Global Value Chain, Technology and Trade

by

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29 August 2014
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Global Value Chain, Technology and Trade

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Abstract

This paper develops a simple general equilibrium model of trade in a global value chain framework. There are two countries, Home and Foreign. Each country specializes in producing only one final good and the two final goods are mutually imperfectly substitutable. The production of each final good is fragmented and some part(s) of one final good could be possibly produced in the other country. This fragmentation could have significant effects on the global economy. The paper not only shows that different international activities could affect the welfare of consumers between different countries in various ways, but also has interesting implications on the relationship between the ratio of domestic value added in gross export and the technology difference, which may explain the stylized fact that the ratio of domestic value added in gross export has been declining over time in recent years for many countries.

Keywords: global value chain, technology, welfare, ratio, declining, trade.
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1 Introduction

1.1 A Picture of Global Value Chain

Global value chain is a pervasive phenomenon in recent years. Its name, to some extent, already explains itself: different stages of production processes are located in different countries. There has been a strong trend towards the international dispersion of value chain activities such as design, production, marketing, distribution, etc. It attracts great attention because people hope to understand the new production features that generate new trade and investment patterns, and also try to find out appropriate policies associated with it. For example, firms would like to know how to optimally locate their production stages globally and governments need to make policies to regulate the international activities. Since global value chain challenges our conventional knowledge, sufficient research needs to be done to uncover the mysterious mask of it.

There is a huge literature addressing global value chain. Recent works, including Antràs and Chor (2013), Costinot, Vogel and Wang (2013), Baldwin and Venables (2013), Yi (2003), Yi (2010), and Grossman and Rossi-Hansberg (2008), suggest that global fragmentation could have important implications.

Antràs and Chor (2013) develop a property-rights model of production in which production is sequential and contracts are incomplete. Based on the information of the complementarity and substitutionary of the stages, the optimal pattern of ownership along the value chain can be determined.

Costinot, Vogel and Wang (2013) model a global supply chain environment where production is sequential and subject to mistakes, which has implications for the country level specialization along the chain, the world income distribution and how shocks spread across countries.

Baldwin and Venables (2013) build a model with two configurations of production process — "snakes" and "spiders". They argue that although the two processes share some features, the responses of trade flows to changes in trade frictions are different.
Yi (2003) emphasizes the importance of "vertical specialization" by developing a two-country dynamic Ricardian trade model in a three-stage setting. The model generates a nonlinear trade response to tariff reductions and can explain over 50% of the growth of world trade. Based on the same framework, Yi (2010) is able to explain 40% of the Canada border effect, about two-and-one-half times that a model with one production stage can explain.

Grossman and Rossi-Hansberg (2008) propose a theory of the global production process that focuses on tradeable tasks. They demonstrate that a decline in the cost of task trade, whose effects are similar to factor-augmenting technological progress, boosts the productivity of the factor whose tasks become easier to move offshore.

1.2 Motivation and Research

The existing global value chain literature has explained various phenomena in reality, and has cast numerous hypotheses how different international activities affect the economy, but none of them has offered the welfare implications on the emergence of these international activities. Besides, even though some literature has studied the ratio of domestic value added in gross export, for example, Hummels, Ishii and Yi (2001) and Yi (2003) attribute "vertical specialization" to the reduction of a country’s domestic value added ratio; Johnson and Noguera (2012) analyse the ratio of value added to gross exports (VAX ratio) across countries and across industries, they somehow explain the difference in the VAX ratios by whether the exported goods are intermediate goods or not. They also argue that the U.S.–China imbalance in 2004 is 30–40% smaller when measured in value added; Koopman, Zhi, and Wei (2012) propose a method for computing domestic and foreign contents that allows for processing trade, and find that the share of domestic content in China’s manufactured exports has risen from about 50% to about 60% since China entered WTO, and the sophisticated sectors would have low share of domestic content, such as the electronic devices sector; Kee and Tang (2013) explain the increase of China’s domestic value added ratio by the substitution of imported materials with domestic materials, instead of
changes in the composition of firms or industries.

However, no literature has uncovered the relationship between the productivities of tradable and internationally mobile sectors and the ratio of domestic value added in gross export for the countries that are related to these sectors, which may potentially explain the declining trend of this ratio for many countries.

