Essays on Credit Constraints, Bubbles and Unemployment

by

XU Lifang

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June 2013, Hong Kong
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XU Lifang

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This is to certify that I have examined the above PhD thesis
and have found that it is complete and satisfactory in all respects,
and that any and all revisions required by
the thesis examination committee have been made.

Professor Pengfei WANG, Thesis Supervisor

Professor Francis T. LUI, Head of Department

Department of Economics

June 2013
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Chapter 1 introduces endogenous credit constraints in a search model of unemployment. These constraints generate multiple equilibria supported by self-fulfilling beliefs. A stock market bubble exists through a positive feedback loop mechanism. The collapse of the bubble tightens the credit constraints, causing firms to reduce investment and hirings. Unemployed workers are hard to find jobs generating high and persistent unemployment.

Chapter 2 documents a hump-shaped empirical relationship between financial development and the national savings rate across 12 East Asian and 31 OECD economies. An incomplete-market model featuring both heterogeneous households and heterogeneous firms is provided to explain this hump-shaped relationship. The key insight of the model is that financial development tends to reduce the precautionary saving incentives of households but increase firms’ ability to borrow and invest. As a result, the aggregate savings rate may rise initially with financial development because of greater investment by firms, but then it declines with further financial development because of substantially reduced precautionary savings by households. The model also predicts that the market interest rate lies substantially below the rate of return to capital in emerging economies, but the gap diminishes with financial development, as observed in the data.

Chapter 3 studies the effects of endogenous participation on wage dynamics and hence the unemployment fluctuations in a search theoretic model. It shows that endogenous participation makes the expected outside options of workers countercyclical, which induces a countercyclical component in the wage equation under Nash bargaining. As a result, the elasticity of wage with respect to productivity becomes smaller and hence the model can generate larger unemployment fluctuations, compared to the textbook model. With careful calibration, I show that the model well capture the wage dynamics and can explain nearly half of the unemployment volatility.
CHAPTER 1

STOCK MARKET BUBBLES AND UNEMPLOYMENT

1.1 Introduction

This paper provides a theoretical study that links unemployment to the stock market bubbles and crashes. Our theory is based on three observations from the U.S. labor, credit, and stock markets. First, the U.S. stock market has experienced booms and busts and these large swings may not be explained entirely by fundamentals. Shiller (2005) documents extensive evidence on the U.S. stock market behavior and argues that many episodes of stock market booms are attributed to speculative bubbles. Second, the stock market booms and busts are often accompanied by the credit market booms and busts. A boom is often driven by a rapid expansion of credit to the private sector accompanied by rising asset prices. Following the boom phase, asset prices collapse and a credit crunch arises. This leads to a large fall in investment and consumption and an economic recession may follow.1 Third, the stock market and unemployment are highly correlated.2 Figure 1.1 plots the post-war U.S. monthly data of the price-earnings ratio (the real Standard and Poor’s Composite Stock Price Index divided by the ten-year moving average real earnings on the index) constructed by Robert Shiller and the unemployment rate downloaded from the Bureau of Labor Statistics (BLS).3 This figure shows that, during recessions, the stock price fell and the unemployment rate rose. In particular, during the recent Great Recession, the unemployment rate rose from 5.0 percent at the onset of the recession to a peak of 10.1 percent in October 2009, while the stock market fell by more than 50 percent from October 2007 to March 2009.

2See Farmer (2012b) for a regression analysis.
Motivated by the preceding observations, we build a search model with credit constraints, based on Blanchard and Gali (2010). The Blanchard and Gali model is isomorphic to the Diamond-Mortensen-Pissarides (DMP) search and matching model of unemployment (Diamond (1982), Mortensen (1982), and Pissarides (1985)). Our key contribution is to introduce credit constraints in a way similar to Miao and Wang (2011a,b,c, 2012a,b). The presence of this type of credit constraints can generate a stock market bubble through a positive feedback loop mechanism. The intuition is the following: When investors have optimistic beliefs about the stock market value of a firm’s assets, the firm wants to borrow more using its assets as collateral. Lenders are willing to lend more in the hope that they can recover more if the firm defaults. Then the firm can finance more investment and hiring spending. This generates higher firm value and justifies investors’ initial optimistic beliefs. Thus, a high stock market value of the firm can be sustained in equilibrium.

There is another equilibrium in which no one believes that firm assets have a high value. In this case, the firm cannot borrow more to finance investment and hiring spending. This makes firm value indeed low, justifying initial pessimistic beliefs. We refer to the first type of equilibrium as the bubbly equilibrium and to the second type as the bubbleless equilibrium. Both types can coexist due to self-fulfilling beliefs. In the bubbly equilibrium, firms can hire more workers and hence the market tightness is higher, compared to the bubbleless

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4The modeling of credit constraints is closely related to Kiyotaki and Moore (1997), Alvarez and Jermann (2000), Albuquerque and Hopenhayn (2004), and Jermann and Quadrini (2012).
equilibrium. In addition, in the bubbly equilibrium, an unemployed worker can find a job more easily (i.e., the job-finding rate is higher) and hence the unemployment rate is lower.

After analyzing these two types of equilibria, we follow Weil (1987), Kocherlakota (2009) and Miao and Wang (2011a,b,c, 2012a,b) and introduce a third type of equilibrium with stochastic bubbles. Agents believe that there is a small probability that the stock market bubble may burst. After the burst of the bubble, it cannot re-emerge by rational expectations. We show that this shift of beliefs can also be self-fulfilling. After the burst of the bubble, the economy enters a recession with a persistent high unemployment rate. The intuition is the following. After the burst of the bubble, the credit constraints tighten, causing firms to reduce investment and hiring. An unemployed worker is then harder to find a job, generating high unemployment.

Our model can help explain the high unemployment during the Great Recession. Figures 1.2 and 1.3 plot the hires rate and the job-finding rate from the first month of 2001 to the last month of 2011 using the Job Openings and Labor Turnover Survey (JOLTS) data set.\(^5\)

\(^5\)To be consistent with our model and the Blanchard and Gali (2010) model, we define the job-finding rate as the ratio of hires to unemployment. We first use the hires rate in the private sector from JOLTS and total employment in the private sector from BLS to calculate the number of hires, then use the unemployment rate and civilian employment from BLS to calculate the unemployed labor force, and finally derive the job-finding rate by dividing hires by unemployment. Our construction is different from that in Shimer (2005) for the DMP model.
These figures reveal that both the job-finding rate and the hires rate fell sharply following the stock market crash during the Great Recession. In particular, the hires rate and the job-finding rate fell from 4.4 percent and 0.7, respectively, at the onset of the recession to about 3.1 percent and 0.25, respectively, in the end of the recession.

While it is intuitive that unemployment is related to the stock market bubbles and crashes, it is difficult to build a theoretical model that features both unemployment and the stock market bubbles in a search framework. To the best of our knowledge, we are aware of two approaches in the literature. The first approach is advocated by Farmer (2010a,b,c,d, 2012a,b). The idea of this approach is to replace the wage bargaining equation by the assumption that employment is demand determined. In particular, Farmer assumes that the stock market value is determined by an exogenously specified belief function, rather than the present value of future dividends. For any given beliefs, there is an equilibrium which makes the beliefs self-fulfilling. A shift in beliefs that lower stock prices reduces aggregate demand and raises unemployment. This approach of modeling stock prices seems ad hoc since anything can happen. The second approach is proposed by Kocherlakota (2011). He

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6As shown by Santos and Woodford (1997), rational bubbles can typically be ruled out in infinite-horizon models by transversality conditions. Bubbles can be generated in overlapping-generations models (Tirole (1985)) or in infinite-horizon models with borrowing constraints (Kocherlakota (1992,2009) and Wang and Wen (2011)). See Brunnermeier (2009) for a short survey of the literature on bubbles.

7One motivation of replacing the wage bargaining equation follows from Shimer’s (2005) finding that Nash bargained wages make unemployment too smooth. Hall (2005) argues that any wage in the bargaining set can be supported as an equilibrium.
combines the overlapping generations model of Samuelson (1958) with the DMP model. The overlapping generations model can generate bubbles in an intrinsically useless asset. As in Farmer’s approach, Kocherlakota also assumes that output is demand determined by removing the job creation equation in the DMP model. He then separates labor markets from asset markets. The two are connected only through the exchange of the goods owned by asset market participants (or households) and the different goods produced by workers. He assumes that both households and workers have finite lives, but firms are owned by infinitely lived people not explicitly modeled in the paper.

Our approach is different from the previous two approaches in three respects. First, we introduce endogenous credit constraints into an infinite-horizon search model. The presence of credit constraints generates multiple equilibria through self-fulfilling beliefs. Unlike the Kocherlakota (2011) model, we focus on bubbles in the stock market value of the firm, but not in an intrinsically useless assets. A distinctive feature of stocks is that dividends are endogenous. Unlike Farmer’s approach, our approach implies that the stock price is endogenously determined by both fundamentals and beliefs. In addition, in our model the crash of bubbles makes the stock price return to the fundamental value often modeled in the standard model. Second, unlike Kocherlakota (2011), we study both steady state and transitional dynamics. We also introduce stochastic bubbles and show that the collapse of bubbles raises unemployment. Kocherlakota (2011) does not model stochastic bubbles. But he shows that the unemployment rate is the same in a bubbly equilibrium as it is in a bubbleless equilibrium, as long as the interest rate is sufficiently low in the latter. He then deduces that labor market outcomes are unaffected by a bubble collapse, as long as monetary policy is sufficiently accommodative.

Third, our model has some policy implications different from Farmer’s and Kocherlakota’s models. In our model, the root of the existence of a bubble is the presence of credit constraints. Improving credit markets can prevent the emergence of a bubble so that the economy cannot enter the bad equilibrium with high and persistent unemployment driven by self-fulfilling beliefs. Our model also implies that raising unemployment insurance benefits during a recession may exacerbate the recession because an unemployed worker is reluctant to search for a job. This result is consistent with the prediction in the DMP model. However, it is different from Kocherlakota’s result that an increase in unemployment insurance benefits funded by the young lowers the unemployment rate. We also show that the policy of hir-

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Gu and Wright (2011), He, Wright, and Zhu (2011), Rocheteau and Wright (2012) also introduce credit constraints into search models and show that bubbles may appear. But they do not study the relation between stock market bubbles and unemployment.
ing subsidies after the stock market crash can help the economy recovers from the recession faster. However, this policy cannot solve the inefficiency caused by credit constraints and hence the economy will enter a steady state with unemployment rate higher than that in the steady state with stock market bubbles.

The remainder of the paper proceeds as follows. Section 1.2 presents the model. Section 1.3 presents the equilibrium system and analyzes a benchmark model with a perfect credit market. Sections 1.4 and 1.5 study the bubbleless and bubbly equilibria, respectively. Section 1.6 introduces stochastic bubbles and show how the collapse of bubbles generates a recession and persistent and high unemployment. Section 1.7 discusses some policy implications focusing on the unemployment benefit and hiring subsidies. Section 1.8 concludes. Appendix 1.A contains technical proofs. In Appendix 1.B, we show that the Blanchard-Gali setup is isomorphic to the DMP setup, even with credit constraints. The key difference is that in the Blanchard-Gali setup vacancies are immediately filled by paying hiring costs, while in the DMP setup it takes time to fill a vacancy and employment is generated by a matching function of vacancy and unemployment. Since vacancy is not the focus of our study, we adopt the Blanchard-Gali framework.

1.2 The Model

Consider a continuous-time setup without aggregate uncertainty, based on the Blanchard and Gali (2010) model in discrete time. We follow Miao and Wang (2011a) and introduce credit constraints into this setup. To facilitate exposition, we sometimes consider a discrete-time approximation in which time is denoted by $t = 0, dt, 2dt, ...$. The continuous-time model is the limit when $dt$ goes to zero.

1.2.1 Households

There is a continuum of identical households of measure unity. Each household consists of a continuum of members of measure unity. The representative household derives utility according to the following utility function:

$$\int_0^\infty e^{-rt}C_t dt,$$  

(1.2.1)
where $r \in (0,1)$ represents the subjective discount rate, $C_t$ represents consumption. As in Merz (1995) and Andolfatto (1996), we assume full risk sharing within a large family. For simplicity, we do not consider disutility from work, as is standard in the search literature (e.g., Pissarides (2000)).

The representative household receives wages from work and unemployment benefits from the government and chooses consumption and share holdings so as to maximize the utility function in (1.2.1) subject to the budget constraint:

$$
\dot{X}_t = rX_t - C_t + w_tN_t + c(1 - N_t) - T_t, \ X_0 \ \text{given},
$$

where $X_t$ represents wealth, $N_t$ represents employment, $w_t$ represents the wage rate, $c > 0$ represents the constant unemployment compensation, and $T_t$ represents lump-sum taxes. Suppose that the unemployment compensation is financed by lump sum taxes $T_t$. Define the unemployment rate by

$$
U_t = 1 - N_t.
$$

Since we have assumed that there is no aggregate uncertainty and that each household has linear utility in consumption, the return on any asset is equal to the subjective discount rate $r$.

### 1.2.2 Firms

There is a continuum of firms of measure unity, owned by households. Each firm $j \in [0,1]$ hires $N^j_t$ workers and purchases $K_t^j$ machines to produce output $Y_t^j$ according to a Leontief technology $Y_t^j = A \min\{K_t^j, N^j_t\}$, which means that each worker requires one machine to produce. We further assume that one unit of capital costs $\kappa$. Each firm $j$ meets an opportunity to hire $H_t^j$ new workers in a frictional labor market with Poisson probability $\pi dt$ in a small time interval $[t, t + dt]$. The Poisson shock is independent across firms. Employment in firm $j$ evolves according to

$$
N^j_{t+dt} = \begin{cases} 
(1 - sdt)N^j_t + H_t^j & \text{with probability } \pi dt \\
(1 - sdt)N^j_t & \text{with probability } 1 - \pi dt
\end{cases}
$$

---

9 One can introduce disutility from work by adopting the utility function in Blanchard and Gali (2010).
10 We introduce physical capital in the model so that it can be used as collateral.
where $s > 0$ represents the exogenous separation rate. Define aggregate employment as $N_t = \int N^j_t \, dj$ and total hires as $H_t = \int H^j_t \, dj$, we can then write the aggregate employment dynamics as

$$N_{t+dt} = (1 - sdt) N_t + H_t dt. \quad (1.2.5)$$

In the continuous-time limit, this equation becomes

$$\dot{N}_t = -sN_t + H_t. \quad (1.2.6)$$

Following Blanchard and Gali (2010), define an index of market tightness as the ratio of aggregate hires to unemployment:

$$\theta_t = \frac{H_t}{U_t}. \quad (1.2.7)$$

It also represents the job-finding rate. Assume that the total hiring costs for firm $j$ are given by $G_t H^j_t$, where $G_t$ is an increasing function of market tightness $\theta_t$:

$$G_t = \psi \theta^\alpha_t, \quad (1.2.8)$$

where $\psi > 0$ and $\alpha > 0$ are parameters. Intuitively, if total hires in the market are large relative to unemployment, then workers will be relatively scarce and a firm’s hiring will be relatively costly.

Let $V_t(N^j_t)$ denote the market value of firm $j$ before observing the arrival of an hiring opportunity. It satisfies the following Bellman equation in the discrete-time approximation:

$$V_t(N^j_t) = \max_{H^j_t} \left( A - w_t \right) N^j_t dt - \left( \kappa H^j_t + G_t H^j_t \right) \pi dt + e^{-r dt} V_{t+dt} \left( (1 - sdt) N^j_t + H^j_t \right) \pi dt + e^{-r dt} V_{t+dt} \left( (1 - sdt) N^j_t \right) (1 - \pi dt), \quad (1.2.9)$$

where $\kappa$ represents the price of capital. Note that the discount rate is $r$ since firms are owned by the risk-neutral households with the subjective discount rate $\rho$.

Assume that hiring and investment are financed by internal funds and external debt:

$$(\kappa + G_t) H^j_t \leq (A - w_t) N^j_t dt + L^j_t, \quad (1.2.10)$$

where $L^j_t$ represents debt. We abstract from external equity financing. Our key insights still

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11The continuous-time Bellman equation is given by (1.A.1) in the appendix.
go through as long as external equity financing is limited. Following Carlstrom and Fuerst (1997), Jermann and Quadrini (2012), and Miao and Wang (2011a,b,c, 2012a,b), we consider intra-period debt without interest payments for simplicity. As in Miao and Wang (2011a,b,c, 2012a,b), we assume that the firm faces the following credit constraint:12

$$L_t^j \leq e^{-r dt} V_{t+dt}(\xi N_t^j),$$

(1.2.11)

where $\xi \in (0, 1]$ is a parameter representing the degree of financial frictions. This constraint can be justified as an incentive constraint in an optimal contracting problem with limited commitment. Because of the enforcement problem, lenders require the firm to pledge its assets as collateral. In our model, the firm has assets (or capital) $N_t^j$ due to the Leontief technology. It pledges assets $N_t^j$ as collateral. If the firm defaults on debt, lenders can capture $\xi N_t$ assets of the firm and the right of running the firm. The remaining fraction $1 - \xi$ accounts for default costs. Lenders and the firm renegotiate the debt and lenders keep the firm running in the next period. Thus lenders can get the threat value $e^{-r dt} V_{t+dt}(\xi N_t^j)$. Suppose that the firm has all the bargaining power as in Jermann and Quadrini (2012). Then the credit constraint in (1.2.11) represents an incentive constraint so that the firm will never default in an optimal lending contract. In the continuous-time limit as $dt \to 0$, (1.2.11) becomes

$$L_t^j \leq V_t(\xi N_t^j),$$

(1.2.12)

It follows from (1.2.10) and (1.2.11) that we can write down the combined constraint:

$$(\kappa + G_t) H_t^j \leq V_t(\xi N_t^j).$$

(1.2.13)

Note that our modeling of credit constraints is different from that in Kiyotaki and Moore (1997). In their model, when the firm defaults lenders immediately liquidate firm assets. The collateral value is equal to the liquidation value. In our model, when the firm defaults, lenders reorganize the firm and renegotiate the debt. Thus, the collateral value is equal to the going concern value of the firm.

---

12 One may introduce intertemporal debt with interest payments as in Miao and Wang (2011a). This modeling introduces an additional state variable (i.e., debt) and complicates the analysis without changing our key insights.
1.2.3 Nash Bargaining

Suppose that the wage rate can be negotiated continually and is determined by Nash bargaining at each point of time as in the DMP model. Because a firm employs multiple workers in our model, we consider the Nash bargaining problem between a household member and a firm with existing workers \( N_t \). We need to derive the marginal values to the household and to the firm when an additional household member is employed.

We can show that the marginal value of an employed worker \( V_t^N \) satisfies the following asset-pricing equation:

\[
    r V_t^N = w_t + s (V_t^U - V_t^N) + \dot{V}_t^N.
\]  

(1.2.14)

The marginal value of an unemployed \( V_t^U \) satisfies the following asset-pricing equation:

\[
    r V_t^U = c + \theta_t (V_t^N - V_t^U) + \dot{V}_t^U.
\]  

(1.2.15)

The marginal household surplus is given by

\[
    S_t^H = V_t^N - V_t^U.
\]  

(1.2.16)

It follows (1.2.14) and (1.2.15) that

\[
    r S_t^H = w_t - c - (s + \theta_t) S_t^H + \dot{S}_t^H.
\]  

(1.2.17)

The marginal firm surplus is given by

\[
    S_t^F = \frac{\partial V_t(N_t)}{\partial N_t}.
\]  

(1.2.18)

The Nash bargained wage solves the following problem:

\[
    \max_{w_t} (S_t^H)^\eta (S_t^F)^{1-\eta},
\]  

subject to \( S_t^H \geq 0 \) and \( S_t^F \geq 0 \), where \( \eta \in (0,1) \) denotes the relative bargaining power of the worker. The two inequality constraints state that there are gains from trade between the worker and the firm.
1.2.4 Equilibrium

Let $N_t = \int_0^1 N_j^t dj$, $H_t = \int_0^1 H_j^t dj$, and $Y_t = \int_0^1 Y_j^t dj$ denote aggregate employment, total hires, and aggregate output, respectively. A search equilibrium consists of trajectories of $(Y_t, N_t, C_t, U_t, \theta_t, H_t, w_t)_{t \geq 0}$ and value functions $V_t^N, V_t^U$, and $V_t$ such that (i) firms solve problem (1.2.9), (ii) $V_t^N$ and $V_t^U$ satisfy the Bellman equations (1.2.14) and (1.2.15), (iii) the wage rate solves problem (1.2.19), and (iv) markets clear in that equations (1.2.3), (1.2.6), and (1.2.7) hold and

$$C_t + (\kappa + G_t) H_t = Y_t = AN_t. \quad (1.2.20)$$

1.3 Equilibrium System

In this section, we first study a single firm’s hiring decision problem. We then analyze how wages are determined by Nash bargaining. Finally, we derive the equilibrium system by differential equations.

1.3.1 Hiring Decision

Consider firm $j$’s dynamic programming problem. Conjecture that firm value takes the following form:

$$V_t(N_j^t) = Q_t N_j^t + B_t, \quad (1.3.1)$$

where $Q_t$ and $B_t$ are variables to be determined. Because the firm’s dynamic programming problem does not give a contraction mapping, two types of solutions are possible. In the first type, $B_t = 0$ for all $t$. In the second type, $B_t \neq 0$ for some $t$. In this case, we will impose conditions later such that $B_t > 0$ for all $t$ and interpret it as a bubble. The following proposition characterizes these solutions:

**Proposition 1.1** Suppose

$$\mu_t = \frac{Q_t}{\kappa + G_t} - 1 > 0. \quad (1.3.2)$$

Then firm value takes the form in (1.3.1), where $(B_t, Q_t)$ satisfies the following differential
equations:
\[ \dot{B}_t = rB_t - \pi \mu_t B_t, \quad (1.3.3) \]
\[ \dot{Q}_t = (r + s - \xi \pi \mu_t)Q_t - (A - w_t), \quad (1.3.4) \]

and the transversality condition
\[ \lim_{T \to \infty} e^{-rT} B_T = \lim_{T \to \infty} e^{-rT} Q_T = 0. \quad (1.3.5) \]

The optimal hiring is given by
\[ H_t^i = \frac{Q_t \xi N_t^i + B_t}{\kappa + G_t}. \quad (1.3.6) \]

We use \( \pi \mu_t \) to denote the Lagrange multiplier associated with the credit constraint (1.2.13). The first-order condition for problem (1.2.9) with respect to \( H_t^i \) gives
\[ (1 + \mu_t) \left( \kappa + G_t \right) = Q_t. \quad (1.3.7) \]

If \( \mu_t = 0 \), then the borrowing constraint does not bind and the model reduces to the case with perfect capital markets. Condition (1.3.2) ensures that the credit constraint binds so that we can derive the optimal hiring in equation (1.3.6). Equation (1.3.3) is an asset-pricing equation for the bubble \( B_t \). It says that the rate of return on the bubble, \( r \), is equal to the sum of capital gains, \( \dot{B}_t / B_t \), and collateral yields, \( \pi \mu_t \). The intuition for the presence of collateral yields is similar to that in Miao and Wang (2011a): One dollar bubble raises collateral value by one dollar, which allows the firm to borrow one more dollar to finance hiring and investment costs. As a result, the firm can hire more workers and firm value rises by \( \pi \mu_t \).

