES2,dSTRES3,dSTRES4,
4 delN1,delM1,delN,delLM,delE,delC,
5 SIGMAT,strE,strB,curV,T,A,Y,AREA,
6 Ac,AcD,AcDD,Acc,AcDe,AcDDe,Ad,AdD,AdDD,TAd,
7 Apr,Apri,ApriD,ApriDD,TApr,TAprc,
8 Apo,Apod,ApodD,TApo,TApec,
9 As,AsD,AsDd,AsDe,AsDDe,TAs,TAsc,Hc,Hd,Ee,
1 RNc,RNs,RNP,RNc,RNc,RNes,RNep,RNepo,
2 TAc,TBco,Tlco,Cl,CIO,
3 SIGMAY1,SIGMAY2,SIGMAS,SIGMAPR,SIGMAPO,
4 Rpr,Rpo,dSTRESS,dSTRESSpr,dSTRESSpo,
5 Dy,Dh,delH,deNT,deMT,deET,deCT,
6 deSic,deSigs,deSigpr,deSigpo,
7 Yi,bi,Ai,Et,deNTc,deMTc,
8 deNTs,deMTs,deNTpr,deMTpr,deNTpo,deMTpo,
9 Sigco,Sigst,Siggr,Sigpo

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CALL SHORT
CALL TERM
CALL T
CLOSE (nfo, Status="keep")
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    2 Ad(M),AdD(M),AdDD(M),TAd(M),
    3 Apr(M),AprD(M),AprDD(M),AprDe(M),AprDDe(M),TApr(M),TApr(M),
    4 Apo(M),ApoD(M),ApoDD(M),ApoDe(M),ApoDDe(M),TApb(M),TApb(M),
    5 As(M),AsD(M),AsDD(M),AsDe(M),AsDDe(M),TAs(M),TAsc(M),
    6 Hc(M),Hs(M),Hd(M),Hpo(M),HpM(M),
    7 Ec(M),Es(M),Eps(M),Epr(M),Ppr(M),Ppo(M),
    8 RNC(M),RNS(M),RNpr(M),RNec(M),RNes(M),RNpr(M),RNeM(M),
    9 TAc0(M),TBco(M),TLco(M),CI(M),DH(M),Rpr(M),Rpo(M)

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    1 W,D,PHI,X,SH,Ad,Apr,Apo,As,Hs,Hp,H,Apb,
    2 Ec,Es,Epo,Epr,Ppr,Ppo,HH,Alphat,NS1,NS2,Dns,nf,BFileout

* COMMON /INPUT/NN,NJ,NK,NL,
* 1 SUMD,SA,SB,SI,TA,TB,TI,exNs,exMs,EPSILT,CURVA,curve
* 2 W,D,AREA,PHI,X,SH,Ad,Apr,Apo,As,Hs,Hp,H,
* 3 Ec,Es,Epo,Epr,Ppr,Ppo,TAc0,TLco,TBco,CI,Rpr,Rpo,
* 4 DH,HH,Alphat,NS1,NS2,Dns

WRITE (6,10)
Read (5,'(A8)') Filename
WRITE (6,11)
Read (5,'(A8)') Fileout

nfo = 7
nf = 9
Open (nf, File=Filename, Status='old')
Read (nf,*) NN,NJ,NK,NL
Read (nf,*) (D (I), I = 1, NI)
Read (nf,*) (W (I), I = 1, NI)
Read (nf,*) (As (J), J = 1, NJ)
Read (nf,*) (Hs (J), J = 1, NJ)
Read (nf,*) (Apr(K), K = 1, NK)
Read (nf,*) (HpM(K), K = 1, NK)
Read (nf,*) (Apo(L), L = 1, NL)
Read (nf,*) (Hpo(L), L = 1, NL)
Read (nf,*) (Ad (L), L = 1, NL)
Read (nf,*) (Ec (N), N = 1, NN)
Read (nf,*) (Es (J), J = 1, NJ)
Read (nf,*) (Epr(K), K = 1, NK)
Read (nf,*) (Epo(L), L = 1, NL)
Read (nf,*) (PHI(N), N = 1, NN)
Read (nf,*) (X (N), N = 1, NN)
Read (nf,*) (SH (N), N = 1, NN)
Read (nf,*) exMs, exNs
Read (nf,*) (Ppr(K), K = 1, NK)
Read (nf,*) (Ppo(L), L = 1, NL)
Read (nf,*) Dns, Alphat, HH, NS1, NS2
Close (nf, status='keep')
return

98
C Following is the subroutine I-ELE

SUBROUTINE ILE

Parameter (M = 150)

Character Fileout*8
H1M,CH(M),AD(M),ADDD(M),TAD(M),
3 Ac(M),AcD(M),AcDD(M),TAC(M),
4 Apr(M),AprD(M),AprDD(M),TApr(M),
5 Apm(M),ApoD(M),ApoDD(M),TApM(M),
6 As(M),AsD(M),AsDD(M),TAs(M),
7 Hc(M),hs(M),Hpm(M),Hpr(M),
8 Ec(M),Ee(M),Es(M),Epm(M),
9 Rnc(M),RNs(M),Rnpr(M),Rnpo(M),
10 RNc(M),RNes(M),RNepr(M),RNe po(M),
11 Ppr(M),Ppo(M),
12 TACo(M),TBco(M),TicO(M),
13 CIO(M),CIM(M),
4 SIGMAY1(M),SIGMAY2(M),SIGMAS(M),SIGMAPR(M),SIGMAPO(M),
5 Rpr(M),Rpo(M),
6 dSTRESs(M),dSTRESpr(M),dSTRESpo(M),
7 Dy(M),DH(M),
8 deSigm(M),deSig(M),deSigpr(M),deSigpo(M),
9 Yi(M),Bi(M),Ai(M),EtM(M),CNt(M),dMTc(M),
10 dNTs(M),dMTs(M),dNTpr(M),dMTpr(M),dNTPo(M),dMTpo(M),
11 Sigco(M),Sigst(M),Sigr(M),Sigpo(M)

COMMON /INPUT/NN,NI,NJ,NK,NL,exNs,exMs,
2 W,D,PHI,X,SH,Ad,Apr,Apm,AAs,Hs,Hpo,Hpr,
3 Ec,Es,Epr,Epm,Ppr,Ppo,HH,Alphat,NS,NS2,Dns,nsFileout

COMMON /ALL/SUMD,SA,SB,SI,TA,TB,TI,TAC,TACe,
7 Prk,PrkH,Pok,PokH,SuPr,PuPrH,SuPpo,SuPpoH,
9 axNi,axM,SELT,ESILB,CURVA,
1 dSTRES1,dSTRES2,dSTRES3,dSTRES4,
3 delN1,delM1,delN,delLM,delE,delC,
4 SIGMAT,strE,strB,strV,T,A,Y,AREA,
1 Ac,Acd,AcDD,AcCe,Ad,AdD,AdDD,TA,d,
3 Apr,AdD,ApD,AdD,AdDD,TApr,TApre,
4 Apm,ApD,ApD,AdD,AdD,TApo,TApoc,
5 AsD,AsDD,AsDe,AsDD,TA,TAse,Hc,Hd,Ee,
7 Rnc,RNs,Rnpr,Rnpo,RNc,RNes,RNepr,RNepo,
1 TACo,TBco,TicO,CI,CIO,
4 SIGMAY1,SGMAY2,SGMAS,SGMAPR,SGMAPO,
6 Rpr,Rpo,dSTRESs,dSTRESpr,dSTRESpo,
7 Dy,DH,delH,deNT,deMT,deET,deCT,
8 deSigm,deSig,deSigpr,deSigpo,
9 Yi,Bi,Ai,ETM,deNTc,deMTc,
1 deNTs,deMTs,deNTpr,deMTpr,deNTpo,deMTpo,
2 Sigco,Sigst,Sigr,Sigpo
C Take the top surface as reference level
C The width of each element W(I)
C The depth of each element D(I) (Please reference to the figure)
C The area of each layer of non-prestressed steel: As(J)
C THE depth of each layer of non-prestressed steel to top surface: Hs(J)
C The area of each layer of pre-tensioned steel: Apr(K)
C The depth of each layer of pre-tensioned steel to top surface: Hpr(K)
C The area of each layer of post-tensioned steel: Apo(L)
C The depth of each layer of post-tensioned steel to top surface: Hpo(L)
C The area of the duct respectively: Ad(L)
C The depth of duct (post-tensioned) to top surface: Hd(L) (=Hpr(L))
C For composited section, the No. of composited element is NN=2,
C otherwise NN=1