This paper may offer to fill this niche. I develop a general equilibrium model of trade with only two countries. Based on some assumptions, outsourcing (in this model, "outsourcing" only refers to international outsourcing) will occur in one country and offshoring will occur in the other. The model suggests that these two international activities would have different impacts on the welfare of consumers between different countries. Moreover, the model has interesting implications on the relationship between the ratio of domestic value added in gross export and the technology difference, which may explain the stylized fact that the ratio of domestic value added in gross export has been declining over time in recent years for many countries.

The remainder of the paper is in two parts. In section 2, I will describe the models of various economic environments, in which different welfare implications are presented, and the stylized facts of domestic value added ratio are discussed at the end of this part. Section 3 concludes.

2 Model

2.1 The Benchmark Model

I postulate a world with only two countries, Home and Foreign. There are $L^H$ consumers in Home and $L^F$ consumers in Foreign, each consumer, no matter where he is, supplying one unit of labor.

2.1.1 Final Goods

There are two final goods being produced in the world, final good $A$ and final good $B$. Each final good is produced using two intermediate inputs,
capital good \( k \) and labor \( l \). The technologies of producing these final goods are respectively

\[
\begin{align*}
    f_A^H &= \min\{k_A^H, l_A^H\}, \\
    f_B^H &= \theta_A \min\{k_B^H, l_B^H\}, \\
    f_A^F &= \min\{k_A^F, l_A^F\}, \\
    f_B^F &= \min\{k_B^F, l_B^F\},
\end{align*}
\]

where \( f^i_j \), \( k^i_j \) and \( l^i_j \) are respectively the quantity of the output of final good \( i \), the quantity of capital used to produce final good \( i \) and the quantity of labor used to produce final good \( i \) by country \( j \), and \( \theta^i \) represents the productivity of the corresponding final good production sector, \( i = A, B \) and \( j = \text{Home, Foreign} \). The productivities of other final good production sectors are normalized to 1. I assume Leontief production function because in global value chains, intermediate inputs are always complementary to each other. For example, if producing a particular car needs exactly four doors and four wheels, we can not have other kinds of combinations of these two inputs.

I impose assumptions on \( \theta^H \) and \( \theta^F \) that

\[
0 < \theta^H < \frac{L_F}{2L^H} - \frac{1}{\varphi} \quad \text{and} \quad 0 < \theta^F < \frac{L_H - L_F}{2L^F} < \frac{L_H}{2L^F}
\]

to make sure that \textit{Home} specializes in producing final good \( A \) and \textit{Foreign} specializes in producing final good \( B \), which implicitly imply that

\[
0 < \frac{L_H}{L^F} < \frac{\varphi}{3}.
\]

Besides, we also need make the assumptions that

\[
1 + \frac{2}{\varphi} < \frac{L_H}{L^F} < \frac{\varphi}{3} \quad \text{and} \quad \varphi > 6
\]

to guarantee the viability of the whole model. \( 1 + \frac{2}{\varphi} < \frac{L_H}{L^F} \) implies that there are more consumers in \textit{Home} than those in \textit{Foreign} and so is labor. \( \frac{L_H}{L^F} < \frac{\varphi}{3} \) and \( \varphi > 6 \) mean that the productivity of capital good production in \textit{Foreign} is large enough to guarantee that the international activities such as outsourcing or offshoring will be possible to occur in later evolutions of the model. These settings make \textit{Foreign} look like a developed country and \textit{Home} look like a less developed country.

The two final goods are mutually imperfectly substitutable, implying that they are differentiated, and the markets of the two final goods are competitive, implying that the price of each final good will be equal to its unit cost of production. Tradability for these final goods is guaranteed, so that the consumers in either country could enjoy both final goods.
2.1.2 Intermediate Inputs

As mentioned before, there are two intermediate inputs, capital good \( k \) and labor \( l \). Labor is immobile, meaning that I rule out the possibility of migrating in this model. Capital good \( k \) is tradable and the production of it can be easily moved across borders, meaning that both trade and international investment of this good is possible. The technologies of producing the capital good for the two countries are respectively

\[
k^H = l^H_k \quad \text{and} \quad k^F = \varphi l^F_k,
\]

where \( k^j \) and \( l^j_k \) are respectively the quantity of the output of capital good and the labor input to produce the capital good, \( j = \text{Home, Foreign} \), and \( \varphi \) represents the productivity of the capital good production in Foreign and the productivity of capital good production in Home is normalized to 1.