We may interpret \( Q_t \) as the shadow value of capital or labor (recall the Leontief production function). Equation (1.3.2) shows that optimal hiring must be such that the marginal benefit \( Q_t \) is equal to the marginal cost \( (1 + \mu_t) \left( \kappa + G_t \right) \). The marginal cost exceeds the actual costs \( \kappa + G_t \) due to credit constraints. We thus may also interpret \( \mu_t \) as an external financing premium. Equation (1.3.4) is an asset pricing equation. It says that the return on capital \( rQ_t \) is equal to “dividends” \( A - w_t + \pi \mu_t \xi Q_t \), minus the loss of value due to separation \( sQ_t \), plus capital gains \( \dot{Q}_t \). Note that dividends consist of profits \( A - w_t \) and the shadow value of funds \( \pi \mu_t \xi Q_t \).
1.3.2 Nash Bargained Wage

Next, we derive the equilibrium wage rate, which solves problem (1.2.19). To analyze this problem, we consider a discrete-time approximation. In this case, the values of an employed and an unemployed satisfy the following equations:

\[
V_t^N = w_t dt + e^{-r dt} [s dt V_{t+dt}^U + (1 - s dt) V_{t+dt}^N],
\]

\[
V_t^U = c dt + e^{-r dt} \left[ \theta_t dt V_{t+dt}^N + (1 - \theta_t dt) V_{t+dt}^U \right].
\]

Thus, the household surplus is given by

\[
S_t^H = V_t^N - V_t^U
\]

\[
= (w_t - c) dt + e^{-r dt} (1 - s dt - \theta_t dt) (V_{t+dt}^N - V_{t+dt}^U)
\]

\[
= (w_t - c) dt + e^{-r dt} (1 - s dt - \theta_t dt) S_{t+dt}^H.
\]  

(1.3.8)

Turn to the firm surplus. Let \( \mu_t \pi \) be the Lagrange multiplier associated with constraint (1.2.10). If \( \mu_t > 0 \), then both this constraint and constraint (1.2.11) bind. Apply the envelop theorem to problem (1.2.9) to derive

\[
S_t^F = \frac{\partial V_t \left( N_{t}^j \right)}{\partial N_{t}^j} = (A - w_t) dt + e^{-r dt} (\xi \pi \mu_t dt + 1 - s dt) \frac{\partial V_{t+dt} \left( N_{t+dt}^j \right)}{\partial N_{t+dt}^j}
\]

\[
= (A - w_t) dt + e^{-r dt} (\xi \pi \mu_t dt + 1 - s dt) S_{t+dt}^F.
\]  

(1.3.9)

Note that the continuous-time limit of this equation is (1.3.4) since \( S_t^F = Q_t \) by (1.3.1).

Using equations (1.3.8) and (1.3.9), we can rewrite problem (1.2.19) as

\[
\max_{w_t} \left[ (w_t - c) dt + e^{-r dt} (1 - s dt - \theta_t dt) S_{t+dt}^H \right]^\eta \times \left[ (A - w_t) dt + e^{-r dt} (\xi \pi \mu_t dt + 1 - s dt) S_{t+dt}^F \right]^{1-\eta}.
\]

The first-order condition implies that

\[
\eta S_t^F = (1 - \eta) S_t^H.
\]  

(1.3.10)

This sharing rule is the same with the standard Nash bargaining solution in the DMP model,
which says in the equilibrium the worker gets \( \eta \) proportion of the total surplus of a match and the firm gets the remaining part.

Since we have assumed that wage is negotiated continually, equation (1.3.10) also holds in rates of change as in Pissarides (2000, p. 28). We thus obtain

\[
\eta \dot{S}_t^F = (1 - \eta) \dot{S}_t^H. \tag{1.3.11}
\]

Substituting equations (1.2.17), (1.3.4), and \( S_t^F = Q_t \) into the above equation yields

\[
\eta [(r + s - \xi \pi \mu_t)Q_t - (A - w_t)]
= (1 - \eta) [(r + \theta_t + s) S_t^H - w_t + c] \]

Using equation (1.3.10) and \( S_t^F = Q_t \), we can solve the above equation for the wage rate:

\[
w_t = \eta [A + (\xi \pi \mu_t + \theta_t) Q_t] + (1 - \eta) c. \tag{1.3.12}
\]

This equation shows that the Nash bargained wage is equal to a weighted average of the unemployment benefit and a term consisting of two components. The weight is equal to the relative bargaining power. The first component is productivity \( \xi \pi \mu_t \). The second component is related to the value from external financing and the threat value of the worker, \( (\xi \pi \mu_t + \theta_t) Q_t \). Workers are rewarded for the saving of external funds to finance hiring costs. Holding everything else constant, a higher external finance premium leads to a higher wage rate. The market tightness or the job-finding rate, \( \theta_t \), affects a household’s threat value. Holding everything else constant, a higher value of market tightness, \( \theta_t \), implies that a searcher can more easily find a job and hence he demands a higher wage. The second component is also positively related to \( Q_t \), holding everything else constant. The intuition is that workers get higher wages when the marginal \( Q \) of the firm is higher.

1.3.3 Equilibrium

Finally, we conduct aggregation and impose market-clearing conditions. We then obtain the equilibrium system.

**Proposition 1.2** Suppose \( \mu_t > 0 \), where \( \mu_t \) satisfies (1.3.2). Then the equilibrium dynamics
for \((B_t, Q_t, N_t, U_t, \theta_t, H_t, w_t)\) satisfy the system of equations (1.3.3), (1.3.4), (1.2.6), (1.2.3), (1.2.7), (1.3.12), and

\[
H_t = \pi \frac{Q_t \xi N_t + B_t}{\kappa + G_t},
\]

where \(G_t\) is given by (1.2.8). The transversality condition in (1.3.5) also holds.

It follows from this proposition that there are two types of equilibrium. In the first type, \(B_t = 0\) for all \(t\). In the second type, \(B_t \neq 0\) for some \(t\). Because firm value cannot be negative, we restrict attention to the case with \(B_t > 0\) for all \(t\). We call the first type of equilibrium the bubbleless equilibrium and the second type the bubbly equilibrium. Intuitively, if \(N_t^f = 0\), the firm has no worker or capital, one may expect its intrinsic value should be zero. Thus, the positive term \(B_t > 0\) represents a bubble in firm value.

### 1.3.4 A Benchmark with Perfect Credit Markets

Before analyzing the model with credit constraints, we first consider a benchmark without credit constraints. In this case, the Lagrange multiplier associated with the credit constraint is zero, i.e., \(\mu_t = 0\). Since Appendix B shows that this model is isomorphic to a standard DMP model as in Chapter 1 of Pissarides (2000), we will follow a similar analysis.

We still conjecture that firm value takes the form given in (1.3.1). Following a similar analysis for Proposition 1.1, we can show that

\[
Q_t = \kappa + G_t = \kappa + \psi \theta_t^a,
\]

\[
rQ_t = A - w_t - sQ_t + \dot{Q}_t,
\]

\[
rB_t = \dot{B}_t.
\]

By the transversality condition, we deduce that \(B_t = 0\). It follows that a bubble cannot exist for the model with perfect credit markets.

The wage rate is determined by Nash bargaining as in Section 1.3.2. We can show that the Nash bargained wage satisfies

\[
w_t = \eta (A + \theta_t Q_t) + (1 - \eta) c.
\]
Using (1.3.14) and (1.3.16), we can rewrite (1.3.15) as
\[
\dot{Q}_t = (r + s) Q_t - A + \eta (A + \theta_t Q_t) + (1 - \eta) c.
\]
(1.3.17)

Using (1.2.3), (1.2.6), and (1.2.7), we obtain
\[
\dot{N}_t = -s N_t + \theta_t (1 - N_t),
\]
(1.3.18)

An equilibrium can be characterized by a system of differential equations (1.3.17) and (1.3.18) for \((Q_t, N_t)\), where we use (1.3.14) to substitute for \(\eta\).

Now, we analyze the steady state for the above equilibrium system. Equations (1.3.14) and (1.3.15) give the steady state relation between \(w\) and \(\theta\) :
\[
A - w = (r + s) (\psi \theta^\alpha + \kappa).
\]
(1.3.19)

We plot this relation in Figure 1.4 and call it the job creation curve, following the literature on search models, e.g., Pissarides (2000). In the \((\theta, w)\) space it slopes down: Higher wage rate makes job creation less profitable and so leads to a lower equilibrium ratio of new hires to unemployed workers. It replaces the demand curve of Walrasian economics.

Equations (1.3.14) and (1.3.16) give another steady state relation between \(w\) and \(\theta\) :
\[
w = \eta (A + \theta (\psi \theta^\alpha + \kappa)) + (1 - \eta) c.
\]
(1.3.20)

We plot this relation in Figure 1.4 and call it the wage curve, as in Pissarides (2000). This curve slopes up: At higher market tightness the relative bargaining strength of market participants shifts in favor of workers. It replaces the supply curve of Walrasian economics.

The steady state equilibrium \((\bar{\theta}, \bar{w})\) is at the intersection of the two curves. Clearly, when \(\theta\) goes to infinity the wage curve approaches positive infinity and the job creation curve approaches negative infinity. When \(\theta\) approaches zero, we impose the assumption
\[
(1 - \eta) (A - c) > (r + s) \kappa,
\]
(1.3.21)
so that the job creation curve is above the wage curve at \(\theta = 0\). The preceding properties of the two curves ensure the existence and uniqueness of the steady state equilibrium \((\bar{\theta}, \bar{w})\).
Figure 1.4: The job creation and wage curves for the steady state equilibrium with perfect credit markets

Once we obtain \((\bar{\theta}, \bar{w})\), the other steady state equilibrium variables can be easily derived. For example, we can determine \((\bar{H}, \bar{U})\) using equations \(H = s(1 - U)\) and \(H = \theta U\). The first equation is analogous to the Beveridge curve and is downward sloping as illustrated in Figure 1.5.

Figure 1.5: Determination of hiring and unemployment for the benchmark model with perfect credit markets
Turn to local dynamics. We linearize the equilibrium system (1.3.17) and (1.3.18) around the steady state, where \( \theta_t \) is replaced by a function of \( Q_t \) using (1.3.14). We then obtain the linearized system:

\[
\begin{bmatrix}
\dot{Q}_t \\
\dot{N}_t
\end{bmatrix} =
\begin{bmatrix}
+ & 0 \\
+ & -
\end{bmatrix}
\begin{bmatrix}
Q_t - \bar{Q} \\
N_t - \bar{N}
\end{bmatrix}.
\]

Given the sign pattern of the matrix, the determinant is negative. Thus, the steady state is a saddle point. Note that \( N_t \) is predetermined and \( Q_t \) is non-predetermined. Since the differential equation for \( Q_t \) does not depend on \( N_t \), \( Q_t \) must be constant along the transition path. This implies that \( \theta_t \) must also be constant along the transition path.

If \( N_0 \) or \( U_0 \) is out of the steady state, say \( U_0 > \bar{U} \), then the market tightness is relatively low. An unemployed worker is harder to find a job and hence he bargains a lower wage. This causes firm value to rise initially, inducing firms to hire more workers immediately. As a result, unemployment falls. During the transition path, firms adjust hiring to maintain the ratio of hires and unemployment constant, until reaching the steady state.

1.4 Bubbleless Equilibrium

From then on, we focus on the model with credit constraints. In this case, multiple equilibria may emerge. In this section, we analyze the bubbleless equilibrium in which \( B_t = 0 \) for all \( t \). We first characterize the steady state and then study transition dynamics.

1.4.1 Steady State

We use Proposition 1.2 to show that the bubbleless steady-state equilibrium \((Q, N, U, \theta, H, w)\) satisfies the following system of six algebraic equations:

\[
0 = (r + s - \pi \mu \xi)Q - (A - w),
\]

\[H = \pi \frac{Q \xi N}{\kappa + G},\]

\[0 = -sN + H,
\]

\[U = 1 - N,
\]

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\[
\theta = \frac{U}{H}, \quad (1.4.5)
\]
\[
w = \eta [A + (\xi \pi \mu + \theta) Q] + (1 - \eta) c, \quad (1.4.6)
\]

where

\[
\mu = \frac{Q}{\kappa + G} - 1, \quad (1.4.7)
\]
\[
G = \psi \theta^* . \quad (1.4.8)
\]

Solving the above system yields:

**Proposition 1.3** If

\[
0 < \xi < \frac{s}{\pi}, \quad (1.4.9)
\]
\[
A - c > \frac{\kappa s}{\pi \xi (1 - \eta)} [\eta (s - \pi \xi) + r + \pi \xi], \quad (1.4.10)
\]

where

\[
\mu^* = \frac{s}{\pi \xi} - 1, \quad (1.4.11)
\]

then there exists a unique bubbleless steady-state equilibrium \((Q^*, N^*, U^*, \theta^*, H^*, w^*)\) satisfying

\[
Q^* = \frac{s}{\pi \xi} (\kappa + \psi \theta^*), \quad (1.4.12)
\]
\[
N^* = \frac{\theta^*}{s + \theta^*}, \quad (1.4.13)
\]

where \(\theta^*\) is the unique solution to the equation for \(\theta\) :

\[
\frac{(1 - \eta) (A - c)}{\kappa + \psi \theta^*} = \frac{s}{\pi \xi} [r + \pi \xi + \eta (s - \pi \xi + \theta)]. \quad (1.4.14)
\]

Condition (1.4.9) ensures that \(\mu^* > 0\) so that we can apply Proposition 1.2 in a neighborhood of the steady state. The steady state can be derived using the job creation and wage curves analogous to those discussed in Section 1.3.4. We first substitute \(H\) in (1.4.2) into (1.4.3) to derive

\[
s = \pi \frac{Q \xi}{\kappa + G}. \quad (1.4.15)
\]
Rearranging terms, we can solve for $Q$:

$$Q = \frac{s}{\pi \xi} (\kappa + G).$$

(1.4.16)

Combining the above equation with (1.4.7), we obtain the solution for $\mu$ in (1.4.11). Plugging this solution and the expression for $Q$ into (1.4.1), we obtain

$$A - w = \frac{s (r + \pi \xi)}{\pi \xi} (\kappa + \psi \theta^\alpha).$$

(1.4.17)

This equation defines $w$ as a function of $\theta$ and gives the job creation curve. It is downward sloping as illustrated in Figure 1.6.

Next, substituting (1.4.16), (1.4.8), and (1.4.11) into equation (1.4.6), we can express $w$ as a function of $\theta$:

$$w = \eta \left[ A + \frac{s (s - \pi \xi + \theta)}{\pi \xi} (\kappa + \psi \theta^\alpha) \right] + (1 - \eta) c. $$

(1.4.18)

This equation gives the upward sloping wage curve. The equilibrium $(\theta^*, w^*)$ is determined by the intersection of the job creation and wage curves as illustrated in Figure 1.6. As in Section 1.3.4, the equilibrium $(H^*, U^*)$ is determined the Figure 1.5.

![Figure 1.6: The job creation and wage curves for the bubbleless steady state equilibrium](image)

What is the impact of credit constraints? Figure 1.6 also plots the job creation and wage curves for the benchmark model with perfect credit markets. It is straightforward to show
that, in the presence of credit constraints, both the job creation and wage curves shift to the left. As a result, credit constraints lower the steady state market tightness. The impact on wage is ambiguous. We can then use Figure 1.5 to show that credit constraints reduce hiring and raise unemployment.

**Proposition 1.4** Suppose that conditions (1.3.21), (1.4.9), and (1.4.10) are satisfied. Then \( \theta^* < \bar{\theta} \), \( H^* < \bar{H} \), and \( U^* > \bar{U} \). Namely, the labor market tightness and hiring are lower, but unemployment is higher, in the bubbleless steady state with credit constraints than in the steady state with perfect credit markets.

### 1.4.2 Transition Dynamics

Turn to transition dynamics. The predetermined state variable for the equilibrium system is \( N_t \) and the nonpredetermined variables are \( (Q_t, U_t, \theta_t, H_t, w_t) \). Simplifying the system, we can represent it by a system of two differential equations for two unknowns \( (Q_t, N_t) \): (1.3.4) and (1.3.18). In this simplified system, we have to represent \( \theta_t \), \( \mu_t \) and \( w_t \) in terms of \( (Q_t, N_t) \).

To this end, we use (1.2.7), (1.3.13) and (1.2.8) to solve for \( \theta_t \), which satisfies:

\[
\theta_t (1 - N_t) = \pi \frac{Q_t \xi N_t}{\kappa + \psi \theta_t^*}.
\]

We then use (1.3.2) to get \( \mu_t \). Finally, we use (1.3.12) to solve the wage \( w_t \).

To study local dynamics around the bubbleless steady state, one may linearize the preceding simplified equilibrium system and compute eigenvalues. Since this system is highly nonlinear, we are unable to derive an analytical result. We thus use a numerical example to illustrate transition dynamics.\(^{13}\) We set the parameter values as follows. Let one unit of time represent one quarter. Normalize the labor productivity \( A = 1 \) and set \( r = 0.012 \). Shimer (2005) documents that the monthly separation rate is 3.5% and the replacement ratio is 0.4, so we set \( s = 0.1 \) and \( c = 0.4 \). As Appendix 1.B shows, the hiring cost corresponds to the matching function in the DMP model (also see Blanchard and Gali (2010)). Following Blanchard and Gali (2010), we set \( \alpha = 1 \). We then choose \( \psi = 0.05 \) to match the average cost of hiring a worker, which is about 4.5% of quarterly wage, according to Gali (2011).\(^{13}\)

\(^{13}\)We use the reverse shooting method to numerically solve the system of differential equations (see, e.g., Judd (1998)).
Set $\xi = 0.75$, which is the number estimated by Liu, Wang and Zha (2012) and is widely used in the literature. Cooper and Haltiwanger (2006) document that the annual spike rate of positive investment is 18%, so we choose $\pi = 4.5%$. Since there is no direct evidence on the bargaining power of workers, we simply choose $\eta = 0.5$ as in the literature. Finally, we choose $\kappa = 0.15$ to match the unemployment rate after the bubble bursts, which is around 10% during the recent Great Recession.\textsuperscript{14} For the preceding parameter values, conditions (1.4.9) and (1.4.10) are satisfied.

We compute the steady state $(Q, N) = (0.5755, 0.8985)$. We find that both eigenvalues associated with the linearized system around the steady state are real. One of them is positive and the other one is negative. The negative eigenvalue corresponds to the predetermined variable $N_t$. Thus, the steady state is a saddle point and the system is saddle path stable.

Figure 1.7 plots the transition paths. Suppose that the unemployment rate is initially low relative to the steady state. Then the market tightness is relatively high. Thus, an unemployed worker is easier to find a job and hence bargains a higher wage. This in turns lowers firm value and marginal $Q$, causing a firm to reduce hiring initially. In addition, because the initial unemployment rate is low, the initial output is high. The firm then gradually increases hiring. However, the increase is slower than the exogenous separation rate, causing the unemployment rate to rise gradually. Unlike the case of perfect credit markets analyzed in Section 1.3.4, the market tightness $\theta_t$ is not constant during adjustment. In fact, it falls gradually. As a result, the job-finding rate falls gradually, leading the wage rate to fall too. Output also falls over time, but firm value rises. The increase in firm value is due to the increase in marginal $Q$. The gradual rise in marginal $Q$ is due to two effects. First, because hires rise over time, the firm uses more external financing and hence the external finance premium $\mu_t$ rises over time. Second, since wage falls over time, the profits rise over time.

\textsuperscript{14}After the bubble bursts, the economy moves gradually to the bubbleless steady state. Equation (1.4.14) and (1.4.13) imply that given all the other parameters, there is a one-to-one mapping between $\kappa$ and unemployment rate $(1 - N^*)$.
1.5 Bubbly Equilibrium

We now turn to the bubbly equilibrium in which $B_t > 0$ for all $t$. We first study steady state and then examine transition dynamics.

1.5.1 Steady State

We use Proposition 1.2 to show that the bubbleless steady state equilibrium $(B, Q, N, U, \theta, H, w)$ satisfies the following system of seven equations: (1.4.1), (1.4.3), (1.4.4), (1.4.5), (1.4.6) and

\[ 0 = rB - \pi \mu B, \quad (1.5.1) \]

\[ H = \pi \frac{Q \xi N + B}{\kappa + G}, \quad (1.5.2) \]

where $\mu$ and $G$ satisfy (1.4.7) and (1.4.8).

Solving the above system yields:
Proposition 1.5 If

\[ 0 < \xi < \frac{s}{r + \pi}, \]  
\[ A - c > \frac{(r + \pi) \kappa}{\pi (1 - \eta)} [\eta \xi r + r + s - r \xi], \]  

where \( \mu_b = r/\pi \), then there exists a bubbly steady-state equilibrium \( (B, Q_b, N_b, U_b, w_b, \theta_b) \) satisfying

\[ \frac{B}{N_b} = (\kappa + \psi\theta_b^\alpha) \left[ \frac{s}{\pi} - \frac{r + \pi}{\pi} \xi \right] > 0, \]  
\[ Q_b = \frac{r + \pi}{\pi} (\kappa + \psi\theta_b^\alpha), \]  
\[ N_b = \frac{\theta_b}{s + \theta_b}, \]  

where \( \theta_b \) is the unique solution to the equation for \( \theta \) :

\[ \frac{(1 - \eta) (A - c)}{\kappa + \psi\theta^\alpha} = \frac{r + \pi}{\pi} [\eta (\xi r + \theta) + r + s - r \xi]. \]  

Condition (1.5.3) ensures that \( B > 0 \). In addition, it also guarantees that condition (1.4.9) holds so that a bubbleless steady-state equilibrium also exists. To see how the steady-state \( \theta_b \) is determined, we derive the job creation and wage curves as in the case of bubbleless equilibrium. First, we plug equation (1.5.2) into (1.4.3) to derive

\[ s = \pi \frac{Q \xi + B/N}{\kappa + \psi\theta^\alpha}. \]  

Then use equation (1.5.1) to derive \( \mu_b = r/\pi \). Using (1.4.7) yields

\[ Q = \frac{r + \pi}{\pi} (\kappa + G). \]  

Plugging equation (1.5.10) into equation (1.5.9) yields the expression for \( B/N \) in (1.5.5). Plugging equation (1.5.10) and (1.4.8) into (1.4.1) yields

\[ A - w = \frac{(r + \pi) (r + s - r \xi)}{\pi} (\kappa + \psi\theta^\alpha). \]  

The above equation gives \( w \) as a function of \( \theta \). In Figure 1.8, we plot this function and call the resulting curve the job creation curve. As in the case for the bubbleless equilibrium, this curve is downward sloping.
Next, substituting $\mu = \mu_b = r/\pi$, (1.5.10), and (1.4.8) into (1.4.6), we can express wage $w$ as a function of $\theta$:

$$w = \eta \left[ A + (\xi r + \theta) \frac{r + \pi}{\pi} (\kappa + \psi \theta^\alpha) \right] + (1 - \eta) c,$$

(1.5.12)

This gives the upward sloping wage curve as illustrated in Figure 1.8. The equilibrium $(\theta_b, w_b)$ is at the intersection of the two curves. As in the case of the bubbleless steady state, condition (1.5.4) ensures the existence of an intersection point. Equation (1.5.8) expresses the solution for $\theta_b$ in a single nonlinear equation.