C Section

C Subroutine CON1 is used to calculated the properties of concrete cross-section

CALL CON1

C The elastic modulus ratio RNc(N), RNs(J), RNpr(K)

C For concrete and steel

\[
\begin{align*}
E_a &= E_c(1) \\
\text{IF} \ (E_c(1), \text{EQ.0.0}) & \text{THEN} \ E_a = E_c(2) \\
\text{DO} \ 9, \ N=1,NN \\
& \text{RNc(N)=Ec(N)/Ea} \\
9 \ & \text{CONTINUE} \\
& \text{DO} \ 10, \ J=1,NJ \\
& \text{RNs(J)=Es(J)/Ea-1} \\
10 \ & \text{CONTINUE}
\end{align*}
\]

C For bonded pre-tensioned steel

\[
\begin{align*}
\text{DO} \ 12, \ K=1,NK \\
& \text{RNpr(K)=Epr(K)/Ea-1} \\
12 \ & \text{CONTINUE}
\end{align*}
\]

C The age-adjusted effective modulus of one of the concrete elements
C (say Ee(1)) is selected as the modulus of the age-adjusted transformed
C section. The concrete in all other elements is transformed into
C equivalent areas of the concrete of element 1 (Ee(1)) multiplying
C by age-adjusted modular ratio RNec(N). The bonded steel area is
C transformed by multiplying by RNes(J) & RNpr(K) or RNepo(L)

C The age-adjusted effective modulus of each concrete element Ee(N)

\[
\begin{align*}
\text{DO} \ 20, \ N=1,NN \\
& \text{Ee(N)=Ee(N)}/(1+\text{PHI(N)*X(N)}) \\
& \text{WRITE} \ (nfo,500) \\
& \text{WRITE} \ (nfo,510) \ Ee(N) \\
20 \ & \text{CONTINUE}
\end{align*}
\]
C The age-adjusted modular ratio RNe(N), RNes(J), Nepr(K), Nepo(L)

\[ \begin{align*}
\text{Eb} &= Ee(1) \\
\text{IF} \ (Ee(1), \text{EQ}, \ 0.0) \ \text{Eb} &= Ee(2) \\
\text{DO} \ 21, \ N=1, \text{NN} \\
\text{RNe(N)} &= Ee(N)/Eb
\end{align*} \]

21 CONTINUE

\[ \begin{align*}
\text{DO} \ 30, \ J=1, \text{NJ} \\
\text{RNes(J)} &= Ee(J)/Eb-1
\end{align*} \]

30 CONTINUE

\[ \begin{align*}
\text{DO} \ 40, \ K=1, \text{NK} \\
\text{RNepr(K)} &= Eepr(K)/Eb-1
\end{align*} \]

40 CONTINUE

\[ \begin{align*}
\text{DO} \ 50, \ L=1, \text{NL} \\
\text{RNepo(L)} &= Epo(L)/Eb-1
\end{align*} \]

50 CONTINUE

c output the ratio of elastic modulus of concrete

\[ \begin{align*}
\text{WRITE} \ (nfo,333) \\
\text{WRITE} \ (nfo,331) \ n, \ \text{rnc}(1), \ \text{rnc}(2) \\
\text{DO} \ 19, \ L=1, \text{NL} \\
\text{Hd(L)} &= \text{Hpo}(L)
\end{align*} \]

19 CONTINUE

*************** FOR SHORT-TERM ANALYSIS **************

C The properties of the transformed section about the top reference level
C The concrete area of each transformed element (deck & girder) is Ac(N)

\[ \text{TAC}=0.0 \]

C The sum area of the transformed concrete section is TAC

\[ \begin{align*}
\text{DO} \ 100, \ N=1, \text{NN} \\
\text{Ac(N)} &= \text{RNe(N)}*\text{AREA(N)} \\
\text{TAC} &= \text{TAC}+\text{Ac(N)}
\end{align*} \]

100 CONTINUE

C The first moment of transformed concrete section is: AcD(N)
C The secondary moment of transformed concrete section is: AcDD(N)

\[ \begin{align*}
\text{AcD1} &= 0.0 \\
\text{AcDD1} &= 0.0 \\
\text{DO} \ 99, \ N=1, \text{NN} \\
\text{AcD(N)} &= \text{AcD1} + \text{RNe(N)}*\text{AREA(N)}*Hc(N) \\
\text{AcDD(N)} &= \text{AcDD1} + \text{RNe(N)}*\text{CI(N)} \\
\text{AcD1} &= \text{AcD(N)} \\
\text{AcDD1} &= \text{AcDD(N)}
\end{align*} \]

99 CONTINUE

CALL SUM (NJ, RNs, As, Hs, TAS, ASD, ASDD)
CALL SUM (NK, RNpr, Apr, Hpr, TAPR, APRD, APRDD)
CALL SUM (NL, RNpo, Apo, Hpo, TAPO, APOD, APODD)
C If there is a part of post-tensioned steel, for the short-time analyses,
C the steel is not bonded to the concrete, the steel area dose not form
C a part of transformed section. The cross-sectional area of the
C hollow duct containing the unbonded tendons should be subtracted from
C the gross cross-section in the determination of the properties of transformed section.

\[
\begin{align*}
AD1 &= 0.0 \\
ADD1 &= 0.0 \\
ADDD1 &= 0.0 \\
DO 140, L=1, NL \\
TAD(L) &= AD1 + Ad(L) \\
ADD(L) &= ADD1 + Ad(L) * Hd(L) \\
ADDD(L) &= ADDD1 + Ad(L) * Hd(L) ** 2 \\
AD1 &= AD(L) \\
ADD1 &= ADD(L) \\
ADDD1 &= ADDD(L)
\end{align*}
\]

140 CONTINUE

************************** Short Time Analysis **************************

C We will consider that the cross-section is subjected to the following
C loading history:
C (1) I-section is cast and cured
C (2) I-section is post-tensioned with a prestressing force
C with the unbonded ducts
C (3) The post-tensions ducts are grouted

***** The following properties of concrete is just
***** at the transfer of prestress

C The post-tensioned steel is unbonded to the to the concrete, so there is not
C area of post-tensioned steel transfer to the concrete
C SA : Area of transformed section for short-term analysis
C SB : First moment of transformed section about top surface
C SI : Secondary moment of transformed section about top surface

\[
\begin{align*}
SA &= TAC + TAS (NJ) + TAPR (NK) - TAD (NL) \\
SB &= AcD(NN) + ASD (NJ) + APRD (NK) - ADD (NL) \\
SI &= AcDD(NN) + ASDD(NJ) + APRDD(NK) - ADDD(NL)
\end{align*}
\]

************************** Time Analysis **************************

C Properties about the top reference level
C The concrete area of each transformed element (deck & girder) is Ace(N)

TACE=0.0

C The sum area of the transformed concrete section (deck & girder) is TACE

DO 150 N=1,NN
Ace(N)=RNec(N)*AREA(N)
TACE=TACE+Ace(N)
150 CONTINUE

C The first moment of transformed concrete section is AcDe(N)
C The secondary moment of the transformed concrete section is AcDDe(N)

\[ \text{AcDe1} = 0.0 \]
\[ \text{AcDDe1} = 0.0 \]
DO 160, N = 1, NN
  \[ \text{AcDe(N)} = \text{AcDe1} + \text{RNec(N)} \times \text{AREA(N)} \times \text{Hc(N)} \]
  \[ \text{AcDDe(N)} = \text{AcDDe1} + \text{RNec(N)} \times \text{Cl(N)} \]
  \[ \text{AcDe1} = \text{AcDe(N)} \]
  \[ \text{AcDDe1} = \text{AcDDe(N)} \]
160 CONTINUE

CALL SUM (NJ, RNes, As, Hs, TAsc, AsDe, AsDDe)
CALL SUM (NK, RNepr, Apr, Hpr, TApr, AprDe, AprDDe)

C With the time being, the duct is grouted and the tendons are thereby
C bonded to the surrounding concrete, the properties of the transformed
C section must be changed for all subsequent analysis by the inclusion
C of the transformed area of the grouted tendons and area of the now
C solid duct

CALL SUM (NL, RNepo, Apo, Hpo, TApo, ApoDe, ApoDDe)