The markets of these intermediate inputs are competitive, so that the price of each intermediate input will be equal to its unit cost of production.

One more thing to clarify that the technologies of producing final goods and intermediate inputs are nontradable, meaning that I rule out the possibility of technology purchasing.

2.1.3 Free Trade

I assume free trade in this model for two major reasons. One is that in the real world, tariffs are already extremely low in many countries, and thus there is little space to adjust them even if they may have nontrivial effects on consumers’ welfare. The other is that given a productivity level of capital good production in Foreign, \( \varphi \), trade costs are likely to hinder some international activities such as outsourcing, which I do not expect to appear in the model. Consequently, consumers in both countries are facing the same price for each final good.

2.1.4 Consumers and Wages

Consumers in Home have the same preferences and will maximize their utility
subject the budget constraint

\[ p_Ac_H^A + p_Bc_H^B = w^H, \]

and consumers in \textit{Foreign} also have the same preferences and will maximize their utility

\[ U = (c_A^F)^{\frac{1}{2}}(c_B^F)^{\frac{1}{2}} \]

subject the budget constraint

\[ p_Ac_F^A + p_Bc_F^B = w^F, \]

where \( c_n^m \) is the quantity of consumption of final good \( m \) by a consumer in \( n \), \( w^n \) is the nominal wage paid to labor in \( n \) and \( p_q \) represents the price of final good \( q \), \( m = A, B \), \( n = \text{Home, Foreign} \) and \( q = A, B \). Note that the existence of two intermediate input sectors in a country implies that the nominal wages paid to labor are equalized across sectors in that country. I let \( w^H \) to be the numeraire, so that \( w^H = 1 \), and denote \( w^F \) by \( w \). Thus \( w \) also represents the \textit{Foreign-Home} relative nominal wage. The Cobb–Douglas utility function \( U = (c_A^F)^{\frac{1}{2}}(c_B^F)^{\frac{1}{2}} \) implies that it is better to consume both final goods, which gives rise to trade.

Solving the consumers’ problems, we obtain the demands of the two final goods by each consumer in \textit{Home} and \textit{Foreign} respectively

\begin{align*}
&c_H^A = \frac{1}{2p_A}, \quad \text{and} \quad c_F^A = \frac{w}{2p_A}, \\
&c_H^B = \frac{1}{2p_B}, \quad \text{and} \quad c_F^B = \frac{w}{2p_B},
\end{align*}

and the aggregate price \( P \), which represents the price of one unit of utility, is given by

\[ P = 2\sqrt{p_Ap_B}. \]
Note that consumers either in Home or Foreign face the same aggregate price because they have the same preferences. This will consequently expand the meaning of \( w \) that \( w \) will also represent the Foreign-Home relative real wage.

In the benchmark case, I rule out all the international activities except trading final goods. Thus by assuming full employment, all the labor in Home will be used to produce final good A and all the labor in Foreign will be used to produce final good B. First, I impose the assumption that Home specializes in producing final good A and Foreign specializes in producing final good B, and later I will justify this. Then the prices of the two final goods are respectively given by

\[
p_A = 2 \quad \text{and} \quad p_B = (1 + \frac{1}{\xi})w.
\]

Because \( p_A < \frac{p_B}{w} \) and \( p_B < \frac{p_A}{w} \) imply that, in equilibrium, Home will specialize in producing final good A and Foreign will specialize in producing final good B, which justifies the assumption imposed before.

### 2.1.5 Equilibrium

Based on the settings in this section, the labor market clearing conditions are as follows

\[
\begin{align*}
 f_A^H + f_A^H &= L^H, \\
 f_B^F + \frac{1}{\xi} f_B^F &= L^F.
\end{align*}
\]

The final goods market clearing conditions are as follows

\[
\begin{align*}
 L^H c_A^H + L^F c_A^F &= f_A^H, \\
 L^H c_B^H + L^F c_B^F &= f_B^F.
\end{align*}
\]