How does the stock market bubble affect steady-state output and unemployment? To answer this question, we compare the bubbleless and the bubbly steady states. In the appendix, we show that both the job creation curve and the wage curve shift to the right in the presence of bubbles as illustrated in Figure 1.8. The intuition is the following: In the presence of a stock market bubble, the collateral value rises and the credit constraint is relaxed. Thus, a firm can finance more hires and create more jobs for a given wage rate. This explains why the job creation curve shifts to the right. Turn to the wage curve. For a given level of market tightness, the presence of a bubble puts the firm in a more favorable bargaining position because more jobs are available. This allows the firm to negotiate a lower wage rate.

The above analysis shows that the market tightness is higher in the bubbly steady state than in the bubbleless steady state. This in turn implies that hires and output are higher and unemployment is lower in the bubbly steady state than in the bubbleless steady state by Figure 1.5. Note that the comparison of the wage rate is ambiguous depending on the magnitude of the shifts in the two curves. If the job creation curve shifts more than the wage curve, then the wage rate should rise in the bubbly steady state. Otherwise, the wage rate should fall in the bubbly steady state.
Figure 1.8: The job creation and wage curves for the bubbly steady state equilibrium

We summarize the above result in the following:

**Proposition 1.6** Suppose that conditions (1.3.21), (1.4.10), (1.5.3), and (1.5.4) hold. Then in the steady state, $\bar{\theta} > \theta_b > \theta^*$, $\bar{H} > H_b > H^*$, and $\bar{U} < U_b < U^*$.

How is the bubbly steady-state equilibrium with credit constraints compared to the steady-state equilibrium with perfect credit markets analyzed in Section 1.3.4? We can easily check that the presence of bubbles in the model with credit constraints shifts the job creation curve in Figure 1.5 to the right, but it shifts the wage curve to the left in Figure 1.4. It seems that the impact on the market tightness is ambiguous. In the appendix, we show that the effect of the wage curve shift dominates so that $\bar{\theta} > \theta_b$. As a result, $H_b < \bar{H}$, and $U_b > \bar{U}$. The intuition is that even though the presence of bubbles can relax credit constraints and allows the firm to hire more workers, wages absorb the rise in firm value and reduce the firm’s incentive to hire.

### 1.5.2 Transition Dynamics

Turn to transition dynamics. As in the bubbleless equilibrium, the predetermined state variable for the equilibrium system is still $N_t$. But we have one more nonpredetermined
variable, which is the stock price bubble $B_t$. Following a similar analysis for the bubbleless equilibrium in Section 1.4.2, we can simplify the equilibrium system and represent it by a system of three differential equations for three unknowns $(B_t, Q_t, N_t)$. We are unable to derive an analytical result for stability of the bubbly steady state. We thus use a numerical example to illustrate local dynamics. We still use the same parameter values given in Section 1.4.2. We note that the conditions in Proposition 1.5 are satisfied. Thus, both bubbleless and bubbly equilibria exist. In addition, one can check that these conditions are also satisfied for $\xi = 1$, implying that multiple equilibria can exist, even though there is no efficiency loss at default.

We find the steady state $(B, Q, N) = (0.2873, 0.3021, 0.9465)$. We then linearize around this steady state and compute eigenvalues. We find that two of the eigenvalues are positive and real and only one of them is negative and real and corresponds to the predetermined variable $N_t$. Thus, the steady state is a saddle point and the system is saddle path stable.

![Figure 1.9: Transitional dynamics for the bubbly equilibrium](image)

Figure 1.9 plots the transition dynamics. Suppose the unemployment rate is initially low relative to the steady state. For a similar intuition analyzed before, the initial hiring rate must be lower than the steady state level and then gradually rises to the steady state. Other equilibrium variables follow similar patterns to those in Figure 1.7 during adjustment, except for bubbles. Bubbles rise gradually to the steady state value. By (1.3.3), the growth rate of bubbles is equal to the interest rate minus the shadow value of funds, $r - \pi \mu_t$. As the shadow
value of external funds rises over time, the growth rate of bubbles decreases and until it reaches zero.

1.6 Stochastic Bubbles

So far, we have studied deterministic bubbles. In this section, we follow Blanchard and Watson (1982), Weil (1987), and Miao and Wang (2011a) and introduce stochastic bubbles. Suppose that initially the economy has a stock market bubble. But the bubble may burst in the future. The bursting event follows a Poisson process and the arrival rate is given by $\lambda > 0$. When the bubble bursts, it will not reappear in the future by rational expectations. After the burst of the bubble, the economy enters the bubbleless equilibrium studied in Section 1.4. We use a variable with an asterisk to denote its value in the bubbleless equilibrium. In particular, let $V^* (N^j_t, Q^*_t)$ denote the value function for firm $j$ with employment $N^j_t$ and the shadow price of capital $Q^*_t$. As we show in Proposition 1.1, $V^* (N^j_t, Q^*_t) = Q^*_t N^j_t$. We can also represent $Q^*_t$ in a feedback form in that $Q^*_t = g(N_t)$ for some function $g$.

We denote by $V (N^j_t, B_t, Q_t)$ the stock market value of firm $j$ at date $t$ before the bubble bursts. This value function satisfies the continuous-time Bellman equation:

$$r V (N^j_t, B_t, Q_t) = \max_{H^j_t} \left( A - w_t \right) N^j_t - s N^j_t V_N (N^j_t, B_t, Q_t)$$

$$+ \delta \left[ V^* (N^j_t, Q^*_t) - V (N^j_t, B_t, Q_t) \right]$$

$$+ \pi \left[ V (N^j_t + H^j_t, B_t, Q_t) - V (N^j_t, B_t, Q_t) - (\kappa + G_t) H^j_t \right]$$

$$+ \dot{Q}_t V_Q (N^j_t, B_t, Q_t) + \dot{B}_t V_B (N^j_t, B_t, Q_t),$$

subject to the borrowing constraint

$$(\kappa + G_t) H^j_t \leq V (\xi N^j_t, B_t, Q_t).$$

As in Section 1.3, we conjecture that the value function takes the following form:

$$V (N^j_t, B_t, Q_t) = Q_t N^j_t + B_t,$$

where $Q_t$ and $B_t$ are to be determined variables. Here $B_t$ represents the stock price bubble. Following a similar analysis in Section 1.3, we can derive the Nash bargaining wage and
characterize the equilibrium with stochastic bubbles in the following:

**Proposition 1.7** Suppose that \( \mu_t > 0 \) where \( \mu_t \) is given by (1.3.2). Before the bubble bursts, the equilibrium with stochastic bubbles \((B_t, Q_t, N_t, U_t, \theta_t, H_t, w_t)\) satisfies the following system of differential equations: (1.2.3), (1.2.6), (1.2.7), (1.3.13), (1.3.12)

\[
\dot{B}_t = (r + \delta) B_t - \pi \mu_t B_t, \tag{1.6.4}
\]

\[
\dot{Q}_t = (r + s + \delta) Q_t - \delta Q^* - (A - w_t) - \pi \mu_t \xi Q_t, \tag{1.6.5}
\]

where \( G_t \) satisfies (1.2.8) and \( Q^*_t = g(N_t) \) is the shadow price of capital after the bubble bursts.

As Proposition 1.6 shows, the system for the equilibrium with stochastic bubbles is similar to that for the bubbly equilibrium with two differences. First, the equations for bubbles in (1.3.3) and (1.6.4) are different. Because bubbles may burst, (1.6.4) says that the expected return on bubbles is equal to \( r \). Second, the equations for \( Q \) in (1.3.4) and (1.6.5) are different. In particular, immediately after the collapse of bubbles, \( Q_t \) jumps to the saddle path in the bubbleless equilibrium, \( Q^*_t = g(N_t) \).

Following Weil (1987), Kocherlakota (2009), and Miao and Wang (2011a), we focus on a particular type of equilibrium with stochastic bubbles. In this equilibrium, \( B_t, N_t, Q_t, U_t, H_t, \theta_t, \) and \( w_t \) are constant before the bubble bursts. We denote the constant values by \( B^*, N^*, Q^*, U^*, H^*, \theta^*, \) and \( w^* \). These 7 variables satisfy the system of 7 equations: (1.4.3), (1.4.4), (1.4.5), (1.4.6), (1.5.2) and

\[
0 = (r + \delta) B^* - \pi \mu^* B^*,
\]

\[
0 = (r + s + \delta) Q^* - \delta g(N^*) - (A - w^*) - \pi \mu^* \xi Q^*. \tag{1.6.6}
\]

After the burst of bubbles, the economy enters the bubbleless equilibrium. Immediately after the collapse of bubbles, \( B^* > 0 \) jumps to zero and \( Q^*, H^*, \theta^*, \) and \( w^* \) jump to the bubbleless equilibrium \( Q^*_t, H^*_t, \theta^*_t, \) and \( w^*_t \), respectively. But \( N^* \) and \( U^* = 1 - N^* \) continuously move to \( N_t^* \) and \( U_t^* = 1 - N_t^* \) because \( N_t \) is a predetermined state variable.
Figure 1.10: Transition paths for the equilibrium with stochastic bubbles

Figure 1.10 plots the transition paths for the equilibrium with stochastic bubbles. We still use the parameter values given in Section 1.4.2. We suppose that households believe that, with Poisson arrival rate $\delta = 0.95\%$, the bubble can burst.\footnote{We choose $\delta = 0.95\%$ so that the annual bursting rate is 3.8\%, which is the disaster risk estimated by Barro and Ursua (2008) and Barro and Jin (2011). Using this number, the model implies that the unemployment rate before the bubble bursts is 5.9\%, which is close to the historical average 5.8\% in the US data from 1948m1 to 2011m12.} We also suppose that the bubble bursts at time $t = 10$. Because the unemployment rate is predetermined, it rises continuously to the new higher steady state level. Output falls continuously to the new lower steady state level. Other equilibrium variables jump to the transition paths for the bubbleless equilibrium analyzed in Section 1.4.2. In particular, the stock market crashes in that the stock market value of the firm falls discontinuously. The hiring rate and the job-finding rate also fall discontinuously.

The wage rate rises immediately after the crash and then gradually falls to the new higher steady state level. The immediate rise in the wage rate reflects three effects. First, the job-finding rate falls on impact, leading to a fall of wage. Second, the external finance premium and marginal Q rise on impact, leading to the rise in the wage rate. Overall the second effect dominates. The rise in wage may seem counterintuitive. Figure 1.11 plots the data of U.S. real hourly wages from BLS and the price-earnings ratio from Robert Shiller’s website. The sample is from the first month of 1964 to the last month of 2011. We find that during the recession in the early 2000 and the recent Great Recession, real hourly wages actually rose.
However, during other recessions, they fell. Though our model does not intend to explain wage dynamics, it gives an explanation of rising wages during a recession based on the fact that firms that hire during recessions are those that are more profitable and hence can pay workers higher wages.

![Figure 1.11: Real hourly wages and the stock market](image)

1.7 Policy Implications

Our model features two types of inefficiency: credit constraints and search and matching. We have shown that bubbles cannot emerge in an economy with perfect credit markets. They can emerge in the presence of credit constraints. Thus, it is important to improve credit markets in order to prevent the formation of bubbles. Miao and Wang (2011a, 2012a) have discussed credit policy related to the credit market. In this section, we shall focus on policies related to the labor market and study how these policies affect the economy.

1.7.1 Unemployment Benefits

In response to the Great Recession, the U.S. government has expanded unemployment benefits dramatically. Preexisting law provided for up to 26 weeks of benefits, plus up to 20
additional weeks of “Extended Benefits” in states experiencing high unemployment rates. Starting in June 2008, Congress enacted a series of unemployment benefits extensions that brought statutory benefit durations to as long as 99 weeks. In addition to the moral hazard problem, unemployment insurance extensions can lead recipients to reduce their search effort and raise their reservation wages, slowing the transition into employment.

We now use the model in Section 1.6 to conduct an experiment in which the unemployment benefit is raised from 0.40 to 0.50 permanently immediately after the burst of the bubble. This policy experiment resembles a 25 percent of increase in the unemployment benefits. Figure 1.12 plots the transition paths for the parameter values given in Section 1.4.2. This figure reveals that this policy makes the recession more severe. In particular, the fall of the job-finding rate, hires, and the stock market value is larger on impact and these variables gradually move to their lower steady state values. In addition, the unemployment rate rises and gradually move to a higher steady state level. The new steady state unemployment rate is about 2 percentage point higher than the steady state level without the policy.

![Figure 1.12: The impact of raising unemployment benefits](image)

1.7.2 Hiring Subsidies

A potentially powerful policy to bring the labor market back from a recession is to subsidize hiring. In March 2010, Congress enacted the Hiring Incentives to Restore Employment Act,
which essentially provided tax credit for private businesses to hire new employees. We now use the model in Section 1.6 to conduct a policy experiment in which the parameter $\psi$ is reduced from 0.05 to 0.0375 permanently immediately after the burst of the bubble. This policy experiment resembles a 25 percent hiring subsidy.

Figure 1.13 plots the transition paths for the parameter values given in Section 1.4.2. This figure shows that hiring subsidies make the recession less severe and help the economy move out of the recession faster. In particular, immediately after the collapse of the bubble, the policy helps firms start hiring more workers. It also helps the job-finding rate rise to a higher level. As a result, the unemployment rate rises to a lower level after the stock market crash, compared to the case without the hiring subsidy policy.

![Figure 1.13: The impact of hiring subsidies](image)

1.8 Conclusion

In this paper, we have introduced endogenous credit constraints in a search model of unemployment. We have shown that the presence of credit constraints can generate multiple equilibria. In one equilibrium, there is a bubble in the stock market value of the firm. The bubble helps relax the credit constraints and allows firms to make more investment and hire more workers. The collapse of the bubble tightens the credit constraints, causing firms to
cut investment and reduce hiring. Consequently, workers are harder to find a job, generating high and persistent unemployment. In the model, there is no aggregate shock to the fundamentals. The stock market crash and subsequent recession are generated by shifts in households beliefs.

In terms of policy implications, the policymakers should fix the credit market since it is the root cause of bubbles. Extending unemployment insurance benefits will exacerbate unemployment and recession, while hiring subsidies can help the economy recover faster. But the economy will converge to a steady state with unemployment still higher than that in the bubbly steady state.
Appendix

1.A Proofs

Proof of Proposition 1.1: Let the value function be $V \left( N_t^i, Q_t, B_t \right)$. The continuous-time limit of the Bellman equation (1.2.9) is given by

$$
r V \left( N_t^i, Q_t, B_t \right) = \max_{H_t^i} \left( (A - w_t) N_t^i - s N_t^i V_N \left( N_t^i, Q_t, B_t \right) \right)$$

$$+ \pi \left( V \left( N_t^i + H_t^i, Q_t, B_t \right) - V \left( N_t^i, Q_t, B_t \right) - (\kappa + G_t) H_t^i \right)$$

$$+ V_Q \left( N_t^i, Q_t, B_t \right) \dot{Q}_t + V_B \left( N_t^i, Q_t, B_t \right) \dot{B},$$

subject to (1.2.10) and (1.2.12). Conjecture that the value function takes the form in (1.3.1). Substituting this conjecture into the above Bellman equation yields:

$$r Q_t N_t^i + r B_t = \max_{H_t^i} \left( (A - w_t) N_t^i - s N_t^i Q_t \right)$$

$$+ \pi \left( Q_t - (\kappa + G_t) \right) H_t^i + N_t^i \dot{Q}_t + \dot{B},$$

subject to

$$(\kappa + G_t) H_t^i \leq \xi Q_t N_t^i + B_t.$$

Let $\mu_t \pi$ be the Lagrange multiplier associated with the above constraint. Then the first-order condition for $H_t^i$ implies that

$$Q_t = (\kappa + G_t) \left( 1 + \mu_t \right).$$

Clearly, if $\mu_t > 0$, then the credit constraint (1.4.2) binds so that $H_t^i$ is given by (1.3.6). Matching coefficients of $N_t^i$ and the other terms unrelated to $N_t^i$ yields equations (1.3.3) and (1.3.4). Q.E.D.

Proof of Proposition 1.2: We have derive the wage equation in (1.3.12). Aggregating $H_t^i$ in equation (1.3.6) yields (1.3.13). Other equations in the proposition follow from definitions. Q.E.D.
Proof of Proposition 1.3: Part of the proof is contained in Section 1.4.1. Equation (1.4.12) follows from the substitution of (1.4.11) and (1.4.8) into (1.4.16). Equation (1.4.13) follows from (1.4.3), (1.4.4), and (1.4.5). Finally, (1.4.14) follows from equations (1.4.17) and (1.4.18). Q.E.D.

Proof of Proposition 1.4: The proof uses Figures 1.6 and 1.7 and simple algebra. Q.E.D.

Proof of Proposition 1.5: Part of the proof is contained in Section 1.5.1 and the rest is similar to that of Proposition 1.3. Q.E.D.

Proof of Proposition 1.6: First, we show that the job creation curve shifts to the right in the bubbly equilibrium compared to the bubbleless equilibrium. It follows from equations (1.4.17) and (1.5.11) that we only need to show that

\[ \frac{s (r + \pi \xi)}{\pi \xi} > \frac{(r + \pi)(r + s - r \xi)}{\pi}. \]

This inequality is equivalent to

\[ s (r + \pi \xi) > (r + \pi)(r + s - r \xi) \xi. \]

which is equivalent to condition (1.5.3).

Next, we show that the wage curve also shifts to the right in the bubbly equilibrium compared to the bubbleless equilibrium. It follows from equations (1.4.18) and (1.5.12) that we only need to show that

\[ \frac{s (s - \pi \xi + \theta)}{\pi \xi} > (\xi r + \theta) \frac{r + \pi}{\pi}. \]

This inequality is equivalent to

\[ s (s - \pi \xi + \theta) > (\xi r + \theta) (r + \pi) \xi. \quad (1.A.3) \]
Condition (1.5.3) implies \( s > (r + \pi) \xi \), thus it is sufficient to show that
\[
s - \pi \xi + \theta > \xi r + \theta. \tag{1.A.4}
\]
It is easy to check that (1.A.4) is equivalent to (1.5.3).

Now, we compare the bubbly equilibrium and the equilibrium with perfect credit markets. As discussed in the main text, the above method of proof will give an ambiguous result. We then use a different method. It follows from (1.3.19) and (1.3.20) that \( \bar{\theta} \) satisfies the following equation
\[
\frac{(1 - \eta) (A - c)}{\kappa + \psi \theta^2} = \eta \theta + r + s.
\]
The expression on the left-hand side of the above equation is a decreasing function of \( \theta \), while the expression on the right-side is an increasing function of \( \theta \). The solution \( \bar{\theta} \) is the intersection of the two curves representing the preceding two functions. Comparing with equation (1.5.8), we only need to show that
\[
\eta \theta + r + s < \frac{r + \pi}{\pi} \left[ \eta (\xi r + \theta) + r + s - r \xi \right].
\]
We can show the above inequality is equivalent to
\[
0 < (r + \pi) \eta \xi r + r \eta \theta + r (r + s - r \xi) - \pi r \xi.
\]
This inequality holds for any \( \theta > 0 \) by condition (1.5.3). Thus, we deduce that \( \theta_b < \bar{\theta} \). Using Figure 1.6, we deduce that \( H_b < \bar{H} \) and \( U_b > \bar{U} \). Q.E.D.

**Proof of Proposition 1.7:** Substituting the conjecture in (1.6.3) into (1.6.1) and (1.6.2) and matching coefficients, we can derive (1.6.4) and (1.6.5). The rest of equations follow from a similar argument in the proof of Proposition 1.2. Q.E.D.

### 1.B Isomorphism with a DMP Model

We introduce credit constraints into the large-firm DMP model discussed in Chapter 3 of Pissarides (2000). We shall show that this model is isomorphic to the model studied in Section 1.2. In the DMP framework, we introduce the matching function \( m(u, v) = Bu^\gamma v^{1-\gamma} \), where
\( \gamma \in (0, 1) \) and \( u \) and \( v \) represent aggregate unemployment and vacancy rates, respectively. Define the market tightness as \( \vartheta = v/u \), the job-filling rate as \( q(\vartheta) = m(u, v)/v = B \vartheta^{-\gamma} \), and the job-finding rate as \( q(\vartheta) \vartheta = m(u, v)/u = B \vartheta^{1-\gamma} \). Clearly, the job-filling rate decreases with the market tightness, but the job-finding rate increases with the market tightness.

As in the model in Section 1.2, there is a continuum of firm of measure one. Each firm \( j \) has a Leontief technology and posts vacancies \( v^j_t \) when meeting an employment opportunity with Poisson arrival rate \( \pi dt \). Thus, firm \( j \)'s employment follows dynamics:

\[
N^j_{t+dt} = \begin{cases} 
(1 - sdt)N^j_t + q(\vartheta_t) v^j_t & \text{with probability } \pi dt \\ 
(1 - sdt)N^j_t & \text{with probability } 1 - \pi dt 
\end{cases}.
\]  
(1.B.1)

Posting each vacancy costs \( \epsilon \). One filled job requires to buy a new machine at the cost \( \kappa \). The firm faces the credit constraint:

\[
(c_e + \kappa q(\vartheta_t)) v^j_t \leq (A - w_t) N^j_t dt + e^{-\tau dt} V_{t+dt}(\xi N^j_t).
\]  
(1.B.2)

Firm \( j \)'s problem is to choose \( v^j_t \) to maximize its firm value subject to the above two constraints. The discrete-time approximation of the Bellman equation is given by

\[
V_t(N^j_t) = \max_{v^j_t} (A - w_t) N^j_t dt - (c_e v^j_t + \kappa q(\vartheta_t) v^j_t) \pi dt \\
+ e^{-\tau dt} V_{t+dt} \left( (1 - sdt)N^j_t + q(\vartheta_t) v^j_t \right) \pi dt \\
+ e^{-\tau dt} V_{t+dt} \left( (1 - sdt)N^j_t \right) (1 - \pi dt),
\]

subject to (1.B.2).

The values to the employed and unemployed workers \( V_t^N \) and \( V_t^U \) are given by (1.2.14) and (1.2.15), except that the job-finding rate \( \theta \) is replaced by \( q(\vartheta) \vartheta \). The wage rate is defined by the Nash bargaining problem (1.2.19).