C TA : The area of transformed section for time analysis
C TB : The first moment of transformed section about top surface
C TI : The secondary moment of transformed section about top surface

\[ \text{TA} = \text{TACe} + \text{TAsc} \times \text{NJ} + \text{TApr} \times \text{NK} + \text{TApo} \times \text{NL} \]
\[ \text{TB} = \text{AcDe(N)} + \text{AsDe(N)} \times \text{NJ} + \text{AprDe(N)} \times \text{NK} + \text{ApoDe(N)} \times \text{NL} \]
\[ \text{TI} = \text{AcDDe(N)} + \text{AsDDe(N)} \times \text{NJ} + \text{AprDDe(N)} \times \text{NK} + \text{ApoDDe(N)} \times \text{NL} \]

C Following is the calculation of the properties of concrete section
C with grouted post-tensioned duct.
C
C For non-prestressed steels:

CALL FCON (NJ, As, Hs, D, Asc1, AscH1, AsCH1, Asc2, AsCH2, AschH2)

C For pre-tensioned steels:

CALL FCON (NK, Apr, Hpr, D, Aprc1, AprcH1, AprcHH1, Aprc2, AprcH2, AprcHH2)

C For post-tensioned steels:

CALL FCON (NL, Apo, Hpo, D, Apoc1, ApocH1, ApocHH1, Apoc2, ApocH2, ApocHH2)
C For concrete section:

    Arc1 = 0.0
    ArBc1 = 0.0
    Arlc1 = 0.0
    Arc2 = 0.0
    ArBc2 = 0.0
    Arlc2 = 0.0
DO 803 N=1,NN
  IF ( Hc(N) .LE. D(1)/2. ) THEN
    Arc1 = Arc1 + AREA(N)
    ArBc1 = ArBc1 + AREA(N)*Hc(N)
    Arlc1 = Arlc1 + CI(N)
  ELSE IF ( Hc(N) .GT. D(1)/2. ) THEN
    Arc2 = Arc2 + AREA(N)
    ArBc2 = ArBc2 + AREA(N)*Hc(N)
    Arlc2 = Arlc2 + CI(N)
  ENDIF
803  CONTINUE

C The properties of concrete section:
C
C For element 1: TAc01, TBco1, TLco1
C The area of element 1: TAc01
C The first moment of element 1: TBco1
C The secondary moment of element 1: TLco1

    TAc01(1) = Arc1 - Asc1 - Aprc1 - Apoc1
    TBco1(1) = ArBc1 - Asch1 - Aprch1 - ApocH1
    TLco1(1) = Arlc1 - AschH1 - AprchH1 - ApocH1

C For element 2: TAc02, TBco2, TLco2
C The area of element 2: TAc02
C The first moment of element 2: TBco2
C The secondary moment of element 2: TLco2

    TAc02(2) = Arc2 - Asc2 - Aprc2 - Apoc2
    TBco2(2) = ArBc2 - Asch2 - Aprch2 - ApocH2
    TLco2(2) = Arlc2 - AschH2 - AprchH2 - ApocH2

C output the No. of element, area and the depth of central axis to top
C surface of each element

    WRITE (nfo, 200)
    DO 111, N = 1, NN
    WRITE (nfo,201) N, AREA(N), Hc(N)
111  CONTINUE

C output the No. of layer of steel, area and depth to top surface

    WRITE (nfo,210)
    DO 112, J = 1, NJ
    WRITE (nfo, 211) J, As(J), Hs(J)
112  CONTINUE
C output the No. of prestressed steel, area and depth to top surface

    WRITE (nfo,220)
    DO 113, K = 1, NK
        WRITE (nfo, 233) K, Apr(K), Hpr(K)
    113 CONTINUE

C output the No., area and depth of each layer of post-tensioned steel

    WRITE (nfo,230)
    DO 114, L = 1, NL
        WRITE (nfo, 222) L, Apo(L), Hpo(L)
    114 CONTINUE

C output the properties of transformed section for short-term analysis

    WRITE (nfo,240)
    WRITE (nfo,241) SA, SB, SI

C output the properties of transformed section for time analysis

    WRITE (nfo,250)
    WRITE (nfo,241) TA, TB, TI

C output the properties of the concrete section for time analysis

C For element 1:

    WRITE (nfo,251)
    WRITE (nfo,241) TAco(1), TBco(1), TIco(1)

C For element 2:

    WRITE (nfo,252)
    WRITE (nfo,241) TAco(2), TBco(2), TIco(2)

RETURN

200 FORMAT (4X, 'ELEMENT' = N AREA(N) Hc(N))
201 FORMAT (14X, I2, 2 (2X, E10.4))
210 FORMAT (2X, 'NO-PRESTRESSED STEEL' = J As(J) Hs(J))
211 FORMAT (25X, I2, 2 (2X, E10.4))
220 FORMAT (5X, 'PRESTRESSED STEEL' = K Apr(K) Hpr(K))
222 FORMAT (25X, I2, 2 (2X, E10.4))
230 FORMAT (1X, 'POST-TENSIONED STEEL' = L Apo(L) Hpo(L))
233 FORMAT (25X, I2, 2 (2X, E10.4))
240 FORMAT (2X, 'SHORT TERM' = SA SB SI)
241 FORMAT (15X, 3 (2X, E10.4))
250 FORMAT (2X, 'Term = TA TB TI')
251 FORMAT (2X, 'Termc1 = TAco(1) TBco(1) TIco(1))
252 FORMAT (2X, 'Termc2 = TAco(2) TBco(2) TIco(2))
333 FORMAT (2X, 'CONCRETE RATIO' = N Rnc(1) Rnc(2))
331 FORMAT (17X, I2, 2(2X, E10.4))
500 FORMAT (2X, 'pls output the Ee(N)')
510 FORMAT (2X, E10.4)

END
SUBROUTINE SUM (N, R1, R2, R3, R11, R12, R13)

PARAMETER ( M = 150 )

DIMENSION R1(M), R2(M), R3(M), R11(M), R12(M), R13(M)
A = 0.
B = 0.
C = 0.
DO 10 I = 1, N
   A = A + R1(I) * R2(I)
   B = B + R1(I) * R2(I) * R3(I)
   C = C + R1(I) * R2(I) ** 2
R11(I) = A
R12(I) = B
R13(I) = C
10  CONTINUE
RETURN
END

SUBROUTINE FCON (N,R1,R2,D,A1,B1,C1,A2,B2,C2)
PARAMETER ( M = 150 )
DIMENSION R1(M),R2(M),D(M)

A1 = 0.0
B1 = 0.0
C1 = 0.0
A2 = 0.0
B2 = 0.0
C2 = 0.0
DO 10 I = 1, N
   IF (R2(I).LE.D(I)) THEN
      A1 = A1 + R1(I)
      B1 = B1 + R1(I) * R2(I)
      C1 = C1 + R1(I) * R2(I)**2
   ELSE IF (R2(I).GT.D(I)) THEN
      A2 = A2 + R1(I)
      B2 = B2 + R1(I) * R2(I)
      C2 = C2 + R1(I) * R2(I)**2
   ENDIF
10  CONTINUE
RETURN
END

{PAGE[93]}
SUBROUTINE SHORT

In this program, short time responses of concrete bridge
is calculated.
If the elastic modulus of elements are different,
The stresses distribution of whole section is non-linear,
It is necessary to calculate the stresses of top and bottom
surface for each element.

The depth of the acting point of the resultant axial force from
top surface: Dns
The prestressed force at the k-th level of pre-stressed steel: Ppr(K)
The prestressed force at the l-th level of post-tensioned steel: Ppo(L)
The summation of the prestressed force after transfer: SuPpr
The summation of the prestressed force after transfer: SuPpr
The resultant axial force on the transformed section: axNi
The resultant initial moment about the top of the section: axMi
The initial top fibre strain: EPSILT
The initial curvature: CURVA
The initial bottom strain: EPSILB
The stresses of the top surface for the I-th element: SIGMAY1(I)
The stresses of the bottom surface for the I-th element: SIGMAY2(I)
The stresses in the non-pre-stressed steels immediately after
transfer: SIGMAS(I)
The stresses in the pre-stressed steels: SIGMAPR(K)
The stresses in the unbonded tendons: SIGMAPO(L)

Parameter (M = 150)