Simple algebra gives

\[
\begin{align*}
 L^H c_A^H + L^F c_A^F &= \frac{L^H}{2}, \\
 L^H c_B^H + L^F c_B^F &= \frac{L^F}{1+\xi},
\end{align*}
\]

which is equivalent to

\[7\]
\[
L^H \frac{1}{1} + L^F \frac{w}{1} = \frac{L^H}{2}, \\
L^H \frac{1}{2(1+\frac{1}{\varphi})w} + L^F \frac{w}{2(1+\frac{1}{\varphi})} = \frac{L^F}{1+\frac{1}{\varphi}}.
\]

Solving the above equations, we obtain the expression of the nominal wage in Foreign

\[
w = \frac{L^H}{L^F}.
\]

This expression also depends on relative labor supply of the two countries. Given the assumption \(1 + \frac{2}{\varphi} < \frac{L^H}{L^F}\), we have \(w = \frac{L^H}{L^F} > 1 + \frac{2}{\varphi} > 1\), implying that the nominal wage will be higher in Foreign where there is relatively less labor supply. The productivity \(\varphi\) does not matter because labor in each country serves their own final good production only, and thus the effect of \(\varphi\) is totally offset by \(p_B\), which makes the nominal wage in Foreign unchanged.

Trade should always be balanced because there is actually only one factor, labor, for all the productions. To check that, we calculate the total value of export from Home to Foreign as follows

\[
p_AL^Fc_A^F = p_AL^F \frac{w}{2p_A} = p_AL^F \frac{1}{2p_A} \frac{L^H}{L^F} = \frac{L^H}{2},
\]

and the total value of export from Foreign to Home as follows

\[
p_BL^Hc_B^H = p_BL^H \frac{1}{2p_B} = \frac{L^H}{2}.
\]

Clearly, \(p_AL^Fc_A^F = p_BL^Hc_B^H\), and thus balanced trade is verified.

### 2.1.6 Welfare

Given the expression of \(w\), the welfare of each consumer in Home can be calculated as follows

\[
W_{Ben}^H = \frac{1}{p_{Ben}} = \frac{1}{2\sqrt{2(1+\frac{1}{\varphi})\frac{L^H}{L^F}}},
\]

and the welfare of each consumer in Foreign can be calculated as follows

\[
W_{Ben}^F = \frac{w}{p_{Ben}} = \frac{\sqrt{\frac{L^H}{L^F}}}{2\sqrt{2(1+\frac{1}{\varphi})}}.
\]
where $W^n_{Ben}$ and $P_{Ben}$ denote respectively the welfare of each consumer in $n$ and the aggregate price in the benchmark model, $n = \text{Home, Foreign}$. Given $\varphi$, $W^H_{Ben}$ is decreasing in $\frac{L^H}{L^F}$ because $\frac{p_A}{p_B}$ will be lower and the consumer will consume the same amount of final good $A$ but fewer final good $B$, while $W^F_{Ben}$ is increasing in $\frac{L^H}{L^F}$ because $\frac{p_A}{p_B}$ will be higher and the consumer will consume the same amount of final good $B$ but more final good $A$. Given $\frac{L^F}{L^H}$, both $W^H_{Ben}$ and $W^F_{Ben}$ is increasing in $\varphi$ because each consumer is able to consume more the final good produced in the other country while consuming the same amount of the final good produced in their own country.

### 2.2 The Model with Outsourcing

In this section, I extend the benchmark model to incorporating outsourcing only. As mentioned before, this "outsourcing" is actually international outsourcing, which implying the existence of trading intermediate input(s), here namely, the capital good. Importing brings about competition among similar products, which, in the real world, always causes structure change not only in the destination market but also in the source market. These dynamics will definitely have nontrivial impacts on consumers’ welfare in both markets. That is why outsourcing is the international activity that should be paid great attention to.

#### 2.2.1 Outsourcing

Because $\text{Foreign}$ has relatively higher average productivity and less labor, I assume first, to be consistent with the reality, that still $w > 1$, and at the same time, $\frac{w}{\varphi} < 1$ in order to make outsourcing possible and later equilibrium outcomes will justify these assumptions. The patterns of the whole economy are as follows.

- $\text{Home}$ will import the capital good from $\text{Foreign}$ because the capital good is cheaper in $\text{Foreign}$.

- Import competition will shut down the production of the capital good in $\text{Home}$.
As a result, only \textit{Foreign} will produce the capital good.