We now show that our model based on Blanchard and Gali (2010) is isomorphic to the above DMP model. Let

\[
H^j_t = q(\vartheta_t) v^j_t, \quad \psi = c_e B^{-\frac{1}{1-\gamma}}, \quad \text{and } \alpha = \frac{\gamma}{1-\gamma}.
\]

Then, by letting \( H_t = \int H^j_t dj \) and \( U_t = u_t \), we can show that the job-finding rate \( \theta_t = H_t/U_t \)
in the Blanchard-Gali setup is identical to that in the DMP setup, \( q(\vartheta_t) \vartheta_t \). In addition, the vacancy posting costs are equal to the hiring costs:

\[
c_r v_t^j = \psi B^{1-\gamma} H_t^j / q(\vartheta_t) = G_t H_t^j,
\]

where \( G_t = \psi \theta_t^\alpha \). Thus, (1.2.4) is identical to (1.B.1), and (1.2.13) is identical to the continuous time limit of (1.B.2), and hence the firm’s optimization problems in the two setups are identical. Since \( \theta_t = q(\vartheta_t) \vartheta_t \), the values to the employed and unemployed workers in the two setups are also identical. As a result, the two setups give identical solutions.

In particular, when \( \kappa = 0 \) and credit markets are perfect, our model is isomorphic to the DMP model without credit constraints analyzed in Chapters 1 and 3 in Pissarides (2000).
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CHAPTER 2

FINANCIAL DEVELOPMENT AND THE AGGREGATE SAVINGS RATES: A HUMP-SHAPED RELATIONSHIP

2.1 Introduction

How does financial development affect savings rates? Understanding the link between financial development and savings rates is important for several reasons. First, the savings rate is a crucial determinant of a country’s long-run per capita income. The neoclassical exogenous growth model (Solow, 1956) suggests that the higher the rate of saving, the richer the country in terms of per capita income. The endogenous growth theory pioneered by Romer (1986) and Lucas (1988) predicts that the savings rate determines long-run growth: a higher savings rate leads to a higher economic growth rate. Understanding the link between financial development and the savings rate is thus important for understanding how financial development affects economic growth, which is a useful step towards understanding the cross-country dispersion in measured per capita income levels.

Understanding the link between financial development and the savings rate is also important for policy. Aiyagari (1994) has shown that the optimal tax on capital income should be positive instead of zero in a financially underdeveloped economy, which contradicts the well-known recommendation of zero capital income taxation in the long run (Chamley, 1986). Krugger (2010) has further shown in a carefully calibrated general equilibrium model that a positive capital income tax rate has sizeable welfare gains. However, such policy recommendations rely on the prerequisite that financial underdevelopment will lead to oversaving. If financial underdevelopment causes undersaving, such policies may lead to welfare losses.
Finally, understanding the link between financial development and the savings rate is important in itself. The high savings rates in developing Asian countries have drawn significant attention from both academics and policy makers. The high savings rates in emerging Asian economies are claimed by some to be responsible for the low real interest rates worldwide, to have exacerbated asset price bubbles in the developed countries, and even to have triggered the recent financial crisis.\(^1\) One popular explanation provided by the global imbalance literature (e.g., Caballero, Farhi, and Gourinchas, 2008; Mendoza, Quadrini and Rios-Rull, 2009; Song, Kjetil, and Fabrizio, 2011; Wen, 2011) attributes the high savings rates to financial underdevelopment in those countries. However, Wei and Zhang (2011) have recently criticized such explanations, arguing that there has been noticeable financial development in those countries. This continuously rising savings rates despite financial development lead Wei and Zhang (2011) to conclude that there are other culprits, such as a growing sex-ratio imbalance, behind the high savings rate in China and some of the other Asian emerging economies. Again this criticism by Wei and Zhang is valid only if the relationship between financial development and the savings rate is monotonic. Otherwise, financial market development may induce countries with high savings rates to save even more.

The purpose of this study is hence twofold: to examine the relationship between financial development and the aggregate savings rates empirically; and to explain the empirical relationship theoretically in a general equilibrium model.

The existing empirical studies produce mixed results on the relationship between financial development and the savings rates. King and Levine (1993), Loayza, Schmidt-Hebbel and Serven (2000), Horioka and Yin (2010), for example, suggest a negative relationship, while Park and Shin (2009) find the impact of financial development to be insignificant. The above-mentioned studies, however, typically use the aggregate credits to the private sector as a measure of financial development, yet presume that the relationship between financial development and savings is linear, which may not be the case in reality.

\(^1\)On June 3, 2008 in a speech at an international monetary conference, US Federal Reserve chairman Ben Bernanke said, "In the financial sphere, the three longer-term developments I have identified are linked by the fact that a substantial increase in the net supply of saving in emerging market economies contributed to both the U.S. housing boom and the broader credit boom. The sources of this increase in net saving included rapid growth in high-saving East Asian countries and, outside of China, reduced investment rates in that region; large buildups in foreign exchange reserves in a number of emerging markets; and the enormous increases in the revenues received by exporters of oil and other commodities. The pressure of these net savings flows led to lower long-term real interest rates around the world, stimulated asset prices (including house prices), and pushed current accounts toward deficit in the industrial countries—notably the United States—that received these flows."
Beck et al. (2012) find that lending to firms and lending to households have fairly different effects on the outcomes of real sectors. It is also the case for the aggregate savings rate. On the one hand, financially constrained households may save as a precaution, hence extending credits to the households can lead to a decreasing household savings rate. On the other hand, financially constrained firms cannot invest to their optimal level due to limited funds, thus, expanding credits to the firms can lead to an increasing firm investment. As a result, the effects of increasing aggregate private credits on the aggregate savings rate are ambiguous. Hence, non-monotonic relationship is possible at least in theory. Indeed, using data from 12 developing Asian countries during 1996-2007, Horioka and Hagiwara (2010) demonstrate that the relationship between financial development and the savings rate is nonlinear and hump-shaped, which reconciles some of those conflicting findings.

Our empirical analysis extends Horioka and Terada-Hagiwara’s study in three respects. First, we extend the sample to include 31 OECD economies. It is important to confirm the nonlinear relationship with a sample that extends beyond East Asian countries, since these countries have historically had high savings rates. Second, we use various econometric methods to reexamine the statistical relationship, including both parametric and semi-parametric methods. Within parametric methods, we employ both the static panel data regression, as in Horioka and Terada-Hagiwara (2010) and the dynamic panel data regression, as in Loayza, Schmidt-Hebbel and Serven (2000). Finally, we consider four additional measures of financial development, besides the private credits to GDP ratio used in their paper.

Our analysis confirms a hump-shaped relationship between financial development and the aggregate savings rates, after controlling for demographic factors, income level, economic growth, inflation and the interest rate, etc. The existence of a uniform nonlinear relationship between financial development and the aggregate savings rates in a broader sample also calls for a unified theory to explain the relationship. That is the second goal of this study.

The hump-shaped relationship between financial development and the aggregate savings rate cannot be easily reconciled with the existing models of precautionary saving. The precautionary saving models developed by Leland (1968), Kimball (1990), Weil (1993), Aiyagari (1994), and many others typically predict a negative monotonic relationship between financial development and the savings rate. In these models households mainly rely on savings to insure themselves against idiosyncratic risks due to incomplete financial markets. Greater

\footnote{In Appendix 2.D, we use a sample of 26 OECD economies to show that the household savings rate does decrease when the household credit expands.}
financial development can hence reduce households’ reliance on saving for self-insurance and thus reduce the savings rate. Such a negative relationship has been used to explain the high savings rates in some financially underdeveloped countries. For example, Wen (2009, 2011) applies the precautionary savings explanation to the puzzle of high Chinese household savings. He shows that with large uninsured risks and severe borrowing constraints, rapid income growth can increase (instead of decrease) the household savings rate, thus explaining why China’s household savings rate is one of the highest in the world despite a 10% per year average income growth rate over recent decades.3

The global imbalance literature also relies on a negative monotonic relationship between financial development and precautionary savings to explain capital outflows from emerging countries to developed ones. However, such a monotonic relationship is contradicted by the data. Take China as an example. The private credits to GDP ratio in China increased from 50.9% in 1977 to 103.7% in 2008, but China’s national savings rate climbed to 51.8% from 28.9% during the same period. A similar pattern can be found in South Korea. In the 1960s, the Koreans saved less than 10 percent (8.6%) of their GDP, with the average private credits to GDP ratio being 20.6%. The average national savings rate in the 1990s rose to 36.3% while the average private credits to GDP ratio increased to 67.5%. But then the Korean savings rate declined to 30% in 2008 as the average private credits to GDP ratio increased further to 108.8%.

On the other hand, the existing models of firm-level financial constraints typically imply a positive relationship between financial development and a firm’s savings rate, in contrast to the literature on household savings. In his seminal work, Myers (1977) points out that debt can prevent a firm from undertaking an investment project with a positive net present value, a problem commonly referred to as the debt overhang or underinvestment problem. Most macroeconomic workhorse models of financial contracts feature such underinvestment problems. Examples include the credit rationing model of Stiglitz and Weiss (1981), the costly state verification models of Townsend (1979), Gale (1985), and Bernanke and Gertler (1989), the limited contract enforcement model of Kiyotaki and Moore (1997), and the moral hazard

3 We do not argue that there is no model of precautionary savings can explain the hump-shaped relationship between financial development and the aggregate savings rate. A hump-shaped relationship is possible if financial development can stimulate economic growth. Wen (2009, 2011) points out that economic growth can increase savings dramatically in a general equilibrium model. Financial development may increase savings initially due to a rising rate of income growth but it will decrease savings eventually when the self-insurance motivation is reduced. The interaction of these two opposite forces may generate a nonlinear relationship between financial development and the aggregate savings rate. Confirming these conjectures with an endogenous growth model is an interesting topic which we leave for future research.

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model of Holmstrom and Tirole (1998). Since investment equals saving at equilibrium, these models imply undersaving due to financial frictions. Financial development and saving can therefore exhibit a positive monotonic relationship. But such a pattern is not fully consistent with the empirical evidence gathered in this study.

We combine the insights generated by the aforementioned two literatures to explain our empirical findings. In particular, we propose that the observed hump-shaped relationship between financial development and the savings rate can be explained by imposing financial frictions on both the households and the firms. Generally speaking, this assumption should complicate a dynamic general equilibrium model substantially, since to compute the general equilibrium in such a model one must keep track of the distribution of household wealth and the distribution of firms’ capital stocks simultaneously, and both distributions have typically infinite dimensionality. In addition, the two distributions interact endogenously at the general equilibrium. For example, the distribution of capital depends on the pricing kernel, which equals the marginal investor’s discounting factor, which in turn depends on the wealth distribution of the households. Yet the households’ wealth distribution is affected by the firms’ capital distribution as the former affects the returns on the households’ portfolios. Solving such a model can be a daunting task.

To overcome such technical difficulties, we borrow the household model of Wen (2009, 2011) assuming a quasilinear preference function. This allows us to characterize the household saving behavior in closed form even though households face uninsurable idiosyncratic liquidity shocks and borrowing constraints. For any given interest rate, households save excessively compared to an economy without uninsurable risks, as Wen (2009, 2011) has shown. A higher level of financial development will thus lead to less incentive to save, all else being equal.

Our major innovation lies in our addition of heterogeneous firms to Wen’s heterogeneous-household model. In the enriched model, the firms discover investment opportunities randomly, which captures the idea that investment at firm and plant levels is lumpy. Hence, only a fraction of firms invest in each period. This creates a need to transfer funds among the firms. However, frictions arise when funds are moved between firms, due to limited enforcement. In other words, the firms are financially constrained. Assuming constant returns to scales allows us to characterize the firms’ investment decision rules analytically and permit exact aggregation, so only the mean of the capital distribution matters for aggregate equilibrium.\footnote{Our modeling of financial frictions at the firm level is motivated by a vast empirical literature that} We show that borrowing constraints on the firms create a gap between the
return on capital and the effective real interest rate for saving. Relaxation of the borrowing constraints on firms will narrow this gap and increase the real interest rate. Hence a high level of financial development will also generate an incentive for firms to increase their saving (or investment) rates. The overall effect on the aggregate savings rate will then depend on which side of the economy (the households or the firms) dominates.

Our model is able to generate a hump-shaped relationship between financial development and the aggregate savings rate with reasonable parameter values. To see this, imagine an extreme case in which firms have to borrow if they want to invest but they cannot borrow at all. In this case, the total investment demand would be zero. Since at equilibrium total savings must be equal to total investment, the aggregate savings rate will always be zero regardless of the saving incentives for the households. Although households have strong precautionary saving incentives under large idiosyncratic risks, the effective rate of return (the interest rate) on household savings is too low to induce them to save. As the borrowing constraints in such an economy gradually relax (for both firms and households), the aggregate savings rate will initially go up as firms start investing heavily. Beyond a critical level, however, the downward trend in household saving will begin to dominate. Thus, further relaxation in financial constraints will reduce the aggregate savings rate.

The rest of the paper is organized as follows. Section 2.2 presents the empirical evidence of a hump-shaped relationship between financial development and the aggregate savings rate. Section 2.3 presents our model in which both households and firms may face financial constraints. Section 2.4 defines and characterizes the general equilibrium. Section 2.5 examines the impact of financial development on the aggregate savings rate under different scenarios in the model. Section 2.6 concludes.
2.2 Empirical Evidence

In this section, we collect annual time series data from 1960 to 2008 on the savings rate and its potential explanatory variables for the same 12 Asian countries studied by Horioka and Terada-Hagiwara (2010) and an extra group of 31 OECD economies. We use various subsamples and estimators to reexamine the nonlinear relationship between financial development and the aggregate savings rate.

2.2.1 Data

Our sample draws from the World Development Indicators and the Penn World Table 7.0. A detailed description of the sample is presented in Appendix 2.B.

Following Loayza, Schmidt-Hebbel and Serven (2000), we set a threshold of ±50% annual inflation rate to exclude the episodes of high inflation in the sample, and employ dynamic panel data regression to accommodate the joint endogeneity problem of explanatory variables. Hence, we choose to work with the original data instead of phase-averaged data. As a result, we have a maximum of 976 observations for 41 countries.

The data show that both the savings rates and the levels of financial development display significant variations across countries and time. The savings rates range from less than 5% in Indonesia in the early 1960s to more than 50% in China today; the private credits to GDP ratios also range from less than 0.1 in the early years in those developing countries to larger than 2 in developed economies today.

Figure 2.1 displays the scatterplot of the aggregate savings rates against the private credits to GDP ratio for the 12 Asian countries, along with the quadratic fit and LOWESS fit. It is evident from the figure that financial development and the aggregate savings rates display a hump-shaped relationship, with financial development initially increasing and then decreasing the savings rate. This pattern is consistent with what Horioka and Terada-Hagiwara (2010) have found.

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5Japan and South Korea belong to both groups, thus the total number of countries studied is 41 instead of 43. The size of the sample is determined by the availability of observations.
Figure 2.1: Scatterplot of the aggregate savings rate and the private credits to GDP ratio, East Asia

It is our main purpose to confirm this hump-shaped relationship using a broader sample and serious econometric methods in the remaining parts of this section.

2.2.2 Specification

Consider the following reduced-form regression equation:

$$s_{it} = \kappa_1 s_{it-1} + \kappa'_2 X_{it} + \alpha_i + u_{it}, \quad (2.2.1)$$

where $s$ denotes the aggregate savings rate, $X$ is a vector of explanatory variables reflecting financial development, age structure, rates of return, uncertainty, fiscal policy, income level and growth, which have all been shown to affect the savings rate in the literature, $\alpha$ denotes the unobserved country fixed effects, and $u_{it}$ is the error term. Note that equation (2.2.1) nests both the dynamic specification with $\kappa_1 \neq 0$ and the static specification with $\kappa_1 = 0$. We consider both of them in this paper.
Various theories have emphasized on different factors in explaining the saving behavior. Life-cycle models imply that demographic factors play a nontrivial role in determining the savings rate, so each economy's aged dependency ratio, youth dependency ratio and life expectancy feature in the regression. Precautionary saving theory emphasizes that people will save against future uncertainties, thus the inflation rate is employed to capture macroeconomic uncertainty and public expenditure on health and education is used to reflect the uncertainty about future health and education expenditures. The permanent income hypothesis suggests that income and its growth determine economic agents’ consumption and savings, so per-capita real GDP and its square, and the growth rate of per capita real GDP all play a part in the regressions. The square of income is included in the model in an attempt to capture the potential nonlinear relationship between income and the savings rate. Both Park and Shin (2009) and Horioka and Terada-Hagiwara (2010) find that the relationship is convex in their samples of Asian countries. Several other common variables such as real interest rate and current account balance are also included as in the literature.

However, the main purpose of our regression is to investigate the presumably nonlinear effects of financial development on the savings rate, after properly controlling for those relevant factors mentioned above. There are various possible measures of financial development, some of which rely on the size of the whole financial sector, while others focus on the role of financial intermediation.\(^6\) We use the private credits to GDP ratio as our main measure of financial development, following the previous studies, such as King and Levine (1993), Loayza, Schmidt-Hebbel and Serven (2000), and Horioka and Terada-Hagiwara (2010) among many others.\(^7\) Nevertheless, for the purpose of robustness check, we also consider four different measures of financial development that are popular in the literature.\(^8\) The four measures are: deposit money bank assets to GDP ratio, stock market capitalization to GDP ratio, M2 to GDP ratio and financial market depth measured by the sum of outstanding domestic private debt securities and stock market capitalization to GDP ratio. In order to capture

\(^6\)See the survey by Cook (2003) and the survey by Schmidt-Hebbel and Serven (2002). Thorsten Beck also wrote about "two concepts of financial development" recently, with one being the "financial intermediation view", and the other being the "financial center view". For details, please refer to his article at http://www.voxeu.com/index.php?q=node/7185

\(^7\)This measure reflects the "financial intermediation" view of financial development in Thorsten Beck’s article. For sure, a larger financial sector does not necessarily imply a higher level of financial development. As an example, Thorsten Beck points to Nigeria in the 1980s, where the expansion of financial sectors was accompanied by "financial dis-intermediation". Another example is present day China, where large-scale state-owned banks provide limited credits to private firms. On the contrary, aggregate credits provided to the private sectors measure directly the activities of financial intermediation. Hence, we consider it as a more appropriate measure of financial development in the context of this paper.

\(^8\)The results of robustness check are reported in Table 2.8 – 2.9 in Appendix 2.C, but not in the main text.
the possible nonlinearity, we employ both the level and its quadratic term as the explanatory variables.\textsuperscript{9}

2.2.3 Methodology

Besides the standard static panel data regressions with fixed effects or random effects, we also employ the dynamic panel data regression with fixed effects, because Wooldridge test for first-order serial correlation of the errors always rejects the null hypothesis of zero autocorrelation in our static panel regressions, which implies that the time series display serious inertia.

Introducing lagged dependent variable in the regression, however, causes endogeneity problem. It is also highly possible that credits, income and its growth are jointly determined with the savings rate, so they might be correlated with the errors as well. As a result, the standard within estimator and IV estimator might produce inconsistent estimations.\textsuperscript{10} Loayza, Schmidt-Hebbel and Serven (2000) suggest using the GMM-IV estimators developed by Arellano and Bond (1991), Arellano and Bover (1995) and Blundell and Bond (1998) to deal with the inertia and the endogeneity problem.

Arellano and Bond (1991) work with the first-differenced model. They describe the later-called Arellano-Bond difference estimator. The estimator assumes that the errors are serially uncorrelated and $E(s_{iT} \Delta u_{it}) = 0$ for $T \leq t - 2$. With this assumption, $\{s_{it-j}\}_{j=2,3,...}$ can be used as instruments in the first-differenced model and 2SLS/GMM estimation using lags as instruments delivers consistent and efficient results. They also develop a test for the serial correlation of the first-differenced errors.

However, lags could be weak instruments in the first-differenced model if the corresponding level variables display serious inertia over time. Arellano and Bover (1995) and

\textsuperscript{9}While the quadratic fit is enough to capture nonlinearity, it is also interesting to see what happens under semiparametric regression, into which financial development enters non-parametrically and other explanatory variables enter parametrically. In Appendix 2.E, we apply the Baltagi and Li’s (2002) estimator for partially linear fixed-effects panel data models to our full sample, and find that the relationship is indeed hump-shaped.

\textsuperscript{10}To eliminate the unobserved country-specific effects in estimation, we need to work with the mean-difference or first-differenced model derived from equation (2.2.1). However, neither the within estimator nor IV estimation using lags is feasible in the mean-difference model, because any lag $s_{it}$ is correlated with $\bar{u}_i$ and hence $(u_{it} - \bar{u}_i)$, which is the error term in the mean-difference model. The OLS estimator is also inconsistent in the first-differenced model, because $\Delta s_{it-1}$ is correlated with $\Delta u_{it}$, which is the error term now.
Blundell and Bond (1998) propose the later-called system estimator, which employs both the level equation (2.2.1) and its first difference, to deal with this problem. They assume $E(\Delta s_{it-1} u_{it}) = 0$ in addition, so $s_{it}$ can be instrumented using its own first lag difference $\Delta s_{it-1}$ in the level equation. The system estimator is shown to be be more precise and to have better finite sample properties, and it can also be applied to control for the endogeneity of the other explanatory variables. Thus, we focus on this estimator in the following regressions.

In the regressions, we follow Loayza, Schmidt-Hebbel and Serven (2000) and assume that the endogenous explanatory variables are "weakly exogenous". Namely, $E(X_{it} u_{iT}) \neq 0$ for $T \leq t$, but $E(X_{it} u_{iT}) = 0$ for $T > t$, since the past realization of explanatory variables is less likely to be influenced by future innovations to the savings rates. With this assumption, weakly exogenous variables in $X_{it}$ can be instrumented using their own lags $\{X_{it-j}\}_{j=2,3,\ldots}$ in the first-differenced model. In the following regressions, we treat the two dependency ratios, life expectancy and public expenditures as strictly exogenous variables, and assume that all the others are weakly exogenous.

The main results are presented in the next subsection. In deriving the results, we instrument the endogenous variables using their first two feasible lags in the first-differenced equation and using the first lag difference in the level equation.\footnote{The main results are robust if we use three lags as instruments in the first-differenced equation, while the results are mixed if only one lag is used. To be specific, when only one lag is used, in all cases the coefficients still imply a concave relationship, but they are only significant in the East Asian subsample, no matter whether the IV matrix is "collapsed" or not. In the full sample, the coefficients are significant only when the IV matrix is not "collapsed". Nevertheless, the Sargan/Hansen tests for joint validity of instruments in Table 2.5-2.7 indicate that using two lags as instruments is valid, even after we've taken care of the problem of "too many instruments" using the method suggested by Roodman (2009a). Thus, we interpret those results as being supportive of a hump-shaped relationship between financial development and the savings rate. Our semiparametric regression in Appendix 2.E further confirms this.} We also employ the financial reform index prepared by Abiad, Detragiache and Tressel (2008) as an external instrument, as Roodman (2009a) finds that lags of several popular financial development measures, including private credits used in this paper, all perform badly in the system GMM estimation when he replicates the exercises conducted by Levine, Loayza and Beck (2000).