Character Fileout*8

DIMENSION W(M),D(M),A(M),Y(M),AREA(M),PHI(M),X(M),SH(M),
1 Ac(M),AcD(M),AcDD(M),Ace(M),AcDe(M),AcDDe(M),
2 Ad(M),AdD(M),AdDD(M),TAd(M),
3 Apr(M),AprD(M),AprDD(M),AprDe(M),AprDDe(M),TApr(M),TAprM,
4 Apo(M),ApoD(M),ApoDD(M),ApoDe(M),ApoDDe(M),TAp(o,M),TAp(o,M),
5 As(M),AsD(M),AsDD(M),AsDe(M),AsDDe(M),TAs(M),TAs(M),
6 Hc(M),Hs(M),Hd(M),Hpo(M),Hpr(M),
7 Ec(M),Ee(M),Es(M),Epo(M),Epr(M),
8 RNC(m),RNS(m),RNpr(m),RNPo(m),
8 RNec(M),RNeM,REnpr(M),RNePo(M),
1 Ppr(M),Ppo(M),
2 TACO(M),TCbo(M),TLCo(M),
3 CLO(M),Cl(M),
4 SIGMAY1(M),SIGMAY2(M),SIGMAS(M),SIGMAPR(M),SIGMAPO(M),
5 Rpr(M),Rpo(M),
6 DSTRESS(M),dSTRESPr(M),dSTRESpo(M),
7 DH(M),DPO(M),
8 deSigc(M),deSigm(M),deSigpr(M),deSigo(M),
9 Yi(M),Bi(M),Ai(M),ETi(M),dETc(M),dMETc(M),
1 deNts(M),deMTs(M),deNtp(M),dEMPr(M),dEMTo(M),dEMTPo(M),
2 Sigco(M),Sigc(M),Sigpr(M),Sigpo(M)

COMMON /INPUT/NN,NI,NJ,NK,NL,exNs,exMs,
2 W,D,PHI,X,SH,Ad,Ap,As,Hs,Hpo,Hpr,
3 Ec,Es,Epo,Epr,Ppr,Ppo,HH,Alpha,NS1,NS2,Dns,info,Fileout
COMMON /ALL/SUMD,SA,SB,SI,TA,TB,TL,TAC,TACe,

7 Prk,PrkH,Pok,PokH,SuPpr,SuPprH,SuPpo,SuPpoH,
9 axNi,axMi,EPSI,L,T,EPSILB,CURV,
1 dSTRES1,dSTRES2,dSTRES3,dSTRES4,
3 delN1,delM1,delN,delLM,delE,delC,
4 SIGMAT,straE,straB,curvT,A,Y,AREA,
1 Ac,AcD,AcDD,AcCe,AcDe,AcDDe,AdD,AdDD,TAd,
3 AprD,AprDD,ApresD,ApDDe,TApr,TAprE,
4 ApoD,ApoDD,ApoDe,ApoDDe,TApoe,TApo,
5 AsD,AsDD,AsDe,AsDDe,TAs,TAsD,He,Hd,Ec,
7 RNc,RNs,RNpr,RNpo,RNec,RNes,RNepr,RNepo,
1 TAc0,TBco,Tico,Ci,CiO,
4 SIGMAY1,SIGMAY2,SIGMAS,SIGMPR,SIGMAPO,
6 Rpr,Rpo,dSTRESS,dSTRESpr,dSTRESpo,
7 Dy,Ddeh,deNT,deMT,delET,delCT,
8 deSige,deSigs,deSigpr,deSigpo,
9 Yi,Bi,Ai,EtI,deNTc,deMTC,
1 deNTs,deMTs,deNTpr,deMTpr,deNTpo,deMTpo,
2 Sigco,Sigst,Sigpr,Sigpo

C The pre-tensioned steels:
Prk = 0.0
PrkH = 0.0

DO 10, K =1,NK
   Prk = Prk + Ppr(K)
   PrkH = PrkH + Ppr(K) * Hpr(K)
10 CONTINUE
SuPpr = Prk
SuPprH = PrkH

C The post-tensioned steels:
Pok = 0.0
PokH = 0.0

DO 12, L =1,NL
   Pok = Pok + Ppo(L)
   PokH = PokH + Ppo(L) * Hpo(L)
12 CONTINUE
SuPpo = Pok
SuPpoH = PokH

axNi = exNs + SuPpr + SuPpo

axMi = exMs - exNs * Dns - SuPprH - SuPpoH
The initial top fibre strain and the curvature are showed as followed

Please reference to Eqn. 5.6 & 5.7

\[ Ed = Ec(1) \]
\[ IF (Ec(1), EQ. 0.0) Ed = Ec(2) \]
\[ EPSILT = (SB * axMi + SI * axNi)/(Ed * (SA * SI - SB ** 2)) \]
\[ CURVA = (SA*axMi+SB*axNi)/(Ed*(SA*SI-SB**2)) \]
\[ EPSILB = (EPSILT - SUMD * CURVA) \]

The stresses are showed as followed

The fibre concrete stresses

\[ SIGMAT = Ed * EPSILT \]

In element 1 (In-situ reinforced concrete deck)

\[ SIGMAY1(I) = Ec(1)*EPSILT \]
\[ SIGMAY2(I) = Ec(1)*(EPSILT-D(I)*CURVA) \]

In element 2 (Precast prestressed-concrete girder)

\[ Dy1 = 0.0 \]
\[ DO 30, I=1,NI \]
\[ Dy(I) = Dy1 + D(I) \]
\[ Dy1 = Dy(I) \]

CONTINUE

\[ DO 33, J=2,NJ \]
\[ SIGMAY1(I) = Ec(2) * (EPSILT - Dy(I-1)*CURVA) \]
\[ SIGMAY2(I) = Ec(2) * (EPSILT - Dy(I) * CURVA) \]

CONTINUE

\[ DO 40 J=1,NJ \]
\[ SIGMAS(J) = Es(J) * (EPSILT - Hs(J) * CURVA) \]

CONTINUE

\[ DO 50 K=1,NK \]
\[ SIGMAPR(K) = Epr(K) *EPSILT - Hpr(K) * CURVA \]

CONTINUE

\[ DO 60 L=1,NL \]
\[ SIGMAPO(L) = - Ppo(L) / Apo(L) \]

CONTINUE

output the strains of the top and bottom surface: EPSILT & EPSILB

WRITE(nfo,100)
WRITE(nfo,101) EPSILT, EPSILB
WRITE (nfo,102)
WRITE(nfo,103) CURVA
output the stresses of concrete: SIGMAY1(I), SIGMAY2(I)

WRITE(nfo,110)
DO 111, I=1,NI
   WRITE(nfo,116) I, SIGMAY1(I), SIGMAY2(I)
111 CONTINUE

output the stresses of non-prestressed steels:

WRITE(nfo,120)
DO 125, J=1,NJ
   WRITE(nfo,126) SIGMAS(J)
125 CONTINUE

output the stresses in the pre-tensioned steels:

WRITE(nfo,130)
DO 135, K=1,NK
   WRITE(nfo,136) SIGMAPR(K)
135 CONTINUE

output the stresses in the unbonded post-tensioned steels:
(prior to the grouting operation)

WRITE(nfo,140)
DO 145, L=1,NL
   WRITE(nfo,146) SIGMAPO(L)
145 CONTINUE

RETURN

100 FORMAT (1X, 'CONCRETE STRAIN= EPSILT  EPSILB')
101 FORMAT (5X, 2X,E10.4,2X,E10.4)
102 FORMAT (1X, 'CURVATURE = CURVA ')
103 FORMAT (15X, E10.4)
110 FORMAT (1X, 'CONCRETE STRESSES= SIGMAY1(I) SIGMAY2(I)'
116 FORMAT (18X, I2, 2(2X,E10.4))
120 FORMAT (' NON-PRESTRESSED STEEL= SIGMAS(J)'
126 FORMAT (23X, E10.4)
130 FORMAT (' PRE-TENSIONED STEEL= SIGMAPR(K)'
136 FORMAT (23X, E10.4)
140 FORMAT (' POST-TENSIONED STEEL= SIGMAPO(L)'
146 FORMAT (23X, E10.4)
END
SUBROUTINE TERM

In this subroutine, time-dependent responses of concrete bridge
are calculated.

We consider that the composite section is included:
(1) element 1: In-situ reinforced concrete deck;
(2) element 2: Prestressed I-section (with bonded tendons).

The restraining actions, which are required to prevent the free
development of creep and shrinkage in each concrete element and
stresses relaxation in the prestressed steel, "DELTAS N" & "DELTAS M".