The patterns exhibit a "snake" feature proposed by Baldwin and Venables (2013). The production of capital good is in upstream, while the final good production is in downstream, and the firms in upstream (in \textit{Foreign}) export the capital good to the final good firms in downstream (in \textit{Home}).

Still, first, I impose the assumption that \textit{Home} specializes in producing final good \textit{A} and \textit{Foreign} specializes in producing final good \textit{B}, and later I will justify this. Then the prices of the two final goods are respectively given by

\[ p_A = 1 + \frac{w}{\varphi} \] and \[ p_B = (1 + \frac{1}{\varphi})w. \]

Because \( p_A < \frac{p_B}{\varphi} \) and \( p_B < \frac{p_A}{\varphi} \) imply that, in equilibrium, \textit{Home} will specialize in producing final good \textit{A} and \textit{Foreign} will specialize in producing final good \textit{B}, the assumption imposed before is justified.

\subsection*{2.2.2 Equilibrium}

Based on the settings in this section, the labor market clearing conditions are as follows

\[ f_H^H = L_H, \]
\[ \frac{1}{\varphi} f_H^H + f_B^F + \frac{1}{\varphi} f_B^F = L_F. \]

The final goods market clearing conditions are as follows

\[ L_H c_A^H + L_F c_A^F = f_A^H, \]
\[ L_H c_B^H + L_F c_B^F = f_B^F. \]

Simple algebra gives

\[ L_H c_A^H + L_F c_B^F = L_H, \]
\[ L_H c_B^H + L_F c_B^F = \frac{L_F - L_H}{1 + \frac{1}{\varphi}}, \]

which is equivalent to
\[ L^H \frac{1}{2(1 + \frac{w}{\varphi})} + L^F \frac{w}{2(1 + \frac{w}{\varphi})} = L^H, \]
\[ L^H \frac{1}{2(1 + \frac{w}{\varphi})w} + L^F \frac{w}{2(1 + \frac{w}{\varphi})w} = \frac{L^F - L^H}{1 + \frac{w}{\varphi}}. \]

Solving the above equations, we obtain the expression of the nominal wage in Foreign

\[ w = \frac{1}{L^F - \frac{2}{\varphi}}. \]

From now on, I will neglect \( L^F \) and focus on \( \varphi \) which is relatively more important. Indeed, \( w > 1 \) and \( \frac{w}{\varphi} < 1 \), which justify the assumptions imposed before. We notice that \( w \) is decreasing in \( \varphi \), meaning that the real wage inequality will be mitigated as the relative productivity of the tradable sectors goes up. Intuitively, outsourcing allocates more and more labor from the tradable sectors to the nontradable sectors in both countries as the relative productivity of the tradable sectors is increased. The mass of labor in nontradable sectors with similarly low productivities between countries will gradually dominate that in the tradable sectors, which gives rise to a smaller difference of real wages between countries.

Trade should always be balanced because there is actually only one factor, labor, for all the productions. To check that, we calculate the total value of export from Home to Foreign as follows

\[ p_A L^F c_A^F = p_A L^F \frac{w}{2p_A} = p_A L^F \frac{1}{2p_A} \frac{1}{L^F - \frac{2}{\varphi}} = \frac{L^F}{2L^F - \frac{2}{\varphi}}, \]

and the total value of export from Foreign to Home as follows

\[ p_B L^H c_B^H + \frac{w}{\varphi} L^H = p_B L^H \frac{1}{2p_B} + \frac{L^H}{\varphi \frac{L^F}{L^F - \frac{2}{\varphi}}} = \frac{L^F}{2L^F - \frac{2}{\varphi}}. \]

Clearly, \( p_A L^F c_A^F = p_B L^H c_B^H + \frac{w}{\varphi} L^H \), and thus balanced trade is verified.

### 2.2.3 Welfare

Given the expression of \( w \), the welfare of each consumer in Home can be calculated as follows

\[ 11 \]
and the welfare of each consumer in \textit{Foreign} can be calculated as follows

\[
W_{Out}^H = \frac{1}{P_{Out}} = \frac{1}{2 \sqrt{(1+ \frac{1}{LH - \frac{1}{LH}})(1+ \frac{1}{LF - \frac{1}{LF}})}},
\]

where \( W_{Out}^n \) and \( P_{Out} \) denote respectively the welfare of each consumer in \( n \) and the aggregate price in the model with outsourcing, \( n = \text{Home}, \text{Foreign} \).