It is also worth mentioning that using lags as instruments typically generates numerous instruments. Roodman (2009a) shows that "too many instruments" can overfit the endogenous variables and weaken the Hansen test for joint validity of instruments, making the results unreliable. Thus, as a robustness check, we also consider reducing the instrument count in our regression exercises by "collapsing" the instrument matrix, as suggested and exemplified...
2.2.4 Results

The coefficients of the most interest in model (2.2.1) are those on the financial development and its quadratic term. Table 2.1 summarizes the regression coefficients on these two variables for our full sample of 41 countries, using three different estimators. Table 2.5 – 2.7 in Appendix 2.C report the full results. We briefly discuss these results now.

All of the regressions show that the relationship between financial development and the aggregate savings rates is hump-shaped. The regressions confirm our observations in Figure 2.1, and are consistent with the results presented by Horioka and Terada-Hagiwara (2010), who only considered the 12 Asian countries. Indeed, the last two columns of Table 2.5 in Appendix 2.C replicate Horioka and Terada-Hagiwara’s results.

Table 2.1: Summary of the coefficients on private credits and its quadratic term, full sample

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimator</td>
<td>GMM-System</td>
<td>GMM-System</td>
<td>Within RE</td>
<td></td>
</tr>
<tr>
<td>Collapsed IV</td>
<td>No</td>
<td>Yes</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Private credits</td>
<td>0.0210***</td>
<td>0.0321**</td>
<td>0.0925***</td>
<td>0.105***</td>
</tr>
<tr>
<td></td>
<td>(0.00676)</td>
<td>(0.0144)</td>
<td>(0.0287)</td>
<td>(0.0308)</td>
</tr>
<tr>
<td>Private credits sq.</td>
<td>-0.00706**</td>
<td>-0.0127***</td>
<td>-0.0296***</td>
<td>-0.0321***</td>
</tr>
<tr>
<td></td>
<td>(0.00310)</td>
<td>(0.00491)</td>
<td>(0.0100)</td>
<td>(0.0101)</td>
</tr>
<tr>
<td>Country effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Observations (No. of cn)</td>
<td>809 (37)</td>
<td>809 (37)</td>
<td>976 (41)</td>
<td>976 (41)</td>
</tr>
</tbody>
</table>

The regression results for other explanatory variables are broadly consistent with the those of Loayza, Schmidt-Hebbel and Serven (2000) and Horioka and Terada-Hagiwara (2010). For example, the two age dependencies show significant negative effects on the savings rate, as senior citizens tend to consume their previous savings and kids usually consume without their

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12 In unreported tables, we also try the Arellano-Bond difference estimator, and the main results are robust.
13 Note that we don’t report the coefficients and statistics on life expectancy, current account balance, and public expenditures in Tables 2.5 – 2.7 to save the space. The full results are available on request.
own income. The per capita GDP growth has a highly significant positive effect in almost all cases. Real interest and inflation rates have ambiguous effects. Higher public expenditure typically has a negative effect, which may reflect households’ precautionary saving to some extent. The current account surplus has a positive and significant effect.

Thus, we confirm what Horioka and Terada-Hagiwara (2010) have recently found in a sample of 12 developing Asian countries. In fact a hump-shaped relationship is not surprising if both households and firms face credit constraints, as our model will show.

2.3 The Model

We consider an infinite-horizon economy. There is no aggregate uncertainty. The economy has two types of agents, consumers and firms with equal mass normalized to unity. We use $i \in [0, 1]$ to index households and $j \in [0, 1]$ to index firms. Firms accumulate capital and combine labor and capital to produce consumption goods. Households supply labor to the firms and own their stock. Both households and firms are subject to uninsurable idiosyncratic risks and financial constraints to be specified below.

2.3.1 Households

The household side is similar to that modelled by Wen (2009, 2011). Household $i$ derives utility from consumption $c_{it}$ and leisure $n_{it}$. The instantaneous utility function is $u_{it} = \theta_i \log c_{it} - \psi n_{it}$, where $\theta_i$ denotes idiosyncratic preference shocks with the distribution function $F(\theta) = \Pr[\theta_i \leq \theta]$ and support $[0, \theta_{\max}]$. The mean of $\theta_i$ is normalized to one, i.e. $E\theta_i = 1$. Each time period is divided into two sub-periods. The idiosyncratic shocks are realized in the second sub-period. Each household $i$ chooses labor supply $n_{it}$ in the first sub-period without observing $\theta_{it}$, and chooses consumption $c_{it}$ and savings in bonds $(s_{it+1})$ and stocks $(a_{it+1})$ in the second sub-period after observing $\theta_{it}$. With such an information and market structure, labor income cannot be used to fully diversify the idiosyncratic risk and savings become a buffer stock to smooth consumption. Taking as given the real interest
rate $R_{ft}$ and real wage $W_t$, the household $i$ solves

$$\max_{\{c,s,a^t\}} \left\{ \max E_i \left[ \sum_{t=0}^{\infty} \beta^t \left( \theta_{it} \log c_{it} - \psi n_{it} \right) \right] \right\},$$

(2.3.1)

where $\beta \in (0,1)$ is the discount factor. Household $i$ faces a period-by-period budget constraint,

$$c_{it} + \frac{s_{it+1}}{R_{ft}} + a_{it+1}Q_t \leq s_{it} + W_{it} + (Q_t + D_t)a_{it},$$

(2.3.2)

where $a_{it+1}$ is the share of a portfolio of stocks purchased by household $i$. Assume each household is subject to a limited borrowing capacity:

$$\frac{s_{it+1}}{R_{ft}} + a_{it+1}Q_t \geq -B_t,$$

(2.3.3)

where $B_t$ is an exogenous borrowing limit. Denote the Lagrangian multipliers for (2.3.2) and (2.3.3) as $\lambda_{it}$ and $\mu_{it}$, respectively. The first-order condition for labor is

$$W_t E_i [\lambda_{it}] = \psi,$$

(2.3.4)

where $E_i$ is an expectation operator over idiosyncratic risk $\theta_i$. Since labor is determined before observing $\theta_i$, the household will take the distribution of $\theta_i$ into account. The first-order conditions for $\{c_{it}, s_{it+1}, a_{it+1}\}$ are given, respectively, by

$$\frac{\theta_{it}}{c_{it}} = \lambda_{it},$$

(2.3.5)

$$\lambda_{it} - \mu_{it} = \beta R_{ft} E_i [\lambda_{it+1}],$$

(2.3.6)

$$\lambda_{it} - \mu_{it} = \beta E_i \left[ \lambda_{it+1} \frac{D_{t+1} + Q_{t+1}}{Q_t} \right].$$

(2.3.7)

### 2.3.2 Firms

Each firm $j$ combines labor ($N_j$) and capital ($K_j$) to produce output employing Cobb-Douglas technology,

$$Y_{jt} = K_j^{\alpha} N_j^{1-\alpha}, \quad \alpha \in (0,1).$$
Following Kiyotaki and Moore (1997, 2008), we assume that firms encounter investment opportunities randomly. In each period, firm $j$ encounters an investment opportunity with probability $\pi$. With probability $1 - \pi$, the firm has no investment opportunity. Assume the random investment opportunities are independent across firms and over time. Denote as $I_{jt}$ the investment level of firm $j$ in period $t$ if it has an opportunity to invest, the capital accumulation rule then follows

$$K_{jt+1} = \begin{cases} (1 - \delta) K_{jt} + I_{jt} & \text{with probability } \pi \\ (1 - \delta) K_{jt} & \text{with probability } 1 - \pi \end{cases}.$$  

(2.3.8)

With heterogeneous households, the firm’s dynamic programming problem becomes slightly more complicated. The first step is to find the right discounting factor. Like Hansen and Richard (1987), Ingersoll (1988), and Cochrane (1991), we assume that a sequence of prices $P_t$ exists such that a firm’s value is determined by

$$J_{jt} = \sum_{\tau=0}^{\infty} \frac{P_{t+\tau}}{P_t} D_{jt+\tau},$$

(2.3.9)

where $\{D_{jt+\tau}\}_{\tau=0}^{\infty}$ is the dividend flow generated by firm $j$. If we define $\Lambda_t = P_t/\rho^t$, where $\rho < 1$, then we can rewrite the firm’s value as

$$J_{jt} = \sum_{\tau=0}^{\infty} \rho^\tau \frac{\Lambda_{t+\tau}}{\Lambda_t} D_{jt+\tau},$$

(2.3.10)

which can be written recursively as

$$J_{jt} = D_{jt} + \rho \frac{\Lambda_{t+1}}{\Lambda_t} J_{jt+1}.$$  

(2.3.11)

Here $\rho$ is a discount factor, but notice that because of heterogeneity on the household side, $\rho$ does not necessarily equal the households’ discount factor $\beta$. With the the firm value given by (2.3.11), each firm’s problem is then to maximize its value $J_{jt}$ by choosing its optimal labor use and investment.

The optimal labor choice is static. It is straightforward to show that the firm’s operating

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14Miao and Wang (2011) use a similar assumption to study asset bubbles in a financially underdeveloped economy with heterogenous firms but a representative household.
profits are linear function of its capital stock,

$$R_t K_{jt} = \max_{N_{jt}} K_{jt}^a N_{jt}^{1-\alpha} - W_t N_{jt},$$  \hspace{1cm} (2.3.12)$$

where $W_t$ is the real wage and $R_t = \alpha \left( \frac{1-\alpha}{W_t} \right)^{\frac{1-\alpha}{\alpha}}$. The optimal labor demand is hence

$$N_{jt} = \left( \frac{1-\alpha}{W_t} \right)^{\frac{1}{\alpha}} K_{jt}.$$  \hspace{1cm} (2.3.13)$$

Denote the firm value in period $t$ as $J_t (K_{jt})$. It can now be defined recursively using the proper discounting factor $\rho^{\frac{\Lambda_{t+1}}{\Lambda_t}}$ as

$$J_t (K_{jt}) = \max_{I_{jt}} R_t K_{jt} - \pi I_{jt} + \rho^{\frac{\Lambda_{t+1}}{\Lambda_t}} [\pi J_{t+1} ((1 - \delta) K_{jt} + I_{jt}) + (1 - \pi) J_{t+1} ((1 - \delta) K_{jt})].$$  \hspace{1cm} (2.3.14)$$

Assume that firm $j$ can use both internal funds $R_t K_{jt}$ and borrowed funds $L_{jt}$ to finance investment. Its maximum investment is thus subject to the constraint

$$I_{jt} \leq L_{jt} + R_t K_{jt}.$$  \hspace{1cm} (2.3.15)$$

For simplicity, assume that the external funds are raised through intra-period loans: firms borrow from financial intermediaries at the beginning of period $t$ and pay them back with zero interest rate at the end of period $t$ through raising additional equity. (Dividends may be negative at the firm level.) One key assumption of this model is that loans are subject to collateral constraints, as in the model of Kiyotaki and Moore (1997). Firm $j$ pledges a fraction $\xi \in (0, 1]$ of its fixed assets $K_{jt}^j$ at the beginning of period $t$ as collateral. The parameter $\xi$ may represent the tightness of the collateral constraint or the extent of financial market imperfections. At the end of period $t$, the market value of the collateral is equal to $\rho^{\frac{\Lambda_{t+1}}{\Lambda_t}} J_{t+1} (\xi K_{jt})$, which is the discounted expected market value of firm $j$ if it owns capital stock $\xi K_{jt}$ at the beginning of period $t + 1$ and faces the same investment and collateral constraints in the future. The amount of loans $L_{jt}^j$ cannot exceed this collateral value, otherwise the firm would choose to default on its debt and lose the collateral value. This leads to the following collateral constraint:

$$L_{jt} \leq \rho^{\frac{\Lambda_{t+1}}{\Lambda_t}} J_{t+1} (\xi K_{jt}).$$  \hspace{1cm} (2.3.16)$$

To sum up, each firm $j$ solves problem (2.3.14) subject to constraints (2.3.15) and (2.3.16).
2.3.3 Financial Intermediation

The financial intermediate holds a portfolio consisting of stocks in all of the firms and collects aggregate dividends from them,

\[ D_t = \int (R_t K_{jt} - \pi I_{jt}) \, dj. \]  

(2.3.17)

The price of the portfolio \( Q_t \) is thus

\[ Q_t = \rho \frac{\Lambda_{t+1}}{\Lambda_t} [Q_{t+1} + D_{t+1}]. \]  

(2.3.18)

Financial intermediation is introduced for the sole purpose of simplifying the notation of the households’ maximization problem. One can also assume that the households directly hold a market portfolio consisting of stocks in all firms, and the equilibrium results will be the same.

2.3.4 General Equilibrium

Let \( K_t = \int_0^1 K^i_t \, dj, \, I_t = \pi \int_0^1 I^i_t \, dj, \, N_t = \int_0^1 N^i_t \, dj, \, Y_t = \int_0^1 Y^i_t \, dj, \, n_t = \int_0^1 n^i_t \, di, \, S_t = \int_0^1 s^i_t \, di \), and \( C_t = \int_0^1 c^i_t \, di \) be the aggregate capital stock, investment, labor demand, output, labor supply, household savings and consumption, respectively. The general equilibrium is then defined as sequences of aggregate variables \((K_t, I_t, N_t, Y_t, D_t, n_t, S_t, C_t)\), individual firms’ choices \((K^i_t, I^i_t, N^i_t, L^i_t, Y^i_t)\), individual households’ choices \((n^i_t, s^i_{t+1}, c^i_t)\) and prices \((Q_t, W_t, R_t, R_{ft})\), such that each firm and each household solve their optimization problems, and all markets clear:

\[ N_t = n_t, \]  

(2.3.19)

\[ S_{t+1} = 0, \]  

(2.3.20)

\[ \int a_i di = 1, \]  

(2.3.21)

\[ C_t + I_t = Y_t, \]  

(2.3.22)

\[ K_{t+1} = (1 - \delta) K_t + I_t. \]  

(2.3.23)
2.4 Characterization of the Equilibrium

Let us first derive the optimal decision rules for a single firm and a single household, then aggregate their individual choices and characterize the competitive equilibrium by a nonlinear equation system.

2.4.1 A Single Firm’s Decision Problem

We conjecture that the value of a firm has the following functional form:

\[ J_t(K_t^f) = v_t K_t^f, \]  

(2.4.1)

where \( v_t \) is a to-be-determined variable that depends only on the aggregate states. Define \( q_t = \rho E_t \frac{\Lambda_t + 1}{\Lambda_t} v_{t+1} \), which we will prove to be the traditional Tobin’s q. With the conjectured value function, the firm’s investment problem becomes

\[ v_t K_{jt} = \max_{I_{jt}} R_t K_{jt} - \pi I_{jt} + q_t [(1 - \delta) K_{jt} + \pi I_{jt}], \]

(2.4.2)

with the liquidity constraint

\[ I_{jt} \leq L_{jt} + R_t K_{jt}, \]

(2.4.3)

and the collateral constraint

\[ L_{jt} \leq q_t \xi K_{jt}. \]

(2.4.4)

The following proposition characterizes the optimal decisions for firm \( j \).

**Proposition 2.1** If \( q_t \) is greater than 1, the firm always chooses to invest the maximum amount whenever it has an investment opportunity. In particular, the optimal investment decision is

\[ I_{jt} = q_t \xi K_{jt} + R_t K_{jt}. \]

(2.4.5)

In addition,

\[ v_t = R_t + (1 - \delta) q_t + \pi (q_t - 1)(q_t \xi + R_t), \]

(2.4.6)
where \( q_t \) evolves according to

\[
q_t = \rho \frac{\Lambda_{t+1}}{\Lambda_t} [R_{t+1} + (1 - \delta)q_{t+1} + \pi(q_{t+1} - 1)(q_{t+1}q_t + R_{t+1})]. \tag{2.4.7}
\]

Here, \( \nu_t \) is the average market value of one unit of capital and \( q_t \) is the ex-dividend value of one unit of installed capital, which is the marginal benefit of new investment. Since the cost of investment is one, the additional gain from investing is positive if \( \nu_t - 1 > 0 \). In this case the firm will want to borrow as much as possible to invest, so its borrowing constraint binds. The value \( \nu_t \) consists of three parts as shown on the right hand side of equation (2.4.6). First, one unit of capital can generate \( R_t \) units of operating profit in period \( t \). Second, one unit of capital can carry \( 1 - \delta \) units of capital to the next period with value \( (1 - \delta)q_t \) after depreciation and paying dividends. Finally, the capital can also be used as collateral. With probability \( \pi \) the firm will have an investment opportunity and with one unit of capital the firm is able to obtain \( q_tq_t \) units of loans, which increases the expected net benefit from investing by \( \pi(q_t - 1)(q_tq_t + R_t) \).

Equation (2.4.7) comes from the definition of \( q_t \). In fact, after multiplying both sides by \( K_{t+1} \), equation (2.4.7) is equivalent to the asset pricing formula (2.3.18). To see this, aggregating (2.4.5) over firms gives the total investment

\[
I_t = \pi(q_tq_t + R_t)K_t. \tag{2.4.8}
\]

The aggregate capital follows is then

\[
K_{t+1} = I_t + (1 - \delta)K_t. \tag{2.4.9}
\]

It is easy to see that the aggregate dividend is given by \( D_t = R_tK_t - I_t \). Multiplying both sides of (2.4.7) by \( K_{t+1} \) gives

\[
q_tK_{t+1} = \rho \frac{\Lambda_{t+1}}{\Lambda_t} [R_{t+1}K_{t+1} + (1 - \delta)q_{t+1}K_{t+1} + (q_{t+1} - 1)\pi(q_{t+1}q_t + R_{t+1})K_{t+1}]
\]

\[
= \rho \frac{\Lambda_{t+1}}{\Lambda_t} [R_{t+1}K_{t+1} - I_{t+1} + (1 - \delta)q_{t+1}K_{t+1} + q_{t+1}I_{t+1}],
\]

\[
= \rho \frac{\Lambda_{t+1}}{\Lambda_t} [R_{t+1}K_{t+1} - I_{t+1} + q_{t+1}K_{t+1}], \tag{2.4.10}
\]

where the second line comes from aggregate investment equation (2.4.8) and the third line comes from the law of motion of aggregate capital (2.4.9). Comparing (2.4.10) with (2.3.18)
yields

\[ q_t K_{t+1} = Q_t. \]  \hfill (2.4.11)

2.4.2 A Single Household’s Decision Problem

For household \( i \), the first-order conditions are given by (2.3.4) to (2.3.7). Following Wen (2009, 2011) by denoting \( H_{it} = (Q_t + D_t) a_{it} + W_{it} n_t (i) + s_{it} \) as the total income of household \( i \) in period \( t \), the following proposition shows that the income distribution is degenerate \( (H_{it} = H_t) \) and there exists a unique cutoff \( \theta^*_t \) such that the borrowing constraint (2.3.3) will bind if and only if \( \theta_{it} \geq \theta^*_t \).

**Proposition 2.2** Given \( W_t \) and \( R_{ft} \), \( H_t \) and \( \theta^*_t \) are jointly determined by the following two equations:

\[ \theta^*_t = \beta R_{ft} (H_t + B_t) E_t \left( \frac{\psi}{W_{t+1}} \right), \]  \hfill (2.4.12)

and

\[ W_t \int \frac{\max(\theta, \theta^*_t)}{H_t + B_t} f(\theta) d\theta = \psi. \]  \hfill (2.4.13)

Given \( \theta^*_t \) and \( H_t \), the optimal consumption of the household can be summarized as

\[ c_{it} = \min \left[ \frac{\theta_{it}}{\theta^*_t}, 1 \right] (H_t + B_t), \]  \hfill (2.4.14)

and Lagrangian multipliers \( \lambda_{it} \) and \( \mu_{it} \) are given by

\[ \lambda_{it} = \max(\theta_{it}, \theta^*_t). \]  \hfill (2.4.15)

\[ \mu_{it} = \max(\theta_{it} - \theta^*_t, 0). \]  \hfill (2.4.16)

Finally the risk-free rate and the asset price are determined, respectively, by

\[ \frac{\theta^*_t}{H_t + B_t} = \beta R_{ft} \frac{1}{H_{t+1} + B_{t+1}} \int \max(\theta, \theta^*_t) f(\theta) d\theta, \]  \hfill (2.4.17)

and

\[ \frac{\theta^*_t}{H_t + B_t} Q_t = \beta \frac{Q_{t+1} + D_{t+1}}{H_{t+1} + B_{t+1}} \int \max(\theta, \theta^*_t) f(\theta) d\theta. \]  \hfill (2.4.18)
Similar to Wen (2009, 2011), with quasilinear preferences, the household can achieve a target wealth $H_t$ in period $t$ to buffer the idiosyncratic preference shocks through adjusting its labor supply. The quasilinear utility function implies that the marginal disutility of acquiring additional labor income is a constant (equal to $\frac{\omega}{W_t}$) for all households. Since the preference shock is i.i.d., the expected marginal utility from consumption depends only on the total wealth in period $t$ (not on the history of idiosyncratic shocks). The household supplies a level of labor such that the marginal disutility equals the expected gains in the marginal utility, which implies a common target wealth for all households.

In the absence of aggregate uncertainty, equations (2.4.17) and (2.4.18) then imply

$$Q_t = \frac{Q_{t+1} + D_{t+1}}{R_{ft}}.$$  \hspace{1cm} (2.4.19)

That is, the risk-free rate is the proper discounting factor for the firms. With the decision rules of the firms and the households in hand, we are now ready to characterize the aggregate equilibrium by a set of nonlinear equations.

### 2.4.3 Aggregation

Equation (2.3.13) implies that the capital-labor ratio is the same for all firms, so $K_{jt}/N_{jt} = K_t/N_t$. It follows that the aggregate production is given by

$$Y_t = K_t^\alpha N_t^{1-\alpha},$$  \hspace{1cm} (2.4.20)

and the two factor prices $R_t$, and $w_t$ are given by

$$R_t = \alpha \frac{Y_t}{K_t}$$  \hspace{1cm} (2.4.21)

and

$$w_t = (1 - \alpha) \frac{Y_t}{N_t}.$$  \hspace{1cm} (2.4.22)

\[15\] Also see Wen (2010).
Since the wealth distribution is degenerate \((H_{it} = H_t)\), aggregating \(H_{it}\) yields

\[
\int H_{id} \, di = H_t = (Q_t + D_t) \int a_{it} \, di + W_t N_t + \int s_{it} \, di = Q_t + D_t + W_t N_t = q_t K_{t+1} + Y_t - I_t.
\] (2.4.23)

The second line comes from the fact that \(\int a_{it} \, di = 1\) and \(\int s_{it} \, di = 0\); the third lines come from the fact that \(Q_t = q_t K_{t+1}\) by equation (2.4.11), \(D_t = R_t K_t - I_t\) and \(R_t K_t + W_t N_t = Y_t\) by equations (2.4.21) and (2.4.22). Aggregating equation (2.4.14) over households yields the aggregate consumption with heterogeneous households and borrowing constraints as

\[
C_t = (H_t + B_t) \int \min\left(\frac{\theta}{\theta_t}, 1\right) f(\theta) d\theta,
\] (2.4.24)

which resembles a Keynesian consumption function, with \(\int \min\left(\frac{\theta}{\theta_t}, 1\right) f(\theta) d\theta < 1\) as the propensity to consumption. The propensity to consume is less than one because idiosyncratic preference shocks and borrowing constraints give the households a precautionary motive to accumulate wealth. Equation (2.4.7) and aggregate (2.4.19) imply

\[
q_t = \frac{1}{R_{ft}} \left[ R_{t+1} + (1 - \delta) q_{t+1} + \pi (q_{t+1} - 1) (q_{t+1} \xi + R_{t+1}) \right]
\] (2.4.25)

in the absence of aggregate uncertainty.