The change of stresses at a point of depth D below the
top fibre is equal to the sum of the stress loss due to
relaxation of the age-adjusted transformed section, when
creep and shrinkage are fully restrained, and the stress
which results when "DELTAS N" & "DELTAS M" are applied to
the cross-section

deHE : time-dependent change in the top fibre strain
deHC : time-dependent change in the top fibre curvature
strasE : the total strain of the top fibre strain
strasB : the total strain of the bottom fibre strain
curvT : the total curvature of the top fibre

Change of stress:

dSTRES1: in the top of the first element
dSTRES2: in the bottom of the first element
dSTRES3: in the top of the second element
dSTRES4: in the bottom of the second element
dSTRESs(I): in the non-prestressed steels
dSTRESpr(K): in the pre-tensioned steels
dSTRESpo(L): in the post-tensioned steels

PARAMETER ( M = 150 )

Character Fileout*8

DIMENSION W(M),D(M),A(M),Y(M),AREA(M),PHI(M),X(M),SH(M),
1 Ac(M),AcD(M),AcDD(M),Ace(M),AcDe(M),AcDDe(M),
2 Ad(M),AdD(M),AdDD(M),TAd(M),
3 Apr(M),AprD(M),AprDD(M),AprDe(M),AprDDe(M),TApr(M),TApr(M),
4 Apo(M),ApoD(M),ApoDD(M),ApoDe(M),ApoDDe(M),TAp(M),TApoe(M),
5 As(M),AsD(M),AsDD(M),AsDe(M),AsDDe(M),TAs(M),TAs(M),
6 Hc(M),Hc(M),Hd(M),Hpo(M),Hpr(M),
7 Ec(M),Ec(M),Es(M),Epo(M),Epr(M),
8 Rnc(M),Rns(M),Rnp(M),RNpo(M),
8 RNe(M),RNe(M),RNe(M),RNeo(M),
1 Ppr(M),Ppo(M),
2 Ta(M),TBo(M),Tco(M),
3 CiO(M),Ci(M),
4 SIGMAY1(M),SIGMAY2(M),SIGMAS(M),SIGMAE(M),SIGMAPO(M),
5 Rpr(M),Rpo(M),
6 DSTRESSs(M),DSTRESSpr(M),DSTRESSpo(M),
7 Dy(M),DH(M),
8 deSigm(M),deSigs(M),deSigm(M),deSigpo(M),
9 Yi(M),Bi(M),Ai(M),ETi(M),deNTc(M),deMTc(M),
The restraining actions which are required to prevent the free development of creep and shrinkage in each concrete element:

\[
\begin{align*}
\text{delN1} &= 0.0 \\
\text{delM1} &= 0.0 \\
\text{DO } 30 & \quad N=1,NN \\
\text{delN1} &= \text{delN1} + \text{Ee(N)}*(\text{PHI(N)}*(\text{TACo(N)}*\text{EPSILT-TCbo(N)}*\text{CURVA}) \\
& \quad + \text{SH(N)}*\text{TACo(N)}) \\
\text{delM1} &= \text{delM1} + \text{Ee(N)}*(\text{PHI(N)}*(-\text{TCbo(N)}*\text{EPSILT+TCo(N)}*\text{CURVA}) \\
& \quad - \text{SH(N)}*\text{TCbo(N)}) \\
\end{align*}
\]

CONTINUE

The calculation of the stress relaxation in prestressed steel (pre-tensioned and post-tensioned)

In the pre-tensioned steel

\[
\begin{align*}
\text{RELpr} &= 0.0 \\
\text{RELprD} &= 0.0 \\
\text{DO } 31 & \quad K=1,NK \\
\text{RELpr} &= \text{RELpr} + \text{Rpr(K)} \\
\text{RELprD} &= \text{RELprD} + \text{Rpr(K)}*\text{Hpr(K)} \\
\end{align*}
\]

CONTINUE

In the post-tensioned steel

\[
\begin{align*}
\text{RELpo} &= 0.0 \\
\text{RELpoD} &= 0.0 \\
\text{DO } 32 & \quad L=1,NL \\
\text{RELpo} &= \text{RELpo} + \text{Rpo(L)} \\
\end{align*}
\]
RELpoD = RELpoD + Rpo(L)*Hpo(L)
CONTINUE

C The total relaxation is

REL = RELpr + RELpo
RELD = RELprD + RELpoD

C So the restraining actions " DELTA N " & " DELTA M ":

delN = delN1 - REL
delM = delM1 + RELD

C If the cross section is singly cross section Eg=Ee(2)
C For composite section Eg=Ee(1)

Eg = Ee(1)
IF (Ee(1).EQ.0.0) Eg = Ee(2)

delE = (TB*delM+TI*delN) / ((Eg*(TA*TI-TB**2))
delC = (TA*delM+TB*delN) / (Eg*(TA*TI-TB**2))
straE = EPSILT + delE
straB = (EPSILT+delE) - SUMD * (CURVA+delC)
curvT = CURVA + delC

C Change of stresses:

  dSTRES1 = -Ee(1) *( PHI(1)*EPSILT + SH(1) - delE )
dSTRES2 = -Ee(1) *( PHI(1)*EPSILT-D(1)*CURVA)+ SH(1)
  - (delE-D(1)*delC )
dSTRES3 = -Ee(2)*(PHI(2)*(EPSILT-D(1)*CURVA)+SH(2)
  - (delE-D(1)*delC)
dSTRES4 = -Ee(2)*(PHI(2)*(EPSILT-SUMD*CURVA)+SH(2)
  - (delE-SUMD*delC)

C The stress change with time in the non-prestressed steels

DO 1 J=1,NJ
dSTRESs(J) = Es(J) * ( delE - Hs(J)*delC )
CONTINUE

1 C

C The stress change with time in the pre-tensioned steels

DO 2 K=1,NK
dSTRESpr(K) = Epr(K) * ( delE - Hpr(K)*delC ) + Rpr(K)/Apr(K)
CONTINUE

2 C

C The stress change with time in the post-tensioned steels

DO 3 L=1,NL
dSTRESpo(L) = Epo(L) * ( delE - Hpo(L)*delC ) + Rpo(L)/Apo(L)
CONTINUE

3 C

output the restraining actions " DELTA N " & " DELTA M "

WRITE(nfo,10)
WRITE (nfo,11) delN
WRITE(nfo,12)
WRITE (nfo,13) delM
output the time-dependent change in the top fibre strain
WRITE (nfo,14)
WRITE (nfo,15) delE
output the time-dependent change in the top fibre curvature
WRITE (nfo,16)
WRITE (nfo,17) delC
output the time-dependent change in the bottom fibre strain
WRITE (nfo,36)
WRITE (nfo,37) straE
WRITE (nfo,34)
WRITE (nfo,35) straB
WRITE (nfo,38)
WRITE (nfo,39) curvT
output the change of concrete stresses at a depth D below
the top fibre: dSTRES1, dSTRES2, dSTRES3, dSTRES4
WRITE (nfo,18)
WRITE (nfo,19) dSTRES1,dSTRES2,dSTRES3,dSTRES4
output the change of stresses in non-prestressed steels
WRITE (nfo,20)
DO 21, J=1,NJ
   WRITE (nfo,22) dSTRESs(J)
21 CONTINUE
output the change of stresses in pre-tensioned steels
WRITE (nfo,23)
DO 24 K=1,NK
   WRITE (nfo,25) dSTRESpr(K)
24 CONTINUE
output the change of stresses in post-tensioned steels
WRITE (nfo,26)
DO 27 L=1,NL
   WRITE (nfo,28) dSTRESpo(L)
27 CONTINUE
RETURN

FORMAT (3X, 'the restraining force "delta N"')
FORMAT (2X, E10.4)
FORMAT (3X, 'the restraining moment "delta M"')
FORMAT (2X, E10.4)
FORMAT (3X, 'the time dependent change in the top fibre strain')
FORMAT (2X, E10.4)
FORMAT (3X, 'time dependent change in the top fibre curvature')
FORMAT (2X, E10.4)
FORMAT (3X, 'the change of stresses in concrete')
FORMAT (4 (2X, E10.4))
FORMAT (3X, 'the change of stresses in non-prestressed steels')
FORMAT (2X, E10.4)
FORMAT (3X, 'the change of stresses in pre-tensioned steels')
FORMAT (2X, E10.4)
FORMAT (3X, 'the change of stresses in post-tensioned steels')
FORMAT (2X, E10.4)
FORMAT (3X, 'the total strain of the bottom fibre')
FORMAT (2X, E10.4)
FORMAT (3X, 'the total strain of the top fibre')
FORMAT (2X, E10.4)
FORMAT (3X, 'the total curvature of the top fibre')
FORMAT (2X, E10.4)

END
SUBROUTINE CON1

C This program is used to calculate the properties of cross section

C Consider a composite section which is composed of
C in situ concrete slab deck (element 1) and precast prestressed
C concrete girder (element 2)

C Concrete slab cross section dimension
C slab width: W(1)
C slab depth: D(1)

C Prestressed beam cross-section
C top flange width: W(2)
C top flange depth: D(2)

C depth of the portion of splaged top flange: D(3)

C web width: W(3)
C web depth: D(4)

C depth of the portion of splaged bottom flange: D(5)

C depth of bottom flange: W(4)
C width of bottom flange: D(6)

C This program also can be used for the following sections:

C (1) Rectangular section:
C in which, the weight and depth of the section are considered
C AS D2 & W2, AND W1 = W3 = W4 = 0, D1 = D3 = D4 = D5 = D6 = 0.