From now on, when doing welfare comparison, I compare the welfare in the current model to the benchmark model. Interestingly, by simple algebra, we find that when \( \varphi \) is low, there is a welfare loss for consumers in \textit{Home} and when \( \varphi \) is high, there is a welfare gain for consumers in \textit{Home}, while there is always a welfare gain for consumers in \textit{Foreign}. Intuitively, for consumers in \textit{Home}, although the price of final good \( A \) is lower, the price of final good \( B \) is much higher when \( \varphi \) is low and both prices are getting lower and lower relative to their nominal wage as \( \varphi \) goes up, resulting in conditional welfare implications; for consumers in \textit{Foreign}, the reason may be that their nominal wage is always high enough relative to the prices of the two final goods to guarantee a gain in welfare. The result is formalized as follows

\textbf{Proposition 1} \textit{Compared to the benchmark model, there is a cut-off value} \( \varphi^* \) \textit{such that consumers in Home lose from outsourcing when} \( \varphi < \varphi^* \) \textit{and gain from outsourcing when} \( \varphi > \varphi^* \), \textit{while consumers in Foreign always gain from outsourcing.}

\subsection*{2.3 The Model with Offshoring}

In this section, I extend the benchmark model to incorporating offshoring only. Offshoring is an important factor to international trade in reality. For example, the share of intra-firm trade in U.S. total exports is on average around 31\% and the share of intra-firm trade in U.S. total imports is on average around 46\% from 1992 to 2010. Besides, offshoring also facilitates
economic development. For example, offshoring brings international investments to the target country, which may improve the productivities of the firms in that country by technology transfer, and also creates new jobs there. All these imply that offshoring will have great effects on both the domestic economy and the target economy.

2.3.1 Offshoring

Because labor is the only input in the production of capital good and there is no trade cost, the motivation of offshoring is just to take advantage of the lower nominal wage in the foreign market. Thus, I assume first that \( w > 1 \), and later I will justify this. Note that the offshoring firms can actually sell the capital good in the foreign market, which rule out the possibility of outsourcing in this model. The patterns of the whole economy are as follows

- \( \text{Foreign} \) will locate the production of the capital good in \( \text{Home} \) because the nominal wage is lower there.

- Because the capital good will be cheaper by importing from \( \text{Home} \), there will be no production of the capital good in \( \text{Foreign} \).

- As a result, only \( \text{Home} \) will produce the capital good.

The patterns also exhibit a "snake" feature proposed by Baldwin and Venables (2013). The production of capital good is in upstream, while the final good production is in downstream, and the firms in upstream (in \( \text{Home} \)) export the capital good to the final good firms in downstream (in \( \text{Foreign} \)). Here, because we are talking about offshoring, the upstream firms and the downstream firms could belong to one multinational firm, and thus the upstream firms could be regarded as the vertical international investments in the foreign market.

Still, first, I impose the assumption that \( \text{Home} \) specializes in producing final good \( A \) and \( \text{Foreign} \) specializes in producing final good \( B \), and later I will justify this. Then the prices of the two final goods are respectively given by
\[ p_A = 1 + \frac{1}{\varphi} \quad \text{and} \quad p_B = w + \frac{1}{\varphi}. \]

Because \( p_A < \frac{p_B}{\varphi} \) and \( p_B < \frac{p_A}{\varphi} \) imply that, in equilibrium, \textit{Home} will specialize in producing final good \( A \) and \textit{Foreign} will specialize in producing final good \( B \), the assumption imposed before is justified.

### 2.3.2 Equilibrium

Based on the settings in this section, the labor market clearing conditions are as follows

\[
\begin{align*}
 f^H_A + \frac{1}{\varphi} f^H_A + \frac{1}{\varphi} f^F_B &= L^H, \\
 f^F_B &= L^F.
\end{align*}
\]

The final goods market clearing conditions are as follows

\[
\begin{align*}
 L^H c_A^H + L^F c_A^F &= f^H_A, \\
 L^H c_B^H + L^F c_B^F &= f^F_B.
\end{align*}
\]