The equilibrium can now be characterized in the model by the following proposition.

**Proposition 2.3** The equilibrium path of the model is characterized by dynamic movements of eight aggregate variables \(\{C_t, I_t, Y_t, N_t, K_{t+1}, R_{ft}, q_t, \varepsilon_t^*\}\), which can be solved by a system of eight nonlinear equations:

\[
\frac{\Psi(\theta^*_t)}{C_t} = \beta R_{ft} \frac{\Psi(\theta^*_{t+1})}{C_{t+1}} \Phi(\theta^*_{t+1})
\] (2.4.26)

\[
\frac{\Psi(\theta^*_t) \Phi(\theta^*_{t+1})}{C_{t+1}} (1 - \alpha) \frac{Y_t}{N_t} = \psi
\] (2.4.27)

\[
Y_t = K_t^\alpha N_t^{1-\alpha}
\] (2.4.28)

\[
k_{t+1} = (1 - \delta) K_t + I_t
\] (2.4.29)

\[
C_t + I_t = Y_t
\] (2.4.30)
\begin{equation}
I_t = \pi \left[ q_t \xi + \frac{Y_t}{K_t} \right] K_t \tag{2.4.31}
\end{equation}

\begin{equation}
q_t = \frac{1}{R_{ft} t} \left[ \alpha Y_{t+1} \frac{K_{t+1}}{K_{t+1}} + (1 - \delta)q_{t+1} + \pi(q_{t+1} - 1)(q_{t+1} + \frac{\alpha Y_{t+1}}{K_{t+1}}) \right] \tag{2.4.32}
\end{equation}

\begin{equation}
C_t = (q_t K_{t+1} + Y_t - I_t + B_t) G(\theta^*_t), \tag{2.4.33}
\end{equation}

where

\begin{equation}
\Psi(\theta^*_t) = \int \min(\theta, \theta^*_t) f(\theta) d\theta, \tag{2.4.34}
\end{equation}

\begin{equation}
\Phi(\theta^*_{t+1}) = \int \max(\frac{\theta}{\theta^*_t}, 1) f(\theta) d\theta > 1, \tag{2.4.35}
\end{equation}

and

\begin{equation}
G(\theta^*_t) = \int \min(\frac{\theta}{\theta^*_t}, 1) f(\theta) d\theta. \tag{2.4.36}
\end{equation}

Equation (2.4.26) follows from equation (2.4.24). Equation (2.4.27) is derived from (2.4.13) by replacing $H_t + B_t$ by the aggregate consumption and replacing the real wage by equation (2.4.22). Equation (2.4.28) is identical to equation (2.4.20). Equation (2.4.29) describes how aggregate capital evolves, and equation (2.4.30) is the aggregate resource constraint. Equations (2.4.26) to (2.4.30) are similar to those of a standard RBC model. Equation (2.4.31) is the aggregate investment equation. The financial frictions introduce two new variables, $q_t$ and $\theta^*_t$, which are determined by equations (2.4.32) and (2.4.33). Equation (2.4.32) is obtained by substituting out $R_{t+1}$ on the right hand side of equation (2.4.25) using equation (2.4.21). Finally combining equation (2.4.23) and equation (2.4.24) yields equation (2.4.33).

2.5 The Savings Rate and Financial Development

In this section, we discuss the relationship between the aggregate savings rate and financial development in the steady state. The savings rate is measured by the investment-to-output ratio, $s = \frac{I}{Y}$. The parameter $\xi$ and the steady state ratio $b = B/qK$ measure the financial development on the firm side and on the household side, respectively. To better understand the hump-shaped relationship between financial development and the savings rate, we consider four different scenarios: (i) the case without any financial constraints (the first-best allocation), (ii) the case with financial constraints on firms only, (iii) the case with financial
constraints on households only, and (iv) the case with both firms and households subject to financial constraints.

2.5.1 The First-Best Allocation

Consider first an economy without any financial frictions. The two borrowing constraints (2.3.3) and (2.3.16) then never bind. In this case, equations (2.3.4), (2.3.5), and (2.3.6) imply that

\[
\frac{\theta_{it}}{c_{it}} = \frac{\theta_{jt}}{c_{jt}},
\]

for any \(i\) and \(j\). The aggregate consumption can be obtained by integrating \(c_{jt}\) over \(j\), which gives

\[
C_t = \int_0^1 c_{jt} dj = \frac{1}{\theta_{it}} c_{it} \int_0^1 \theta_{jt} dj = \frac{1}{\theta_{it}} c_{it},
\]

or \(c_{it} = \theta_{it} C_t\). Each individual’s consumption is proportional to the idiosyncratic shock he faces. Equation (2.3.6) then becomes \(\frac{1}{C_t} = \beta R_{ft} \frac{1}{\theta'_{t+1}}\) and equation (2.3.4) can be rewritten as \(\frac{1}{C_t} W_t = \psi\). It is easy to see that these two equations are equivalent to equations (2.4.26) and (2.4.27) if \(\theta'_{t}\) and \(\theta'_{t+1}\) are equal to \(\theta_{max}\). On the other hand, if the borrowing constraint (2.4.4) does not bind, then \(q_t = 1\). Equation (2.4.32) would then imply that \(R_{ft} = \frac{\alpha Y_{t+1}}{K_{t+1}} + (1-\delta) q_{t+1}\). Appendix A.3 shows that such an allocation is also the first-best allocation for a central planner. In the steady state,

\[
R_f = \beta^{-1} = \frac{\alpha}{\beta' K} + 1 - \delta.
\]

The steady state savings rate can then be determined as

\[
s^* = \frac{I}{Y} = \frac{\delta K}{Y} = \frac{\alpha \delta \beta}{1-(1-\delta) \beta'},
\]

which is identical to that of a standard RBC model.

The financial constraints on households and firms create two wedges between the discount factor \(\beta\), the interest rate \(R_f\), and the marginal product of capital. First, notice that equation (2.4.26) gives a steady state interest rate of

\[
R_f = \frac{1}{1 \Phi(\theta^*)} < \frac{1}{\beta'},
\]

since \(\Phi(\theta^*) > 1\) as long as the borrowing of some households is constrained. As we will show
in Section 2.5.3, this tends to increase the savings rate above the first-best savings rate due to precautionary saving (as Aiyagari (1994) has shown). If \( q > 1 \) in the steady state, then financial constraints on the firm side create a gap between the interest rate and the marginal product of capital. In the steady state, with \( q > 1 \) equation (2.4.32) implies
\[
R_f = \frac{\alpha Y}{K} \left( 1 + \frac{\pi(q - 1)}{q} \right) + 1 - \delta + \pi(q - 1)\xi \\
\neq \frac{\alpha Y}{K} + 1 - \delta. \tag{2.5.6}
\]

We will show in Section 2.5.2 that financial constraints on the firm side alone will reduce the savings rate to below the first-best level. Equation (2.5.6) suggests why: financial constraints on firms only may reduce the interest rate for a given marginal product of capital, so households have less incentive to save.

### 2.5.2 Financial Constraints only on Firms

Assume that only firms but not households are subject to financial constraints. As a result, \( R_f = \beta^{-1} \) in the steady state as discussed in Section 2.5.1. We will show that the aggregate savings rate in this case increases with the level of financial development measured by \( \xi \). Given that \( R_f = \beta^{-1} \) at the steady state, equation (2.4.32) becomes
\[
q = \beta \left[ \frac{Y}{K} + (1 - \delta)q + \pi(q - 1)(q\xi + \frac{Y}{K}) \right]. \tag{2.5.7}
\]

The following proposition shows that the savings rate is an increasing function of financial development level \( \xi \) if it is below some critical value \( \xi^* \).

**Proposition 2.4** There exists a critical value \( \xi^* = \delta \frac{1-\pi}{\pi} - \frac{1}{\beta} + 1 \), such that
\[
q = \begin{cases} 
1 & \text{if } \xi > \xi^* \\
\delta \frac{1-\pi}{\pi} \xi^* + (1-\beta) > 1 & \text{if } \xi \leq \xi^* 
\end{cases}, \tag{2.5.8}
\]

and the steady state savings rate is given by
\[
s = \frac{I}{Y} = \begin{cases} 
\frac{\alpha \delta}{\frac{1}{\beta(1+\pi)}} & \text{if } \xi > \xi^* \\
\frac{\alpha}{\pi} \frac{1}{\beta(1+\pi\xi)} & \text{if } \xi \leq \xi^* 
\end{cases}. \tag{2.5.9}
\]

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Recall that a firm’s borrowing constraint (2.4.4) will be binding with \( q > 1 \). It is then clear that the savings rate is increasing in \( \xi \) according to equation (2.5.9). Borrowing constraints on firms will prevent them from undertaking investments that yield a positive net present value, a problem commonly referred to as the underinvestment problem as discussed in the introduction. To see this, first calculate the gross marginal product of capital

\[
MPK = \alpha \frac{Y}{K} + 1 - \delta = (1 - \beta) \frac{\delta^{1-\alpha}}{\xi \beta + 1 - \beta} + 1,
\]

(2.5.10)

which is the social gross return on one unit of capital. One unit of investment can yield an output of \( \alpha Y/K \) in the next period, and after depreciation the consumption value of this unit is \( 1 - \delta \). Notice that

\[
MPK > \frac{1}{\beta} = R_f,
\]

(2.5.11)

if \( \xi \leq \xi^* \).

This difference between the social return of capital and the interest rate prevents households from investing in capital to its socially optimal level, even though capital yields high social return. In the steady state, the households need to pay \( q \) for one unit of capital (indirectly through buying stocks), while the social cost of building one unit of capital is one. In other words, the households pay a premium to acquire capital, which weakens their incentive to save. An increase in \( \xi \) leads to a decrease in \( q \), which increases the households’ incentive to save.

### 2.5.3 Financial Constraints only on Households

When only households are subject to the financial constraints, the Tobin’s \( q_t \), equals to 1, and equation (2.4.32) becomes

\[
R_f = MPK = \alpha \frac{Y}{K} + 1 - \delta
\]

in the steady state. Solve for the savings rate as a function of the interest rate

\[
s = \frac{I}{Y} = \frac{\alpha \delta}{R_f - 1 + \delta}.
\]

(2.5.12)
The savings rate has a negative slope with respect to the user’s cost of capital (the interest rate). Since firms are the demanders for savings, equation (2.5.12) can be considered the demand function for loanable funds. Equations (2.4.26) and (2.4.33) then lead to another relationship between savings and the real interest rate from the suppliers of loanable funds. Equation (2.4.26) implies that

\[ Z_{\max}(\phi, 1) \Phi(\phi) \delta = 1 \]

which defines an implicit function for the cutoff \( \theta^* \) as a function of the interest rate \( R_f \). With a slight abuse of notation, define such function as \( \theta^* = \theta^*(R_f) \). It is easy to see that \( \frac{\partial \theta^*}{\partial R_f} > 0 \). A higher interest rate induces households to save more and hence reduces the probability of being liquidity constrained. Equation (2.4.33) implies that in the steady state the savings rate is

\[ s = \frac{\delta}{1/c(\phi^*) - (1 + b) + \delta} \]

where \( \phi^* = \phi^*(R_f) \) is increasing in \( R_f \), and \( G(\phi^*) \) defined in Proposition 2.3 is the marginal propensity to consume. The savings rate increases with the interest rate. Since the households are the suppliers of loanable funds, equation (2.5.14) describes the supply curve for funds with respect to the interest rate. The demand curve (2.5.12) and the supply curve (2.5.14) are plotted in Figure 2.2. The steady state value of the interest rate \( R_f \) and the savings rate \( s \) can be found at the intersection of these two curves. An increase in \( b \), which relaxes the households’ borrowing constraints and reduces the incentive for households to save, shifts the supply curve to the left and moves the equilibrium from point A to point B. So the steady state savings declines and the interest rate increases. The following proposition summarizes the result.

**Proposition 2.5** If only households are subject to financial constraints, the savings rate in the steady state will decrease with increasing financial development.

### 2.5.4 Financial Constraints on both Households and Firms

When both households and firms are subject to financial constraints, the aggregate equilibrium is characterized by the system of equations from (2.4.26) to (2.4.33). In order to study the impact of financial constraints on the savings rate in the steady state, we must
Figure 2.2: The effect of relaxing the households’ borrowing constraints on the savings rate

first derive the relationships between the savings/investment rate and the interest rate from the perspective of both households and firms. As in the previous section, the demand and supply curves can again be used to study the impact of these two borrowing constraints on savings. Derive first the demand curve for savings using equations (2.4.29), (2.4.31), and (2.4.32). This gives the relationship between the savings rate and the interest rate from the demand side:

\[ s = \frac{I}{Y} = \alpha - \alpha \frac{\frac{1}{\pi} - 1}{\frac{1}{\pi} + \xi / (R_f - 1)} \]  
(2.5.15)

which decreases with interest rate \( R_f \). An increase in \( R_f \), representing a higher user’s cost of capital, depresses demand for investment (savings). The supply curve can be derived as in Section 2.5.3. First, equation (2.5.13) still holds, so \( \theta^* = \theta^*(R_f) \) and \( \frac{\partial \theta^*}{\partial R_f} > 0 \) for reasons discussed in Section 2.5.3. The next step is to use equation (2.4.33) to solve for the savings rate as a function of the interest rate from the supply side. After rearranging terms, as an intermediate step, we write

\[ \frac{Y}{K} = \frac{G(\theta^*)}{1 - G(\theta^*)} q (1 + b) + \delta, \]  
(2.5.16)
Figure 2.3: The effect of relaxing the households’ borrowing constraints on the savings rate

where $\theta^* = \theta^*(R_f)$, and $G(\theta^*)$ is defined in Proposition 2.3. $q$ can then be derived as a function of the interest rate from equation (2.4.32)

$$q = q(R_f) = \frac{\delta^{1-\pi}}{\xi + R_f - 1},$$

(2.5.17)

which decreases with the interest rate. The intuition is simple: from the households’ point of view, the value of one unit of capital, $q$, is the sum of the present value of all future dividend flows. An increase in the interest rate reduces the present value and hence reduces $q$. Finally replace $K$ by $I/\delta$ in the steady state and $q$ by (2.5.17) to obtain the supply of saving $s$ as

$$(s)^{-1} = \frac{G(\theta^*)/\delta}{1 - G(\theta^*)} q(R_f) (1 + b) + 1.$$ (2.5.18)

Recall that $\theta^*$ is increasing with $R_f$ and $G(\theta^*)$ is decreasing with $\theta^*$, so $G(\theta^*)$ decreases with $R_f$. $\frac{G(\theta^*)/\delta}{1 - G(\theta^*)}$ thus decrease with $R_f$. By equation (2.5.17), $q$ is also decreasing with $R_f$. So the right hand side as a whole is decreasing with $R_f$. We can therefore conclude that the total savings rate, the inverse of the right hand side, is an increasing function of the interest rate.

Figure 2.3 shows the effect of relaxing the households’ borrowing constraints on the savings
Figure 2.4: The effect of changing the firms’ borrowing constraints on the savings rate.

The relationship between the interest rate and the savings rate implied by equation (2.5.15) is represented by the downward sloping curve labeled $D$. The real interest rate, $R_f$, is shown on the vertical axis. The savings rate $s$ is measured on the horizontal axis. Equation (2.5.18) is represented by the upward sloping curve labeled $S$. The equilibrium interest rate and savings rate are at the intersection of the demand and supply curves. The initial steady state equilibrium is at point $E$. According to equation (2.5.18), an increase in $b$ reduces the savings rate for any given interest rate $R_f$. As the borrowing constraints are relaxed, there is less need to save to self-insure against the idiosyncratic preference shocks. In other words, an increase in $b$ shifts the supply curve to the left from $S$ to $S'$. The new steady state equilibrium is at point $E'$. The interest rate $R_f$ increases, but the savings rate $s$ decreases.

The effect of changing the firms’ borrowing constraints is illustrated in Figure 2.4. First, by equation (2.5.15), an increase in $\xi$ increases firm investment at any given interest rate. This is easy to understand since firms’ investments are constrained by their borrowing capacities. In other words, an increase in $\xi$ shifts the demand curve to the right from $D$ to $D'$. But an increase in $\xi$ also changes the supply curve. By equation (2.5.17), an increase in $\xi$ reduces Tobin’s $q$, the households’ cost of obtaining one unit of capital, so it increases the households’ incentive to save. As is more evident from equation (2.5.18), an increase in $\xi$ increases the savings rate from the supply side. In other words, an increase in $\xi$ also shifts the supply curve to the right from $S$ to $S'$. The new steady state equilibrium is at point $E'$. The savings rate increases for sure, but the effect on the interest rate is not clear. Proposition 2.6 summarizes
these results.

**Proposition 2.6** The equilibrium savings rate can be expressed as a function of $b$ and $\xi$, namely, $s = s(\xi, b)$ with $\frac{\partial s}{\partial \xi} > 0$ and $\frac{\partial s}{\partial b} < 0$ if borrowing constraints on both households and firms bind.

The above proposition also suggests that if financial development increases $b$ and $\xi$ simultaneously, the overall effect on the savings rate will be ambiguous. An increase in the households’ borrowing reduces the savings rate but an increase in the firms’ borrowing increases it. A numerical example in Section 2.5.5 will illustrate how it is possible for this model to explain the observed hump-shaped relationship between financial development and the savings rate.

### 2.5.5 Explaining the Hump-Shaped Relationship: A Numerical Example

In this numerical example,\(^\text{16}\) we set the parameters in such a way that the model economy is best interpreted as an emerging economy with a high average savings rate. Set capital share in production $\alpha$ to 0.42, depreciation rate $\delta$ to 0.012 and the discount factor $\beta$ to 0.999. These parameter values generate a high savings rate and a low interest rate in the steady state, which are broadly consistent with the data on emerging Asian economies. Set the probability that each firm receives an investment opportunity $\pi$ to 0.1 and assume the idiosyncratic preference shock $\theta(i)$ follows Pareto distribution with shape parameter $\sigma = 1.15$. These two assumptions make firms’ and households’ demand for borrowing large, so they can potentially face severe financial constraints if the financial market is imperfect. We assume $b$ and $\xi$ are driven by a common force $f$ that indicates the financial development level:

$$ b = f, \quad (2.5.19) $$

$$ \xi = a_1 f + a_2 \quad (2.5.20) $$

where parameters $\{a_1, a_2\}$ are non-negative. Set $a_1 = 0.15$ and $a_2 = 0.005$ to induce a hump-shaped relationship in the data.

\(^{16}\)A quantitative exercise would be more desirable if it were possible. A careful calibration exercise, however, requires micro data on the dispersion of household consumption and incomes to measure the idiosyncratic risks and the ratio of consumer credit to GDP to measure to what extent households are financially constrained. Due to data unavailability, we leave this as a future research topic.
Figure 2.5: Financial development and the savings rate

Figure 2.5 plots the steady state savings rate for different values of $f$. The dashed line shows the first-best savings rate. It is evident from Figure 2.5 that financial underdevelopment can induce either oversaving or undersaving. Figure 2.5 shows a clear hump-shaped relationship between financial development and the aggregate savings rate. The savings rate is about 25% when both households and firms can barely borrow. The savings rate starts to climb as the borrowing constraints are relaxed. The peak savings rate is close to 51%. Interestingly, with a higher level of financial development, the savings rate can double. This model thus explains the puzzle of China’s savings rate discussed by Wen (2011). The savings rate in China was less than 30% in the late 70s, but it climbed to 51.8% in 2008 as financial markets improved.

Figure 2.6 shows that a hump-shaped relationship does not exist if only firms or only households are subject to credit constraints. As expected, the left panel of figure 2.6 shows that the savings rate increases monotonically with the level of financial development if only firms are constrained, while the right panel shows that the savings rate decreases monotonically with financial development. Neither prediction is validated by the data, nor are they consistent with the recent pattern seen in emerging Asian economies. Although the left

---

17 Notice that $f$ has a one-to-one mapping with the credit-to-GDP ratio.
18 Financial markets in China has developed in many ways. Ma and Wang (2010) document that bank loans to Chinese households have expanded substantially, reaching 15% of the total outstanding bank loans recently from less than 1% in the late 1990s.
panel is consistent with the fact that the savings rates increase with financial development in emerging Asian countries, it suggests that savings rates in these countries are systemically lower than in economies with similar cultures but better financial development, which is counterfactual. It is well known that China’s savings rate is much higher than those of South Korea, Japan and Taiwan in recent years. The right panel shows that financial underdevelopment can indeed generate very high savings rates. But with only households being financially constrained, such predictions are prone to the criticism by Wei and Zhang (2011) using time series evidence.

2.6 Conclusion

This study has documented a hump-shaped relationship between financial development and the national savings rate. It has shown that financial development increases saving if only firms are financially constrained, but reduces saving if only households are borrowing constrained. When both firms and households are financially constrained, our theoretical model predicts an observed hump-shaped relationship between financial development and the aggregate savings rate. This model also shows that financial frictions can cause either undersaving
or oversaving depending on the stage of financial development. A natural policy recommendation would be that the optimal taxation on capital income may either be negative (in the case of undersaving) or positive (in the case of oversaving). Given its enormous welfare consequences, a more complete characterization of the optimal capital income taxation should be pursued in the future. With both firms and households being financially constrained, the model also generates another interesting prediction: the coexistence of high returns on capital and low real interest rates, which is consistent with the evidence from emerging economies.\textsuperscript{19} An interesting implication of this prediction is that if these economies were to open up to countries with more advanced financial markets, financial capital would flow out from the former to the latter, while physical capital would flow (in terms of FDI) in the opposite direction to enjoy high capital returns (see evidence by Ju and Wei (2011)). Wang, Wen and Xu (2012) recently explain such two-way flows.

\textsuperscript{19}See Wen (2009) for an alternative model to explain this fact. Wen (2009) assumes an imperfectly competitive (state-owned) banking sector generating a spread between the deposit rate and the loan rate. Here we obtain the same effect by allowing financial constraints on the firm side.
Appendix

2.A Proofs

Proof of Proposition 2.1  Suppose \( q_t > 1 \), firm j then always chooses to invest as much as possible when investment opportunities arise. The investment amount is

\[
I_{jt} = q_t \xi K_{jt} + R_t K_{jt}.
\]  (2.A.1)

Substituting this decision rule into equation (2.4.2) and comparing terms gives equation (2.4.6). Using the definition \( q_t = \rho \frac{A_{t+1}}{A_t} \xi_t \) easily yields

\[
q_t = \rho \frac{A_{t+1}}{A_t} [R_{t+1} + (1 - \delta)q_{t+1} + \pi (q_{t+1} - 1)(q_{t+1} + R_{t+1})],
\]  (2.A.2)

which is equation (2.4.7). Q.E.D.