C (2) I section:
C there is only concrete girder (i.e. element 2), and W1=D1=0.0

C (3) T section:
C in this case, we consider that W3 = W4.

C Consider that the cross section is combined by several elements,
C area and depth from top surface to central axis of each element are
C A(I) AND Y(I) respectively

C From this program, it can be obtained

C Area of slab (element 1): AREA(1)
C Hc(1) : depth to central axis for element 1 from top surface
C Area of prestressed beam (element 2): AREA(2)
C Hc(2) : depth to central axis for element 2 from top surface
C
C ClO(1) : the secondary moment of element 1 about central axis
C Cl(1) : the secondary moment of element 1 about top surface
C Cl(2) : the secondary moment of element 2 about central axis
C ClO(2) : the secondary moment of element 2 about top surface
Parameter (M = 150)

Character Fileout*8
DIMENSION W(M),D(M),A(M),Y(M),AREA(M),PHI(M),X(M),SH(M),
1 Ac(M),AcD(M),AcDD(M),Ace(M),AcDe(M),AcDDDe(M),
2 Ad(M),AdD(M),AdDD(M),TAd(M),
3 Apr(M),AprD(M),AprDD(M),AprDe(M),AprDDDe(M),TApr(M),TAprre(M),
4 Apo(M),ApoD(M),ApoDD(M),ApoDe(M),ApoDDDe(M),TPapo(M),TPapoe(M),
5 As(M),AsD(M),AsDD(M),AsDe(M),AsDDDe(M),TAs(M),TAsre(M),
6 Hc(M),Hs(M),Hd(M),Hpo(M),Hpr(M),
7 Ec(M),Ee(M),Es(M),Epo(M),Epr(M),
8 Rnc(M),RNs(M),RNP(M),RNpo(M),
9 RNec(M),RNes(M),RNPre(M),RNPeo(M),
1 Ppr(M),Ppo(M),
2 TAco(M),TBCo(M),TLco(M),
3 CIO(M),C1(M),
4 SIGMAY1(M),SIGMAY2(M),SIGMAS(M),SIGMAPR(M),SIGMAPO(M),
5 Rpr(M),Rpo(M),
6 dSTRESs(M),dSTRESpr(M),dSTRESpo(M),
7 Dy(M),DH(M),
8 dSigc(M),dSigs(M),dSigpr(M),dSigpo(M),
9 Yi(M),Bi(M),Ai(M),ETi(M),dENNc(M),dENTc(M),
1 deNTs(M),deMTs(M),deNTpr(M),deMTpr(M),deNTpo(M),deMTpo(M),
2 Sigco(M),Sigst(M),Sigpr(M),Sigpo(M)

COMMON /INPUT/,NN,NI,NJ,NK,NL,EXNs,EXMs,
2 W,D,PHI,X,SH,Ad,Apr,Apo,As,hs,Hpo,Hpr,
3 Ec,Es,Epo,Epr,Ppr,Ppo,HH,Alphat,NS1,NS2,Dns,nfo,Fileout

COMMON /ALL/,SUMD,SA,SB,SI,TA,TA,B,TI,TAC,TACC,
7 Prk,PrkH,Pok,H,PokH,SUPr,SUPrH,SUPpo,SUPpoH,
9 axNi,axMi,EPSILT,EPISLB,CURVA,
1 dSTRES1,dSTRES2,dSTRES3,dSTRES4,
3 delN1,delM1,delN,delLM,delE,delC,
4 SIGMAT,STRA,STRA,STRA,STRA,STRA,
1 Ac,AcD,AcDD,Acce,AcDe,Ad,AdDD,AdDD,AdDDD,AdDDDe,AdDDDe,
3 AprD,AprrD,AprDe,AprrDe,AdApr,AdAprr,
4 ApoD,ApoDD,ApoDe,ApoDDDe,ApoDDDe,ApoDDDe,
5 AsD,AsDD,AsDe,AsDDDe,AsDDDe,AsDDDe,
7 RNc,RNn,RNnp,RNpo,RNec,RNes,RNPre,RNPeo,
1 TAco,TBCo,TLco,CIO,
4 SIGMAY1,SIGMAY2,SIGMAS,SIGMAPR,SIGMAPO,
6 Rpr,Rpo,dSTRESs,dSTRESpr,dSTRESpo,
7 Dy,DH,delH,deNT,deMT,deNTc,deMTc,
8 dSigc,dSigs,dSigpr,dSigpo,
9 Yi,Bi,Ai,ETi,deNTc,deMTc,
1 deNTs,deMTs,deNTpr,deMTpr,deNTpo,deMTpo,
2 Sigco,Sigst,Sigpr,Sigpo

SUMD = 0.0
DO 5 I = 1, NI
    SUMD = SUMD + D(I)
    DH(I) = SUMD
5 CONTINUE
C The area of each element is

\[
\begin{align*}
A(1) &= W(1) \cdot D(1) \\
A(2) &= W(2) \cdot (D(2) + D(3)) \\
A(3) &= -1/2 \cdot (W(2) - W(3)) \cdot D(3) \\
A(4) &= W(3) \cdot D(4) \\
A(6) &= W(4) \cdot (D(5) + D(6)) \\
A(5) &= -1/2 \cdot (W(4) - W(3)) \cdot D(5)
\end{align*}
\]

C Depth from top surface to central axis is

\[
\begin{align*}
Y(1) &= D(1)/2 \\
Y(2) &= D(1) + 1/2 \cdot (D(2) + D(3)) \\
Y(3) &= D(1) + D(2) + 2 \cdot D(3) \\
Y(4) &= D(1) + D(2) + D(3) + 1/2 \cdot D(4) \\
Y(5) &= SUMD - D(6) - 2 \cdot D(3) \\
Y(6) &= SUMD - 1/2 \cdot (D(5) + D(6))
\end{align*}
\]

\[
\begin{align*}
\text{AREA}(1) &= A(1) \\
Hc(1) &= Y(1)
\end{align*}
\]

\[
\begin{align*}
\text{AREA2} &= 0.0 \\
A2Y2 &= 0.0 \\
\text{DO 11 I=2,NI}
\end{align*}
\]

\[
\begin{align*}
\text{AREA}(2) &= \text{AREA2} + A(I) \\
A2Y2 &= A2Y2 + A(I) \cdot Y(I) \\
\text{AREA2} &= \text{AREA}(2)
\end{align*}
\]

\[
\begin{align*}
1 \text{ CONTINUE}
\end{align*}
\]

\[
\begin{align*}
Hc(2) &= A2Y2 / \text{AREA}(2) \\
CIO(1) &= 1/12 \cdot W(1) \cdot D(1)**3 \\
CI(1) &= CI(1) + \text{AREA}(1) \cdot Hc(1)**2 \\
CIO(2) &= 1/12 \cdot (W(2) \cdot (D(2) + D(3))**3 + W(3) \cdot D(4)**3 + \\
& \quad \cdot W(4) \cdot D(5)**3) - 1/36 \cdot (W(2) - W(3)) \cdot D(3)**3 + \\
& \quad \cdot (W(4) - W(3)) \cdot D(5)**3 \\
CI(2) &= CI(2) + A(2) \cdot Y(2)**2 + A(3) \cdot Y(3)**2 + A(4) \cdot Y(4)**2 + \\
& \quad A(5) \cdot Y(5)**2 + A(6) \cdot Y(6)**2
\end{align*}
\]

C output the width, depth, area and depth of central axis of each element

\[
\begin{align*}
\text{WRITE}(nfo,12) \\
\text{DO 10 I=1,NI}
\end{align*}
\]

\[
\begin{align*}
\text{WRITE}(nfo,*) \quad 1, W(I), D(I), A(I), Y(I) \\
10 \text{ CONTINUE}
\end{align*}
\]

\[
\begin{align*}
\text{WRITE} (nfo,700) \\
\text{WRITE} (nfo,710) \text{ SUMD}
\end{align*}
\]

C output the area, depth to central axis from top surface,

C the secondary moment about top surface of deck & girder element

C (element 1 & 2)

\[
\begin{align*}
\text{DO 33, N=1,NN} \\
\text{WRITE} (nfo,32) \\
\text{WRITE} (nfo,42) \quad \text{AREA}(N), Hc(N), CI(0)(N), CI(N)
\end{align*}
\]