Simple algebra gives

\[
\begin{align*}
 L^H c_A^H + L^F c_A^F &= \frac{L^H - L^F}{1 + \varphi}, \\
 L^H c_B^H + L^F c_B^F &= L^F,
\end{align*}
\]

which is equivalent to

\[
\begin{align*}
 L^H \frac{1}{2(1 + \frac{1}{\varphi})} + L^F \frac{w}{2(1 + \frac{1}{\varphi})} &= \frac{L^H - L^F}{1 + \frac{1}{\varphi}}, \\
 L^H \frac{1}{2(w + \frac{1}{\varphi})} + L^F \frac{w}{2(w + \frac{1}{\varphi})} &= L^F.
\end{align*}
\]

Solving the above equations, we obtain the expression of the nominal wage in \textit{Foreign}

\[ w = \frac{L^H}{L^F} - \frac{2}{\varphi}. \]

Indeed, \( w > 1 \), which justifies the assumption imposed before. Now, \( w \) is increasing in \( \varphi \), meaning that the real wage inequality will be enlarged as the productivity of the offshoring firms goes up. Intuitively, because the supply
of final good $B$ is unchanged but the supply of final good $A$ will be higher, as the productivity of the offshoring firms is increased. Then the final good $B$-final good $A$ relative price will be higher, which bids the Foreign-Home relative real wage up.

Trade should always be balanced because there is actually only one factor, labor, for all the productions. To check that, we calculate the total value of export from Home to Foreign as follows

$$p_A L^F c^F_A + L^F \frac{1}{\varphi} = p_A L^F \frac{1}{2p_A} \left( \frac{L^H}{L^F} - \frac{\varphi}{2} \right) + L^F \frac{1}{\varphi} = \frac{L^H}{2},$$

and the total value of export from Foreign to Home as follows

$$p_B L^H c^H_B = p_B L^H \frac{1}{2p_B} = \frac{L^H}{2}.$$ 

Clearly, $p_A L^F c^F_A + L^F \frac{1}{\varphi} = p_B L^H c^H_B$, and thus balanced trade is verified.

2.3.3 Welfare

Given the expression of $w$, the welfare of each consumer in Home can be calculated as follows

$$W^H_{\text{off}} = \frac{1}{P_{\text{off}}} = \frac{1}{2\sqrt{(1+\frac{1}{\varphi})(\frac{L^H}{L_F^F} - \frac{\varphi}{2})}},$$

and the welfare of each consumer in Foreign can be calculated as follows

$$W^F_{\text{off}} = \frac{w}{P_{\text{off}}} = \frac{\frac{L^H}{L^F} - \frac{\varphi}{2}}{2\sqrt{(1+\frac{1}{\varphi})(\frac{L^H}{L^F^F} - \frac{\varphi}{2})}},$$

where $W^n_{\text{off}}$ and $P_{\text{off}}$ denote respectively the welfare of each consumer in $n$ and the aggregate price in the model with offshoring, $n = \text{Home, Foreign}$. By simple algebra, we find that there is always a welfare gain for consumers both in Home and Foreign. Two reasons might give rise to that. One is that the offshoring firms boost the production of final good $B$ by moving the production of capital good to the target country, because there is no production of capital good in the domestic country. The other is that the offshoring firms make better use of the labor in the target country, which boosts the production of final good $A$. As a result, there are more both final goods than before, which makes everyone better off. The result is formalized as follows
Proposition 2 Compared to the benchmark model, there is always a welfare gain for consumers both in Home and Foreign.

2.4 Trade in Value Added

Trade in value added traces the value added by each industry or country to the final export. How much domestic value added of a country embedded in its gross export could to a great extent reveal its position in the global trade, which is an important indicator not only for international business activities but also for policy makers. Thus, related research is considerably necessary.

In this section, we now look at the ratio of domestic value added in gross export in the model with outsourcing and the model with offshoring respectively. To simplify writing, I use "the domestic value added ratio" to equivalently represent "the ratio of domestic value added in gross export".

2.4.1 The Outsourcing Case

In this case, the domestic value added ratio of Foreign is always 1 because Foreign does not import intermediate inputs.