Proof of Proposition 2.2  Prove first that the total wealth \( H_{it} = (Q_t + D_t) a_{it} + W_t n_t (i) + s_{it} \) is degenerate. In the second sub-period, the households’ consumption, savings, and stock holdings can be written as a function of their wealth \( H_{it} \), liquidity shock \( \theta_{it} \), and the aggregate variables,

\[
\lambda_{it} = \frac{\theta_{it}}{c_t} = \frac{\theta_{it}}{c_t(H_{it}, \theta_{it})}
\]  (2.A.3)

Equation (2.3.6) can then be written as

\[
\frac{\theta_{it}}{c_t(H_{it}, \theta_{it})} = \beta R_f t E_t [\lambda_{it+1}] + \mu_{it}
\]  (2.A.4)

\[
= \beta R_f t \frac{\psi}{W_{t+1}} + \mu_{it},
\]

where the second line has made use of equation (2.3.4). Define \( \theta^*_{it} \) such that

\[
\frac{\theta^*_{it}}{H_{it} + B_t} = \beta R_f t \frac{\psi}{W_{t+1}}.
\]  (2.A.5)

Since \( c_{it} \leq H_{it} + B_t \), then \( \frac{\theta_{it}}{c_t(H_{it}, \theta_{it})} \geq \frac{\theta^*_{it}}{H_{it} + B_t} \) for \( \theta_{it} \geq \theta^*_{it} \). By (2.A.4) and (2.A.5), \( \mu_{it} > 0 \). Or the borrowing constraint (2.3.3) binds, and the household’s consumption \( c_{it} = H_{it} + B_t \).
On the other hand, $\mu_{it} = 0$, so

$$\frac{\theta_{it}}{c_t(H_{it}, \theta_{it})} = \frac{\theta^*_t}{H_{it} + B_t}, \tag{2.A.6}$$

or $c_{it} = (H_{it} + B_t) \theta^*_t$. Since $c_{it} \leq H_{it} + B_t$, then $\theta_{it} \leq \theta^*_t$. Finally, using the consumption rule derived above, we can rewrite equation (2.3.4) as

$$\frac{W_t}{\psi} \left[ \int_{\theta < \theta^*_t} \frac{\theta^*_t}{H_{it} + B_t} f(\theta) d\theta + \int_{\theta > \theta^*_t} \frac{\theta}{H_{it} + B_t} f(\theta) d\theta \right] = 1. \tag{2.A.7}$$

Equations (2.A.5) and (2.A.7) jointly determine $\theta^*_t$ and $H_{it}$. It is evident that $H_{it}$ and $\theta^*_t$ only depend on aggregate variables in the economy. Hence $H_{it} = H_t$, and $\theta^*_t = \theta^*_t$. Dropping the subscripts from equation (2.A.5) yields equation (2.4.12). Writing equation (2.A.7) more compactly and dropping the subscripts gives equation (2.4.13). Equations (2.4.14) to (2.4.18) are straightforward to obtain. Q.E.D.

The First Best Allocation The social planner’s problem is to maximize the total welfare:

$$\max_{c_{it}, n_{it}, I_t} \left\{ \max_{n_{it}} \int E_i \left[ \sum_{t=0}^{\infty} \beta^t \theta_{it} \log c_{it} - \psi n_{it} \right] di \right\} \tag{2.A.8}$$

with the following resource constraint

$$\int c_{it} di + I_t \leq K_t^a \left( \int n_{it} di \right)^{1-\alpha}, \tag{2.A.9}$$

and the aggregate capital evolves according to

$$K_{t+1} = (1 - \delta)K_t + I_t. \tag{2.A.10}$$

The social planner decides individual $i$’s labor supply $n_{it}$ before observing individual $i$’s idiosyncratic shock $\theta_{it}$ as in a decentralized economy. Since the labor decision is made before the realization of idiosyncratic shock, for each household $i$, $n_{it}$ is the same. Denote the optimal aggregate labor as $N_t$, which satisfies

$$\psi = (1 - \alpha) \lambda_t \frac{Y_t}{N_t}. \tag{2.A.11}$$
For the consumption $c_{it}$, 
\[
\frac{\theta_{it}}{c_{it}} = \lambda_t, \tag{2.A.12}
\]
for all $i$. Defining the aggregate consumption $\int c_{it} di = C_t$ and integrating the last equation with respect to $i$, we obtain
\[
\frac{1}{C_t} = \lambda_t. \tag{2.A.13}
\]
Here we assume $E(\theta_{it}) = 1$. The first order condition with respect to capital is then
\[
1 = \beta \left[ \frac{C_t}{C_{t+1}} \left( \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right) \right]. \tag{2.A.14}
\]
In the steady state, the savings rate can be calculated as
\[
\frac{I}{Y} = \frac{\delta K}{\delta K} = \frac{\alpha \delta \beta}{1 - (1 - \delta) \beta}. \tag{2.A.15}
\]

Proof of Proposition 2.4  Suppose $\xi > \xi^*$. We aim to show that in equilibrium, the steady state $q$ equals 1, and the liquidity constraint on investment (2.4.3) is never binding. Equation (2.5.7) implies the savings rate is
\[
\frac{I}{Y} = \frac{\delta K}{\delta K} = \frac{\alpha \delta}{1 - (1 - \delta) \beta}. \tag{2.A.16}
\]
In addition, it is easy to show that if $\xi > \xi^*$, $\frac{L}{K} < \pi (\xi + R)$, i.e. the liquidity constraint is not binding. In contrast, if $\xi < \xi^*$, $q > 1$ and aggregating the firms’ optimal investment decisions gives
\[
I_t = \pi [q_t \xi + R_t] K_t \tag{2.A.16}
\]
\[
K_{t+1} = I_t + (1 - \delta) K_t. \tag{2.A.17}
\]
These last three equations together with (2.5.7) imply that in the steady state
\[
R = \alpha \frac{Y}{K} = \frac{\delta \frac{1 - \pi}{\pi}}{\xi \frac{1 - \beta}{1 - \beta} + 1} + \delta. \tag{2.A.18}
\]
We can immediately obtain the savings rate
\[
\frac{I}{Y} = \alpha \frac{\xi \frac{1 - \beta}{\beta} + 1}{\frac{1 - \pi}{\pi} + \xi \frac{1 - \beta}{1 - \beta} + 1}. \tag{2.A.19}
\]
and

\[ q = \frac{1}{\xi} \left( \frac{\delta}{\pi} - R \right) = \frac{\delta(1 - \pi)}{\pi} \frac{\beta}{\xi \beta + 1 - \beta}. \]  

(2.A.20)

This establishes the proof. Q.E.D.
2.B Sample Description

Table 2.2: Variables used in the aggregate savings rate regression

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data source</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross domestic savings rate</td>
<td>WDI</td>
<td>% of GDP</td>
</tr>
<tr>
<td>Domestic credit to private sector</td>
<td>WDI</td>
<td>% of GDP</td>
</tr>
<tr>
<td>Aged dependency ratio</td>
<td>WDI</td>
<td>Pop. aged 65 and above/total pop.</td>
</tr>
<tr>
<td>Youth dependency ratio</td>
<td>WDI</td>
<td>Pop. aged 0-14/total pop.</td>
</tr>
<tr>
<td>Consumer price inflation</td>
<td>WDI</td>
<td>Annual rate</td>
</tr>
<tr>
<td>GDP deflator inflation</td>
<td>WDI</td>
<td>Annual rate</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>WDI</td>
<td>Annual rate</td>
</tr>
<tr>
<td>Per capita GDP</td>
<td>WDI</td>
<td>In constant 2000 US dollar</td>
</tr>
<tr>
<td>Per capita GDP growth</td>
<td>WDI</td>
<td>Annual rate</td>
</tr>
<tr>
<td>Public expenditure</td>
<td>PWT 7.0</td>
<td>% of GDP</td>
</tr>
<tr>
<td>Current account balance</td>
<td>WDI</td>
<td>% of GDP, positive if surplus</td>
</tr>
<tr>
<td>Life expectancy</td>
<td>WDI</td>
<td>Life expectancy at birth, in years</td>
</tr>
</tbody>
</table>

Table 2.3: Variables used in the household savings regression

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data source</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household savings rate</td>
<td>OECD</td>
<td>% of household disposable income</td>
</tr>
<tr>
<td>Household liabilities</td>
<td>OECD</td>
<td>Loans</td>
</tr>
<tr>
<td>Aged dependency ratio</td>
<td>WDI</td>
<td>Pop. aged 65 and above/total population</td>
</tr>
<tr>
<td>Youth dependency ratio</td>
<td>WDI</td>
<td>Pop. aged 0-14/total population</td>
</tr>
<tr>
<td>Consumer price inflation</td>
<td>WDI</td>
<td>Annual rate</td>
</tr>
<tr>
<td>GDP deflator inflation</td>
<td>WDI</td>
<td>Annual rate</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>WDI</td>
<td>Annual rate</td>
</tr>
<tr>
<td>P-c household disp. income growth</td>
<td>OECD</td>
<td>Annual rate</td>
</tr>
<tr>
<td>P-c household disp. income</td>
<td>OECD</td>
<td></td>
</tr>
<tr>
<td>Social expenditures</td>
<td>OECD</td>
<td>% of GDP</td>
</tr>
<tr>
<td>Life expectancy</td>
<td>WDI</td>
<td>Life expectancy at birth, in years</td>
</tr>
</tbody>
</table>
Table 2.4: Economies in the sample

<table>
<thead>
<tr>
<th>Country</th>
<th>Code</th>
<th>Region/Group</th>
<th>Country</th>
<th>Code</th>
<th>Region/Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>AUS</td>
<td>OECD</td>
<td>Norway</td>
<td>NOR</td>
<td>OECD</td>
</tr>
<tr>
<td>Belgium</td>
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<td>OECD</td>
<td>Poland</td>
<td>POL</td>
<td>OECD</td>
</tr>
<tr>
<td>Canada</td>
<td>CAN</td>
<td>OECD</td>
<td>Portugal</td>
<td>PRT</td>
<td>OECD</td>
</tr>
<tr>
<td>Czech Rep.</td>
<td>CZE</td>
<td>OECD</td>
<td>Slovak Rep.</td>
<td>SVK</td>
<td>OECD</td>
</tr>
<tr>
<td>Denmark</td>
<td>DNK</td>
<td>OECD</td>
<td>Slovenia</td>
<td>SVN</td>
<td>OECD</td>
</tr>
<tr>
<td>Estonia</td>
<td>EST</td>
<td>OECD</td>
<td>Spain</td>
<td>ESP</td>
<td>OECD</td>
</tr>
<tr>
<td>Finland</td>
<td>FIN</td>
<td>OECD</td>
<td>Sweden</td>
<td>SWE</td>
<td>OECD</td>
</tr>
<tr>
<td>France</td>
<td>FRA</td>
<td>OECD</td>
<td>Switzerland</td>
<td>CHE</td>
<td>OECD</td>
</tr>
<tr>
<td>Germany</td>
<td>DEU</td>
<td>OECD</td>
<td>United Kingdom</td>
<td>GBR</td>
<td>OECD</td>
</tr>
<tr>
<td>Greece</td>
<td>GRC</td>
<td>OECD</td>
<td>United States</td>
<td>USA</td>
<td>OECD</td>
</tr>
<tr>
<td>Hungary</td>
<td>HUN</td>
<td>OECD</td>
<td>China</td>
<td>CHN</td>
<td>East Asia</td>
</tr>
<tr>
<td>Iceland *</td>
<td>ISL</td>
<td>OECD</td>
<td>Hong Kong</td>
<td>HKG</td>
<td>East Asia</td>
</tr>
<tr>
<td>Ireland *</td>
<td>IRL</td>
<td>OECD</td>
<td>India</td>
<td>IND</td>
<td>East Asia</td>
</tr>
<tr>
<td>Israel *</td>
<td>ISR</td>
<td>OECD</td>
<td>Indonesia</td>
<td>IDN</td>
<td>East Asia</td>
</tr>
<tr>
<td>Italy</td>
<td>ITA</td>
<td>OECD</td>
<td>Malaysia</td>
<td>MYS</td>
<td>East Asia</td>
</tr>
<tr>
<td>Japan *</td>
<td>JPN</td>
<td>OECD/East Asia</td>
<td>Pakistan</td>
<td>PAK</td>
<td>East Asia</td>
</tr>
<tr>
<td>Korea</td>
<td>KOR</td>
<td>OECD/East Asia</td>
<td>Philippines</td>
<td>PHL</td>
<td>East Asia</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>LUX</td>
<td>OECD</td>
<td>Singapore</td>
<td>SGP</td>
<td>East Asia</td>
</tr>
<tr>
<td>Mexico *</td>
<td>MEX</td>
<td>OECD</td>
<td>Thailand</td>
<td>THA</td>
<td>East Asia</td>
</tr>
<tr>
<td>Netherlands</td>
<td>NLD</td>
<td>OECD</td>
<td>Vietnam</td>
<td>VNM</td>
<td>East Asia</td>
</tr>
<tr>
<td>New Zealand</td>
<td>NZL</td>
<td>OECD</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: OECD countries with an asterisk are not included in the household savings regression, due to unavailability of data.
2.C Aggregate Savings Rate Regression

This appendix presents the main results of aggregate savings rate regressions. We use three different estimators: within estimator and random-effect estimator that apply to the static models, and the system estimator that applies to the dynamic model. We also consider three different samples: the sample of Asian countries studied by Horioka and Terada-Hagiwara (2010), the sample of 31 OECD economies, and finally the full sample consisting of both the Asian and OECD economies. Table 2.5 – 2.7 present the results on the three different samples.

In all of the following tables, we report the estimated coefficients, their robust standard errors and the significance levels. However, please note that we don’t report the coefficients and statistics on life expectancy, current account balance and public expenditures to save the space in Table 2.5 – 2.7. The full results are available on request. The standard errors are displayed in the brackets under the coefficients. ***, ** and * indicates significance at 1%, 5% and 10%, respectively. We also report several tests: Wald test for the joint significance of the coefficients, Sargan test for the validity of the instruments, Wooldridge test for the first-order autocorrelation of the residuals in the static model and Arellano-Bond test for the autocorrelations of the first difference of the residuals in the dynamic model.

In deriving the results, we instrument the endogenous variables using their first two feasible lags in the first-differenced equation and using the first lag difference in the level equation. We also employ the financial reform index prepared by Abiad, Detragiache and Tressel (2008) as an external instrument in the level equation. As a robustness check, we also consider reducing the instrument count in our regression exercises by "collapsing" the instrument matrix, as suggested and exemplied by Roodman (2009a, 2009b).

To further check the robustness of the hump-shaped relationship, we consider four additional measures of financial development in Table 2.8 – 2.9 by running fixed-effect and random-effect regressions. The four measures are: deposit money bank assets to GDP ratio, financial market depth, stock market capitalization to GDP ratio and M2 to GDP ratio, where we measure the financial market depth by the sum of outstanding domestic private debt securities and stock market capitalization to GDP ratio. The tables show that there does exist a significant hump-shaped relationship between the aggregate savings rates and financial development, regardless of which measure we employ.
Table 2.5: Aggregate savings rate regression: dynamic panel, East Asia subsample

<table>
<thead>
<tr>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimator</td>
<td>GMM-System</td>
<td>GMM-System</td>
<td>Within RE</td>
<td>RE</td>
</tr>
<tr>
<td>Number of IV</td>
<td>312</td>
<td>32</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Collapsed IV</td>
<td>No</td>
<td>Yes</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Lagged savings rate</td>
<td>0.851***</td>
<td>0.805***</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>(0.0411)</td>
<td>(0.0948)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private credits</td>
<td>0.0327**</td>
<td>0.109**</td>
<td>0.140***</td>
<td>0.261***</td>
</tr>
<tr>
<td></td>
<td>(0.0160)</td>
<td>(0.0531)</td>
<td>(0.0206)</td>
<td>(0.0263)</td>
</tr>
<tr>
<td>Private credits sq.</td>
<td>-0.00808*</td>
<td>-0.0271*</td>
<td>-0.0318***</td>
<td>-0.0816***</td>
</tr>
<tr>
<td></td>
<td>(0.00446)</td>
<td>(0.0160)</td>
<td>(0.00817)</td>
<td>(0.0115)</td>
</tr>
<tr>
<td>Inflation rate</td>
<td>-0.0445</td>
<td>-0.210**</td>
<td>0.0696*</td>
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<td>(0.0682)</td>
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<td>Per capita GDP growth</td>
<td>0.241***</td>
<td>0.209</td>
<td>0.272***</td>
<td>0.673***</td>
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<td>(0.0267)</td>
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<td>(0.0765)</td>
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<td>Per capita GDP (log)</td>
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<td>Per capita GDP (log) sq.</td>
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<td>3rd-order autocorr.</td>
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Table 2.6: Aggregate savings rate regression: dynamic panel, OECD subsample

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<td>GMM-System</td>
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<td>RE</td>
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<td>Number of IV</td>
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<td>Collapsed IV</td>
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|                                |          |           |           |           |
| Lagged savings rate            | 0.794*** | 0.830***  | –         | –         |
| Private credits                | 0.0106** | 0.0241    | 0.0552**  | 0.0513**  |
| Private credits sq.            | -0.00670*** | -0.0132** | -0.0168** | -0.0165*  |
| Inflation rate                 | 0.0231*** | 0.00758   | 0.0304    | 0.0239    |
| Real interest rate             | -0.0817*** | -0.0853   | -0.136*** | -0.169*** |
| Per capita GDP growth          | 0.348***  | 0.279***  | 0.200***  | 0.235***  |
| Per capita GDP (log)           | -0.00769  | -0.0779   | 0.329*    | 0.259     |
| Per capita GDP (log) sq.       | 0.000521  | 0.00413   | -0.0136   | -0.0107   |
| Aged dependency ratio          | -0.101*** | -0.0945*** | -0.612*** | -0.583*** |
| Youth dependency ratio         | -0.0395*** | -0.0155   | -0.278**  | -0.316*** |
| Wald test (p-val)              | 0.0000    | 0.0000    | 0.0000    | 0.0000    |
| Sargan/Hansen test (p-val)     | 1.0000    | 0.492     | –         | –         |
| Wooldridge test (p-val)        | –         | –         | 0.0000    | 0.0000    |
| Arellano-Bond test (p-val):    |           |           |           |           |
| 1st-order autocorr.            | 0.0040    | 0.001     | –         | –         |
| 2nd-order autocorr.            | 0.2748    | 0.316     | –         | –         |
| 3rd-order autocorr.            | 0.1087    | 0.114     | –         | –         |
| Observations (No. of countries)| 603 (27)  | 603 (27)  | 731 (31)  | 731 (31)  |
Table 2.7: Aggregate savings rate regression: dynamic panel, full sample

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<td>Lagged savings rate</td>
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<td>(0.0136)</td>
<td>(0.0514)</td>
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<td>Private credits</td>
<td>0.0210***</td>
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<td>Private credits sq.</td>
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<td>-0.0296***</td>
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<td>(0.0553)</td>
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<td>Per capita GDP growth</td>
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<td>0.288***</td>
<td>0.245***</td>
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<td>Per capita GDP (log)</td>
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<td>0.180***</td>
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<td>Per capita GDP (log) sq.</td>
<td>-0.000566</td>
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<td>-0.212**</td>
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<td>(0.0944)</td>
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Table 2.8: Aggregate savings rate reg: alternative measures, within estimator, full sample

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<td>Per capita GDP (log)</td>
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<td>Per capita GDP (log) sq.</td>
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Table 2.9: Aggregate savings rate reg: alternative measures, RE estimator, full sample

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<td>0.173***</td>
<td>0.230***</td>
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<td>-0.234**</td>
<td>-0.0827</td>
<td>-0.173</td>
<td>-0.289***</td>
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<tr>
<td>Obs. (No. of countries)</td>
<td>966 (41)</td>
<td>519 (36)</td>
<td>636 (41)</td>
<td>874 (40)</td>
</tr>
<tr>
<td>Wald test (p-val)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
2.D Household Savings Rate Regression

In this appendix, we investigate how the household savings rate changes with household credits for the sample of OECD countries. We focus on OECD countries, simply because of data accessibility. For East Asian countries, there does not exist a single data source from which we could retrieve all of the data needed in this regression.

We collect the annual time series data of the household savings rate and its potential explanatory variables for OECD economies from the OECD databases, whenever the data are available. As a result, we have a sample of 26 countries, with the time series ranging from 1995 to 2008, because the earliest observations on household finance begin in 1995.

Consider the following reduced-form regression equation:

$$s^h_{it} = \gamma_1 s^h_{it-1} + \gamma_2 X^h_{it} + \alpha^h_i + u^h_{it},$$  \hspace{1cm} (2.D.1)

where $s^h$ denotes the household savings rate, $X^h$ is a vector of explanatory variables reflecting the volume of household credits, household disposable income and its growth, age structure, social expenditures, the rates of return and uncertainty, all of which are important to household saving behavior, $\alpha^h$ denotes the country-specific effects, and finally $u^h$ is the error term.

For the same reasons as those given in the main text for the aggregate savings rate regression, we choose to work with the dynamic model. We use the ratio of household liabilities to household disposable income to measure the size of credits going to households.

Table 2.10 – 2.12 summarize the results using different estimators and different specifications. We use three estimators: the Arellano-Bond system estimator for the dynamic specification in equation (2.D.1), the within estimator, and the RE estimator for static specification without the lagged dependent variable. Note that in all of the three tables, we don’t report the results on life expectancy to save the space. Full results are available upon request.

Table 2.10 investigates if there exists a hump-shaped relationship between the household savings rates and household credits. To this end, we introduce the quadratic term. In column (1) and (2), we also control for the overall financial development using two different indices: the financial reform index prepared by Abiad, Detragiache and Tressel (2008) in column (1)
and private credits to GDP ratio in column (2). The table shows that there does not exist any hump-shaped relationship between the household savings rate and the household credits. Instead, the household savings rates are monotonically decreasing in household credits.

Table 2.11 and 2.12 confirm the monotonic relationship using different specifications and estimators. In Table 2.11, we employ the dynamic panel data regression without controlling for the overall financial development, while in Table 2.12 we control for the overall financial development using the same two different indices: financial reform index in column (1) and (3), private credits to GDP ratio in column (2) and (4).