33 \text{ CONTINUE}

\[
\begin{align*}
\text{RETURN}
\end{align*}
\]

118
FORMAT (5X, 'ELEMENT =WIDTH DEPTH AREA Y(Y)')
FORMAT (3X, I3, 4 (2X,E8.4))
FORMAT (5X, 'ELEMENT N = AREA(N) Hc(N) CIQ(N) CI(N)')
FORMAT (12X, 4 (3X,E10.4))
FORMAT (2X, 'PLS OUTPUT THE SUMMATION OF THE DEPTH SUMD')
FORMAT (10X, E10.4)

END
SUBROUTINE T

This subroutine is used to calculate the temperature effect to the concrete structures

Parameter ( M = 150 )

DIMENSION W(M),D(M),A(M),Y(M),AREA(M),PHI(M),X(M),SH(M),
1 Ac(M),AcD(M),AcDD(M),Acce(M),AcDe(M),AcDDe(M),
2 Ad(M),AdD(M),AdDD(M),TAd(M),
3 Apr(M),AprD(M),AprDD(M),AprDe(M),AprDDe(M),TApr(M),TAprE(M),
4 Apo(M),ApoD(M),ApoDD(M),ApoDe(M),ApoDDe(M),TApom(M),TApoe(M),
5 As(M),AsD(M),AsDD(M),AsDe(M),AsDDe(M),TAs(M),TAsE(M),
6 Hc(M),Hz(M),Hd(M),Hpoo(M),Hpr(M),
7 Ec(M),Ee(M),Es(M),Epoo(M),Epr(M),
8 RNd(M),RNs(M),RNpr(M),RNpo(M),
9 RNec(M),RNes(M),RNepr(M),RNepo(M),
1 Ppr(M),Ppo(M),
2 TACo(M),TBCo(M),TCo(M),
3 CI0(M),CI(M),
4 SIGMA1(M),SIGMA2(M),SIGMAS(M),SIGMAPR(M),SIGMAPO(M),
5 Rpr(M),Rpo(M),
6 dSTRESs(M),dSTRESpr(M),dSTRESpo(M),
7 Dy(M),DH(M),
8 deSigc(M),deSig(M),deSigp(M),deSigpo(M),
9 Yi(M),Bi(M),Ai(M),ETi(M),deNTc(M),deMTc(M),
1 deNTs(M),deMTs(M),deNTpr(M),deMTpr(M),deNTpo(M),deMTpo(M),
2 Sigic(M),Sig(M),Sigp(M),Sigpo(M)

COMMON /INPUT/NN,NI,NJ,NK,NL,exNs,exMs,
2 W,PHI,X,SH,Ad,Apr,Ap,As,Hs,Hp,Hpr,
3 Ec,Es,Epoo,Ep,Epr,Ppo,HH,Alphat,NS1,NS2,Dns,nfo,Fileout

COMMON /ALL/SUMD,SA,SB,S1,TA,TB,TE,TAC,TACe,
7 Prk,PrkH,Pok,PokH,SuPp,SuPpH,Huppo,Huppoh
9 axni,axmi,EPSILT,EPSILB,CURVA,
1 dSTRES1,dSTRES2,dSTRES3,dSTRES4,
3 delN1,delM1,delN,delLM,delE,delC,
4 SIGMAT,straE,straB,curvTA,Y,AREA,
1 Ac,AcD,AcDD,Acce,AcDe,AcDDe,Ad,Dd,AdDD,TAd,
3 AprD,AprDD,AprDe,AprDDe,TApr,TAPre,
4 ApoD,ApoDD,ApoDe,ApoDDe,TApo,TApoe,
5 AsD,AsDD,AsDe,AsDDe,TAs,TAsE,Hc,Hd,Ee,
7 RNc,RNs,RNpr,RNpo,RNec,RNes,RNepr,RNepo,
1 TACo,TBCo,TCo,CI,CIO,
4 SIGMA12,SIGMAS1,SIGMAPR,SIGMAPO,
6 Rpr,Rpo,dSTRESs,dSTRESpr,dSTRESpo,
7 Dy,DH,delH,deNT,deMT,delET,delCT,
8 deSigc,deSig,deSigp,deSigpo,
9 Yi,Bi,Ai,ETi,deNTc,deMTc,
1 deNTs,deMTs,deNTpr,deMTpr,deNTpo,deMTpo,
2 Sigic,Sig,Sigp,Sigpo

we are going to divided the deck into NS1 elements,
and divided the girder into NS2 elements
The accompanied internal distribution :deSigre

\[ \text{deSigre} = \text{deSigC} + \text{deSigS} + \text{deSigpr} + \text{deSigpo} \]

For concrete: \( \text{deSigC}(I) \)

For non-prestressed steel : \( \text{deSigS}(J) \)

\[
\text{DO 10 } J = 1, NJ \\
\text{deSigS}(J) = \text{Es}(J) \times \text{Et(Hs}(J), \text{Alphat}, \text{HH}, \text{SUMD}) \\
10 \text{ CONTINUE}
\]

For pre-tensioned steel : \( \text{deSigpr}(K) \)

\[
\text{DO 20 } K = 1, NK \\
\text{deSigpr}(K) = \text{Epr}(K) \times \text{Et(Hpr}(K), \text{Alphat}, \text{HH}, \text{SUMD}) \\
\text{write}(nfo,*) \\
\text{write}(nfo,4) \\
\text{write}(nfo,5) \text{deSigpr}(K) \\
20 \text{ CONTINUE}
\]

For post-tensioned steel : \( \text{deSigpo}(L) \)

\[
\text{DO 30 } L = 1, NL \\
\text{deSigpo}(L) = \text{Epo}(L) \times \text{Et(Hpo}(L), \text{Alphat}, \text{HH}, \text{SUMD}) \\
\text{write}(nfo,*) \\
\text{write}(nfo,6) \\
\text{write}(nfo,7) \text{deSigpo}(L) \\
30 \text{ CONTINUE}
\]

The stress resultants which are required to artificially
prevent deformation of the section as the temperature
distribution changes \( \text{deNt} \) & \( \text{deMt} \) are

For concrete section :

\[ \text{deNtc}1 = 0.0 \]
\[ \text{deMtc}1 = 0.0 \]

\[
\text{DO 40 } I=1, NS1+NS2 \\
\text{IF (NS1 .NE. 0) THEN} \\
\text{deltH} = D(I)/NS1 \\
\text{Ei} = Ee(I) \\
\text{Yi}(I) = (1 - 0.5) \times \text{deltH} \\
\text{ENDIF} \\
\text{IF (I .GT. NS1) } \text{deltH} = (\text{SUMD} - D(I))/NS2 \\
\text{IF (I .GT. NS1) } \text{Ei} = Ee(2) \\
\text{IF (I .GT. NS1) } \text{Yi}(I) = D(I) + (I - NS1) \times \text{deltH} - 0.5 \times \text{deltH} \\
\text{Bi}(I) = B(Yi(I), W, DH, D) \\
\text{Ai}(I) = Bi(I) \times \text{deltH} \\
\text{ETi}(I) = ET(Yi(I), Alphat, HH, SUMD) \\
\text{deSigc}(I) = Et \times ETi(I) \\
\text{deNtc}1 = \text{deNtc}1 + \text{deSigc}(I) \times Ai(I) \\
\text{deMtc}1 = \text{deMtc}1 + \text{deSigc}(I) \times Yi(I) \times Ai(I) \\
40 \text{ CONTINUE} \\
\text{write}(nfo,*) \\
\text{write}(nfo,42) \\
\text{write}(nfo,43) \text{deNtc}1 \]
For non-prestressed steel:

\[
\begin{align*}
\mathrm{deNTs1} &= 0.0 \\
\mathrm{deMTs1} &= 0.0 \\
\mathrm{DO} \ 50 \ J = 1, NJ \\
\ &\quad \text{write (nfo,*)} \\
\ &\quad \text{write (nfo,52)} \\
\ &\quad \text{write (nfo,53) deSigS(J)} \\
\ &\quad \text{deNTs1 = deNTs1 + deSigS(J)*As(J)} \\
\ &\quad \text{deMTs1 = deMTs1 + deSigS(J)*Hs(J)*As(J)} \\
50 \ &\quad \text{CONTINUE} \\
\ &\quad \text{write (nfo,*)} \\
\ &\quad \text{write (nfo,44)} \\
\ &\quad \text{write (nfo,45) deNTs1}
\end{align*}
\]