For Home, the total value of gross export is

\[ L^F \frac{w}{p_A} + L^F \frac{w}{p_A} e^A \]

in which the total value of domestic value added is

\[ L^F \frac{w}{p_A} \]

Then the domestic value added ratio is given by

\[ \frac{L^F \frac{w}{p_A}}{L^F \frac{w}{p_A} + L^F \frac{w}{p_A} e^A} = \frac{1}{1 + e^A} = 1 - \frac{L^F}{L^F + e^A} \]

Clearly, this ratio is increasing in \( \varphi \) (Figure 1 presents a numerical result in Appendix). Intuitively, because the capital good-labor relative price, \( \frac{w}{r} \), is
getting lower and lower, as $\varphi$ increases (note that in this case, $w$ is decreasing in $\varphi$), the value of intermediate inputs embedded in final export goes down, implying an increasing domestic value added ratio. The result is formalized as follows

**Proposition 3** In the outsourcing case, the domestic value added ratio of Foreign is always 1, while the domestic value added ratio of Home increases, as the relative productivity increases.

### 2.4.2 The Offshoring Case

In this case, the domestic value added ratio of Home is always 1 because Home does not import intermediate inputs.

For Foreign, the total value of gross export is

$$L^H \frac{1}{2p_B} w + L^H \frac{1}{2p_B} \varphi,$$

in which the total value of domestic value added is

$$L^H \frac{1}{2p_B} w.$$

Then the domestic value added ratio is given by

$$R^F_{Off} = \frac{L^H \frac{1}{2p_B} w}{L^H \frac{1}{2p_B} w + L^H \frac{1}{2p_B} \varphi} = \frac{w}{w + \varphi} = 1 - \frac{1}{\varphi \frac{L^H}{L^F} - 1}.$$

Clearly, this ratio is also increasing in $\varphi$ (Figure 2 presents a numerical result in Appendix). Intuitively, for the same reason as above, because the capital good-labor relative price, $\frac{1}{\varphi w}$, is getting lower and lower, as $\varphi$ increases (note that in this case, $w$ is increasing in $\varphi$), the value of intermediate inputs embedded in final export goes down, implying an increasing domestic value added ratio. The result is formalized as follows

**Proposition 4** In the offshoring case, the domestic value added ratio of Home is always 1, while the domestic value added ratio of Foreign increases, as the relative productivity increases.
2.4.3 Explaining the World

The results may explain the declining trend of the domestic value added ratio for many countries from a new point of view. The positive relationship between the domestic value added ratio and the relative productivity implies that, when the technology difference is smaller, the domestic value added ratio should be lower. I am glad to see this because this is consistent with my intuition. Nowadays, international activities are prevalent around world. Imported goods and international investments in a country can always generate positive externalities that improve the productivities of domestic firms. The most advanced technologies may update slowly, but the speed of imitating these technologies is relatively much higher. For example, Lenovo computer once imported Intel chips from U.S. when there were no plants of Intel in China. Intel had kept innovating and the price of the chips of some certain type had been going down, and thus Lenovo’s the domestic value added (including screens and hard drives) had a higher weight in gross export. But later, Lenovo had much higher productivities in producing screens and some other parts of the computers (Lenovo always imported Intel chips because Intel had a high enough chips-producing technology to capture a significantly large market share), by both imitation and innovation, and thus the domestic value added ratio fell. Therefore, I believe that, the converging trend of productivities between industries in different countries may be a critical reason for the over time decrease in the domestic value added ratio for many countries.

3 Conclusion

I have presented a simple general equilibrium model of trade in a global value chain framework. The paper offers welfare implications for different countries under various economic environments including the economy with outsourcing and the economy with offshoring. Besides, I have also analysed the relationship between the ratio of domestic value added in gross export and the technology difference for different economies, and there are interesting
results. Finally, based on these results, I could to some extent explain the stylized fact that the ratio of domestic value added in gross export has been declining over time in recent year for many countries.

There are many ways for the extension of the model. For example, governments could be involved to make policies such as trade policy, investment policy or development policy. we could also extend this model by considering more industries or more countries to see how they interact with each other, and there must be more interesting implications.
References


Appendix

Figure 1 shows the relationship between the domestic value added ratio of *Home* and the relative productivity in the outsourcing case, where $\frac{L_H}{L_F} = 2$.
Figure 2 shows the relationship between the domestic value added ratio of *Foreign* and the relative productivity in the offshoring case, where $\frac{L^H}{L^F} = 2$.

**y**: the domestic value added ratio

**x**: the relative productivity

$\frac{L^H}{L^F} = 2$.