For pre-tensioned steel:

\[
\begin{align*}
\mathrm{deNTpr1} &= 0.0 \\
\mathrm{deMTpr1} &= 0.0 \\
\mathrm{DO} \ 60 \ K = 1, NK \\
\ &\quad \text{deNTpr1 = deNTpr1 + deSigpr(K)*Apr(K)} \\
\ &\quad \text{deMTpr1 = deMTpr1 + deSigpr(K)*Hpr(K)*Apr(K)} \\
60 \ &\quad \text{CONTINUE} \\
\ &\quad \text{write (nfo,*)} \\
\ &\quad \text{write (nfo,46)} \\
\ &\quad \text{write (nfo,47) deNTpr1}
\end{align*}
\]

For post-tensioned steel:

\[
\begin{align*}
\mathrm{deNTpo1} &= 0.0 \\
\mathrm{deMTpo1} &= 0.0 \\
\mathrm{DO} \ 70 \ L = 1, NL \\
\ &\quad \text{deNTpo1 = deNTpo1 + deSigpo(L)*Apo(L)} \\
\ &\quad \text{deMTpo1 = deMTpo1 + deSigpo(L)*Hpo(L)*Apo(K)} \\
70 \ &\quad \text{CONTINUE} \\
\ &\quad \text{deNT = -1*(deNTc1+deNTs1+deNTpr1+deNTpo1)} \\
\ &\quad \text{deMT = deMTc1+deMTs1+deMTpr1+deMTpo1}
\end{align*}
\]

If the artificial restrain is removed, the section is subjected to the equal and opposite internal forces deNT & deMT, and the resultant change of the top fibre strain and curvature are obtained from:

\[
\begin{align*}
\mathrm{Eg} &= \mathrm{Ee(1)} \\
\mathrm{IF} \ (\mathrm{Ee(1)} \ . \mathrm{EQ.} \ 0.0) \ \mathrm{Eg} &= \mathrm{Ee(2)} \\
\mathrm{deIET} &= (\mathrm{TB}*\mathrm{deMT}+\mathrm{TI}*\mathrm{deNT}) / (\mathrm{Eg}*(\mathrm{TA}*\mathrm{TI}-\mathrm{TB}**2)) \\
\mathrm{deICT} &= (\mathrm{TA}*\mathrm{deMT}+\mathrm{TB}*\mathrm{deNT}) / (\mathrm{Eg}*(\mathrm{TA}*\mathrm{TI}-\mathrm{TB}**2)) \\
\ &\quad \text{write (nfo,*)} \\
\ &\quad \text{write (nfo,66)} \\
\ &\quad \text{write (nfo,67) deIET, deICT}
\end{align*}
\]

The change of stress at any point on the section is the sum of the change of stress which occurred with the section was artificially restrained (deSig) and the change of stress when deNT & deMT were applied to the section

For concrete: (divided into NS1+NS2 elements)
Sigco(I) = 0.0
DO 80 I=1,NS1+NS2
  IF (NS1 .NE. 0) THEN
    deltaH = D(1)/NS1
    Ej  = Ee(1)
    Yi(I) = Yi(I)*delH - 0.5*delH
  ENDIF
  IF (I .GT. NS1) Ej = Ee(2)
  IF (I .GT. NS1) deltaH = (SUMD-D(1))/NS2
  IF (I .GT. NS1) Yi(I) = D(1)+(1-NS1)*delH-0.5*delH
  Sigco(I) = Sigco(I)+Ej*(Et(Yi(I),Alphat,HH,SUMD)
             + delET-Yi(I)*delCT)
80  CONTINUE

C At top of the surface:

  Em=Ee(1)
  IF (Ee(1) .EQ. 0.0) Em=Ee(2)
  Sigco(0)=Em*(Et(0,Alphat,HH,SUMD)+delET)

C At bottom of the surface:

  Sigco(NS1+NS2+1)=Ee(2)*(Et(SUMD,Alphat,HH,SUMD)+delET-SUMD*delCT)
WRITE (nfo,*)
write (nfo,991)
write (nfo,992) deNT,deMt
WRITE (nfo,*)
WRITE (nfo,*)
WRITE (nfo,111)
WRITE (nfo,*)
WRITE (nfo,112) (Yi(i),Sigco(i),i=0,NS1+NS2+1)

C For non-prestressed steel:

DO 90 J=1,NJ
  Sigst(J) = Es(J)*(Et(Hs(J),Alphat,HH,SUMD)
               + delET-Hs(J)*delCT)
90  CONTINUE

C For pre-tensioned steel:

DO 100 K=1,NK
  Sigpr(K) = Epr(K)*(Et(Hpr(K),Alphat,HH,SUMD)
                     + delET-Hpr(K)*delCT)
100 CONTINUE

C For post-tensioned steel:

DO 110 L=1,NL
  Sigpo(L) = Epo(L)*(Et(Hpo(L),Alphat,HH,SUMD)
                   + delET-Hpo(L)*delCT)
110 CONTINUE

WRITE (nfo,113)
WRITE (nfo,114) (J,Sigst(J),J=1,NJ)
WRITE (nfo,115)
WRITE (nfo,114) (K,Sigpr(K),K=1,NK)
WRITE (nfo,116)
WRITE (nfo,114) (L,Sigpo(L),L=1,NL)
RETURN

4 FORMAT (5X, 'Pls output deSigpr(K)')
5 FORMAT (5X, F11.4)
6 FORMAT (5X, 'Pls output deSigpo(L)')
7 FORMAT (5X, F11.4)
42 FORMAT (5X, 'Pls output deNtCI & deMtCI')
43 FORMAT (5X, F11.4)
44 FORMAT (5X, 'Pls output deNtISI')
45 FORMAT (5X, F11.4)
46 FORMAT (5X, 'Pls output deNTpr1')
47 FORMAT (5X, F11.4)
52 FORMAT (5X, 'Pls output deSigS(J)')
53 FORMAT (5X, F11.4)
66 FORMAT (5X, 'Pls output delET & delCT')
67 FORMAT (5X, E11.4,5X,E11.4)
112 FORMAT (2X, 2F11.4)
111 FORMAT (2X, 8X,'Y(i)',4X,'Sigco(I)')
114 FORMAT (2X, I2, 2X, E10.4)
113 FORMAT (2X, 2(7X,'.',4X,'Sigst'))
115 FORMAT (2X, 2(7X,'.',4X,'Sigpr'))
116 FORMAT (2X, 2(7X,'.',4X,'Sigpo'))
991 FORMAT (5X, 'Pls out the value of deNT & deMT')
992 FORMAT (5X, 2F13.2)

END

C  This function is used to determine the temperature distribution

FUNCTION TEM(Y,HH,SUMD)
REAL TEM
C  The thicknness of the black bituminous wearing surface: HH

IF (Y .LE. 1200.) THEN
  TEMo = 32.*0.2*HH
  TEM = TEMo * ((1200.-Y)/1200.)**5
RETURN
ENDIF
C
IF ((SUMD-Y) .GE. 0.0 .and. (SUMD-Y) .LE. 200.) THEN
  TEM = (200.0-(SUMD-Y))*1.5/200.
ENDIF
RETURN
END

C  This function is used to determine the relationship between
C  the width and depth

FUNCTION B(Y,W,DH,D)
PARAMETER ( M = 150 )
REAL B,W(M),DH(M),D(M)
IF (Y .LE. DH(1)) THEN
  B = W(1)
RETURN
ENDIF
IF (Y .GT. DH(1) .and. Y .LE. DH(2)) THEN
  B = W(2)
  RETURN
ENDIF
IF (Y .GT. DH(2) .and. Y .LE. DH(3)) THEN
  B = W(2) - 2.*( (W(2)-W(3))/2 * (Y-DH(1))/D(3) )
  RETURN
ENDIF
IF (Y .GT. DH(3) .and. Y .LE. DH(4)) THEN
  B = W(3)
  RETURN
ENDIF
IF (Y .GT. DH(4) .and. Y .LE. DH(5)) THEN
  B = W(3) + 2.*( (W(4)-W(3))/2 * (Y-DH(4))/D(5) )
  RETURN
ENDIF
IF (Y .GT. DH(5) .and. Y .LE. DH(6)) THEN
  B = W(4)
ENDIF
RETURN
END

C If a concrete fibre at any depth Y below the top of the section
C was free to deform and was not restrained by the surrounding
C concrete, the temperature induced strain in that fibre would be:Et

FUNCTION Et(Y,AlphaT,HH,SUMD)
  Et = AlphaT * TEM(Y,HH,SUMD)
RETURN
END