It is evident from the tables that the household savings rates are indeed decreasing in the volume of household credits. This result implies that households tend to save less when the level of household finance development is high, which may justify our modeling of the household side in this paper.
### Table 2.10: OECD household savings reg: with quadratic terms

<table>
<thead>
<tr>
<th>Estimator</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household credits</td>
<td>-0.155**</td>
<td>-0.0731</td>
<td>-0.0664*</td>
<td>-0.0678*</td>
</tr>
<tr>
<td></td>
<td>(0.0732)</td>
<td>(0.0603)</td>
<td>(0.0374)</td>
<td>(0.0351)</td>
</tr>
<tr>
<td>Household credits sq.</td>
<td>0.0331</td>
<td>0.00395</td>
<td>-0.00328</td>
<td>-0.00102</td>
</tr>
<tr>
<td></td>
<td>(0.0275)</td>
<td>(0.0226)</td>
<td>(0.0149)</td>
<td>(0.0149)</td>
</tr>
<tr>
<td>Financial development</td>
<td>-0.114</td>
<td>-0.00810</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>(0.0734)</td>
<td>(0.0114)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P-c household disp. inc. growth</td>
<td>0.298***</td>
<td>0.296***</td>
<td>0.287***</td>
<td>0.271***</td>
</tr>
<tr>
<td></td>
<td>(0.0514)</td>
<td>(0.0599)</td>
<td>(0.0517)</td>
<td>(0.0569)</td>
</tr>
<tr>
<td>P-c household disp. inc. (log)</td>
<td>-0.135</td>
<td>-0.309</td>
<td>-0.337</td>
<td>-0.603**</td>
</tr>
<tr>
<td></td>
<td>(0.777)</td>
<td>(0.704)</td>
<td>(0.336)</td>
<td>(0.307)</td>
</tr>
<tr>
<td>P-c household disp. inc. (log) sq.</td>
<td>0.0167</td>
<td>0.0244</td>
<td>0.0252</td>
<td>0.0369**</td>
</tr>
<tr>
<td></td>
<td>(0.0418)</td>
<td>(0.0382)</td>
<td>(0.0181)</td>
<td>(0.0164)</td>
</tr>
<tr>
<td>Inflation rate</td>
<td>0.218</td>
<td>0.292*</td>
<td>0.287***</td>
<td>0.328***</td>
</tr>
<tr>
<td></td>
<td>(0.154)</td>
<td>(0.142)</td>
<td>(0.0701)</td>
<td>(0.0769)</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>0.137</td>
<td>0.188*</td>
<td>0.166***</td>
<td>0.225***</td>
</tr>
<tr>
<td></td>
<td>(0.0878)</td>
<td>(0.0923)</td>
<td>(0.0626)</td>
<td>(0.0685)</td>
</tr>
<tr>
<td>Social expenditures</td>
<td>1.661</td>
<td>1.510*</td>
<td>1.494***</td>
<td>0.707***</td>
</tr>
<tr>
<td></td>
<td>(0.983)</td>
<td>(0.819)</td>
<td>(0.345)</td>
<td>(0.235)</td>
</tr>
<tr>
<td>Aged dependency ratio</td>
<td>-0.449</td>
<td>-0.445</td>
<td>-0.381</td>
<td>-0.347*</td>
</tr>
<tr>
<td></td>
<td>(0.398)</td>
<td>(0.348)</td>
<td>(0.247)</td>
<td>(0.183)</td>
</tr>
<tr>
<td>Youth dependency ratio</td>
<td>0.456</td>
<td>0.497**</td>
<td>0.424**</td>
<td>0.265*</td>
</tr>
<tr>
<td></td>
<td>(0.267)</td>
<td>(0.184)</td>
<td>(0.191)</td>
<td>(0.141)</td>
</tr>
<tr>
<td>Wald test (p-val)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Observations (No. of countries)</td>
<td>194 (24)</td>
<td>224 (26)</td>
<td>229 (26)</td>
<td>229 (26)</td>
</tr>
</tbody>
</table>
### Table 2.11: OECD household savings reg: without financial development

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Estimator</strong></td>
<td>GMM-System</td>
<td>GMM-System</td>
<td>Within RE</td>
<td>RE</td>
</tr>
<tr>
<td><strong>Number of IV</strong></td>
<td>19</td>
<td>26</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td><strong>Collapsed IV</strong></td>
<td>Yes</td>
<td>Yes</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Variable</strong></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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</thead>
<tbody>
<tr>
<td>Lagged household savings rate</td>
<td>0.724***</td>
<td>0.791***</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>(0.109)</td>
<td>(0.0598)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Household credits</td>
<td>-0.0262**</td>
<td>-0.0255*</td>
<td>-0.0747***</td>
<td>-0.0705***</td>
</tr>
<tr>
<td></td>
<td>(0.0107)</td>
<td>(0.0138)</td>
<td>(0.0183)</td>
<td>(0.0160)</td>
</tr>
<tr>
<td>P-c household disp. inc. (log)</td>
<td>-0.422**</td>
<td>-0.496**</td>
<td>-0.532</td>
<td>-0.709</td>
</tr>
<tr>
<td></td>
<td>(0.215)</td>
<td>(0.231)</td>
<td>(0.694)</td>
<td>(0.534)</td>
</tr>
<tr>
<td>P-c household disp. inc. (log) sq.</td>
<td>0.0224**</td>
<td>0.0265**</td>
<td>0.0353</td>
<td>0.0422</td>
</tr>
<tr>
<td></td>
<td>(0.0101)</td>
<td>(0.0109)</td>
<td>(0.0374)</td>
<td>(0.0300)</td>
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<td>P-c household disp. inc. growth</td>
<td>-0.184</td>
<td>0.0711</td>
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<td>0.269***</td>
</tr>
<tr>
<td></td>
<td>(0.284)</td>
<td>(0.123)</td>
<td>(0.0653)</td>
<td>(0.0597)</td>
</tr>
<tr>
<td>Inflation rate</td>
<td>0.171</td>
<td>0.233***</td>
<td>0.259**</td>
<td>0.332***</td>
</tr>
<tr>
<td></td>
<td>(0.128)</td>
<td>(0.0795)</td>
<td>(0.119)</td>
<td>(0.0979)</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>0.272***</td>
<td>0.328***</td>
<td>0.289**</td>
<td>0.393***</td>
</tr>
<tr>
<td></td>
<td>(0.0844)</td>
<td>(0.0630)</td>
<td>(0.124)</td>
<td>(0.128)</td>
</tr>
<tr>
<td>Social expenditures</td>
<td>-0.107</td>
<td>-0.0633</td>
<td>1.366</td>
<td>0.618*</td>
</tr>
<tr>
<td></td>
<td>(0.158)</td>
<td>(0.141)</td>
<td>(0.875)</td>
<td>(0.369)</td>
</tr>
<tr>
<td>Aged dependency ratio</td>
<td>0.0234</td>
<td>-0.0349</td>
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<td>-0.296</td>
</tr>
<tr>
<td></td>
<td>(0.131)</td>
<td>(0.112)</td>
<td>(0.320)</td>
<td>(0.250)</td>
</tr>
<tr>
<td>Youth dependency ratio</td>
<td>-0.119</td>
<td>-0.0889</td>
<td>0.425*</td>
<td>0.271</td>
</tr>
<tr>
<td></td>
<td>(0.134)</td>
<td>(0.131)</td>
<td>(0.214)</td>
<td>(0.195)</td>
</tr>
<tr>
<td>Wald test (p-val)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Sargan/Hansen test (p-val)</td>
<td>0.567</td>
<td>0.560</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Wooldridge test (p-val)</td>
<td>–</td>
<td>–</td>
<td>0.0000</td>
<td>0.0000</td>
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<tr>
<td>Arellano-Bond test (p-val):</td>
<td>0.035</td>
<td>0.023</td>
<td>–</td>
<td>–</td>
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<tr>
<td>1st-order autocorr.</td>
<td>0.255</td>
<td>0.313</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>2nd-order autocorr.</td>
<td>0.516</td>
<td>0.394</td>
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<td>–</td>
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<tr>
<td>3rd-order autocorr.</td>
<td>0.170</td>
<td>0.240</td>
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<td>170 (24)</td>
<td>170 (24)</td>
<td>229 (26)</td>
<td>229 (26)</td>
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</table>

93
<table>
<thead>
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<th>(1)</th>
<th>(2)</th>
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</thead>
<tbody>
<tr>
<td>Estimator</td>
<td>GMM-System</td>
<td>GMM-System</td>
<td>Within</td>
<td>Within</td>
</tr>
<tr>
<td>Number of IV</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Collapsed IV</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lagged household savings rate</td>
<td>0.527***</td>
<td>0.805***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.112)</td>
<td>(0.0893)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Household credits</td>
<td>-0.0242*</td>
<td>-0.0274**</td>
<td>-0.0826***</td>
<td>-0.0658***</td>
</tr>
<tr>
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<td>(0.0144)</td>
<td>(0.0136)</td>
<td>(0.0201)</td>
<td>(0.0190)</td>
</tr>
<tr>
<td>Financial developent</td>
<td>-0.0390</td>
<td>0.00423</td>
<td>-0.130</td>
<td>-0.00656</td>
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<td>(0.0366)</td>
<td>(0.0201)</td>
<td>(0.0763)</td>
<td>(0.0112)</td>
</tr>
<tr>
<td>P-c household disp. inc. growth</td>
<td>-0.0557</td>
<td>0.235*</td>
<td>0.294***</td>
<td>0.294***</td>
</tr>
<tr>
<td></td>
<td>(0.125)</td>
<td>(0.128)</td>
<td>(0.0504)</td>
<td>(0.0676)</td>
</tr>
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<td>P-c household disp. inc. (log)</td>
<td>0.345</td>
<td>0.341</td>
<td>-0.305</td>
<td>-0.500</td>
</tr>
<tr>
<td></td>
<td>(0.334)</td>
<td>(0.444)</td>
<td>(0.709)</td>
<td>(0.674)</td>
</tr>
<tr>
<td>P-c household disp. inc. (log) sq.</td>
<td>-0.0195</td>
<td>-0.0152</td>
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<td>0.0340</td>
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<td>(0.0176)</td>
<td>(0.0244)</td>
<td>(0.0384)</td>
<td>(0.0365)</td>
</tr>
<tr>
<td>Inflation rate</td>
<td>0.492***</td>
<td>0.326***</td>
<td>0.212</td>
<td>0.266**</td>
</tr>
<tr>
<td></td>
<td>(0.114)</td>
<td>(0.106)</td>
<td>(0.155)</td>
<td>(0.117)</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>0.403***</td>
<td>0.336***</td>
<td>0.253*</td>
<td>0.316***</td>
</tr>
<tr>
<td></td>
<td>(0.0355)</td>
<td>(0.0634)</td>
<td>(0.144)</td>
<td>(0.111)</td>
</tr>
<tr>
<td>Social expenditures</td>
<td>-0.489***</td>
<td>0.162</td>
<td>1.479</td>
<td>1.360</td>
</tr>
<tr>
<td></td>
<td>(0.180)</td>
<td>(0.170)</td>
<td>(0.894)</td>
<td>(0.837)</td>
</tr>
<tr>
<td>Aged dependency ratio</td>
<td>0.378***</td>
<td>-0.0976</td>
<td>-0.458</td>
<td>-0.386</td>
</tr>
<tr>
<td></td>
<td>(0.121)</td>
<td>(0.166)</td>
<td>(0.376)</td>
<td>(0.325)</td>
</tr>
<tr>
<td>Youth dependency ratio</td>
<td>0.0536</td>
<td>-0.200</td>
<td>0.464</td>
<td>0.494**</td>
</tr>
<tr>
<td></td>
<td>(0.131)</td>
<td>(0.124)</td>
<td>(0.278)</td>
<td>(0.186)</td>
</tr>
<tr>
<td>Wald test (p-val)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Sargan/Hansen test (p-val)</td>
<td>0.570</td>
<td>0.249</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wooldridge test (p-val)</td>
<td></td>
<td></td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Arellano-Bond test (p-val)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st-order autocorr.</td>
<td>0.044</td>
<td>0.016</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd-order autocorr.</td>
<td>0.703</td>
<td>0.585</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3rd-order autocorr.</td>
<td>0.945</td>
<td>0.460</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations (No. of countries)</td>
<td>139 (22)</td>
<td>165 (24)</td>
<td>194 (24)</td>
<td>224 (26)</td>
</tr>
</tbody>
</table>
In this section, we apply the Baltagi-Li (2002) estimator for partially linear panel data models to our full sample. We choose this estimator among many others, simply because there exists a handy user-written Stata command *xtsemipar* to implement it. In the following, we draw from Baltagi and Li (2002) in introducing their methods.

We consider the following semiparametric specification:

\[ s_{it} = \kappa' \tilde{X}_{it} + g(z_{it}) + \tilde{\alpha}_i + \tilde{u}_{it}, \tag{2.E.1} \]

where \( \tilde{X}_{it} \) contains those explanatory variables of the savings rate explained in Section 2.2.2, except the measure of financial development (private credits), \( z_{it} \) denotes the ratio of aggregate private credits to GDP, \( \tilde{\alpha}_i \) is the unobserved country fixed effects, and finally \( \tilde{u}_{it} \) is the error term.

Thus, financial development enters nonparametrically while the other explanatory variables enter parametrically in this partially linear specification. To eliminate the fixed effects, we choose to work with the first-differenced model:

\[ \Delta s_{it} = \kappa' \Delta \tilde{X}_{it} + [g(z_{it}) - g(z_{it-1})] + \Delta \tilde{u}_{it}, \tag{2.E.2} \]

where \( \Delta \) denotes the first difference. Baltagi and Li (2002) propose using series \( p^K(z) \) of dimension \( K \times 1 \) to approximate \( g(z) \), where \( p^K(z) \) denotes the first \( K \) elements of series \( \{p_j(z)\}_{j=1,2,3,...} \). The series of functions have the property that as \( K \) grows there exists a linear combination of \( p^K(z) \) that can approximate any \( g(z) \) belonging to an additive class of functions arbitrarily well in mean square error. As a result, \( g(z_{it}) - g(z_{it-1}) \) can be approximated using \( p^K(z_{it}) - p^K(z_{it-1}) \) and equation (2.E.2) becomes:

\[ \Delta s_{it} = \kappa' \Delta \tilde{X}_{it} + \gamma [p^K(z_{it}) - p^K(z_{it-1})] + \Delta \tilde{u}_{it}. \tag{2.E.3} \]

Baltagi and Li (2002) show that \( \kappa \) and \( \gamma \) can be consistently estimated. Once \( \kappa \) and \( \gamma \) are known, we can get the residuals and hence by equation (2.E.1), we have

\[ \text{residuals} = s_{it} - \kappa' \tilde{X}_{it} - \tilde{\alpha}_i = g(z_{it}) + \tilde{u}_{it}. \tag{2.E.4} \]
Thus, $g(z_{it})$ can be fitted by some standard univariate nonparametric regression of the above residuals on $z_{it}$. Table 2.13 reports the coefficients on the other explanatory variables. It is evident that these results are consistent with what we obtained in Section 2.2, using parametric regression. Figure 2.7 displays the nonparametric fit of the savings rate on private credits, using quartic B-spline smoothing. The graph looks similar if we set the power to 3, 5 or 6. It confirms that the relationship between the gross savings rate and financial development is hump-shaped.

Figure 2.7: Nonparametric fit of the aggregate savings rate on private credits to GDP ratio, full sample
Table 2.13: Aggregate savings rate regression: Baltagi-Li estimator, full sample

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation rate</td>
<td>-0.0245</td>
<td>(0.0381)</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>-0.0978***</td>
<td>(0.0351)</td>
</tr>
<tr>
<td>Per capita GDP growth</td>
<td>0.100***</td>
<td>(0.0297)</td>
</tr>
<tr>
<td>Per capita GDP (log)</td>
<td>0.182</td>
<td>(0.109)</td>
</tr>
<tr>
<td>Per capita GDP (log) sq.</td>
<td>-0.00201</td>
<td>(0.00606)</td>
</tr>
<tr>
<td>Current account surplus</td>
<td>0.305***</td>
<td>(0.0349)</td>
</tr>
<tr>
<td>Public expenditures</td>
<td>-1.110***</td>
<td>(0.307)</td>
</tr>
<tr>
<td>Aged dependency ratio</td>
<td>-0.761***</td>
<td>(0.231)</td>
</tr>
<tr>
<td>Youth dependency ratio</td>
<td>0.125</td>
<td>(0.121)</td>
</tr>
<tr>
<td>Life expectancy</td>
<td>-0.292*</td>
<td>(0.149)</td>
</tr>
</tbody>
</table>

Observations: 920  
R-squared: 0.379

Note: Robust standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1
Bibliography and References


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CHAPTER 3

PARTICIPATION, WAGE AND UNEMPLOYMENT FLUCTUATIONS

3.1 Introduction

The labor market volatility puzzle has been attracting many attentions since Shimer (2005) and Hall (2005). By "puzzle", it means that the textbook DMP model (Pissarides, 2000) fails in explaining the observed labor market fluctuations. The failure is mainly due to the counterfactual high wage elasticity in the model, so introducing certain kind of real wage rigidity becomes a natural way to solve the puzzle. Gertler and Trigari’s (2009) "staggered Nash wage bargaining" is a prominent example along this line.

This paper also looks at the puzzle through the lens of wage, however, it does not assume any exogenous wage rigidity. Instead, it introduces home production in an otherwise standard model. Workers are heterogeneous in time-varying home productivity in the model and hence face endogenous labor market participation choice. The paper investigates how participation choice influences the wage dynamics and hence the unemployment fluctuations.

Here it goes. In the presence of home production, the expected outside options of workers depend on home productivity. Since home production becomes less appealing when the market productivity is high, the expected outside options are countercyclical. Under Nash

\[\text{103}\]
bargaining, the countercyclical expected outside options are translated into a countercyclical component in the wage equation. As a result, the wage elasticity is smaller than that in the textbook model which features constant participation. By calibrating the model to match the U.S. data, the paper shows that the model captures the wage dynamics quite well and hence can explain nearly half of the unemployment fluctuations without assuming any wage rigidity.

Why endogenous participation matters? Besides its important role in explaining the wage dynamics in the model, it is also widely recognized that the "ins and outs" of labor force are non-trivial parts of labor market flows. Krusell et al. (2009), for example, find that the monthly transition rate from nonparticipation to participation is around 10%. If we look at the so-called "marginally attached workers", then the transition rate from nonparticipation to participation is even higher. Jones and Ridell (1999) estimates that the transition rate from marginally attached to employment is 4 times larger, and to unemployment is 6 times larger than from the other nonparticipation force, using the Canadian data.\(^2\) Hence, neither home production nor endogenous participation is new in the literature. Benhabib, Rogerson and Wright (1991) and Greenwood, Rogerson and Wright (1995) have studied the implications of home production for real business cycles. Krusell et al. (2011) have studied a unified model of participation, unemployment and employment to explain the worker flows among the three states.

None of the above-mentioned papers, however, address the labor market volatility puzzle and hence are not directly related to this paper. This paper bears a direct relationship with Tripier (2003), Haefke and Reiter (2006) and Veracierto (2008). Tripier (2003) was the first to study the implications of endogenous participation for unemployment fluctuations in a search model of labor market. Veracierto (2008) studies the implications for the real business cycles using the Lucas-Prescott (1974) islands model. Both models generate a procyclical unemployment rate, which is counterfactual. Haefke and Reiter (2006) solve the procyclicality problem by introducing heterogeneous workers. However, their model cannot explain the unemployment fluctuations without resorting to the real wage rigidity, because they eliminate the important wage channel highlighted in this paper.

As in Haefke and Reiter (2006), I assume that workers are heterogeneous in home pro-

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\(^2\)Jones and Ridell (1999) also show that the marginally attached workers amounts to 25-30% of the unemployment size, and they are systematically different from the other nonparticipants. In the United States, BLS started to collect the data on this group from 1994, the sample shows that they are nearly 30% of the unemployment size, highly countercyclical and almost as volatile as unemployment.
ductivity. However, my model is different from theirs in two main respects. First, I assume that the idiosyncratic home productivity is i.i.d. across time and workers, while Haefke and Reiter assume that a worker redraws her home productivity with a probability less than 1. It is true that their assumption is more realistic, however, the assumption makes the model intractable and also introduces "some unusual incentives". Thus, they further assume that the probability of redrawing home productivity is equal to the exogenous job separation rate. This latter assumption eliminates the "unusual incentives", however, it also eliminates the effects of endogenous participation on wage (or, the wage channel), because the threshold level of home productivity for participation no longer enters the wage equation. By assuming i.i.d. home productivity, this paper highlights the wage channel without introducing the problem of "unusual incentives", because the probability of redrawing home productivity is always larger than the job separation rate. Second, I assume that the unemployed workers do not produce at home but get a fixed amount of unemployment benefits, while they assume that the flow utility of an unemployed worker is increasing in her idiosyncratic productivity at home. I make the assumption to distinguish between the exogenous unemployment benefits and the outside options of workers. In the equilibrium, the expected outside options of workers are endogenously determined. The assumption also makes the wage distribution degenerate. Hence, the calibration strategies are also different. As in Tripier (2003) and Veracierto (2008), I choose the parameters to match the relative volatility of employment, while Haefke and Reiter choose the parameters to match the wage dispersion.

The remainder of the paper proceeds as follows. Section 3.2 presents the model and studies the steady state properties. Section 3.3 parameterizes the model and then evaluates the performance of the model. Section 3.4 concludes. The appendix contains proofs of the propositions in the paper.

3.2 The Model

The model economy is populated by a continuum of workers and firms. Both of them are risk neutral and have unit measure. We first consider the case without capital, then we introduce capital in the last subsection.

\(^3\) To quote their original words, "for example, when \(\chi > \eta\), a nonemployed with low home productivity has an incentive to look for a job not just because she then collects a wage, but also because employment increases her chances for a new redraw of home productivity". In the quoted text, \(\chi\) denotes the probability of redrawing home productivity and \(\eta\) denotes the exogenous job separation rate.
Workers are heterogeneous in their productivity at home. At each point of time, each worker stays in one of the following states: outside of the labor force, unemployed or employed. The house workers stay outside of the labor force and are engaged in home production. The idiosyncratic productivity at home $x$ is $i.i.d.$ across time and workers. The distribution has a cumulative density function $G(x)$ and a nonnegative support over $[0, x_{\text{max}}]$. The unemployed workers search for jobs in the labor market and each receives unemployment benefits $b$, which is a constant. The employed workers earn wages. Neither the unemployed nor employed workers produce at home.

Firms are identical. Using labor as the only input, they produce a homogeneous market good that is a perfect substitute for the home good. The production function is $y_t = z_t m_t$, where $n_t$ denotes the labor demand and $z_t$ denotes the labor productivity. The logarithm of labor productivity follows an $AR(1)$ process. Namely, $\log(z_t) = \rho \log(z_{t-1}) + \varepsilon_t$, where $0 < \rho < 1$ and $\varepsilon_t \sim N(0, \sigma^2)$ denotes the stochastic innovations to the labor productivity.

The labor market is frictional. New hires are formed through random matches between the unemployed workers and job vacancies. The aggregate matching function $m(u_t, v_t)$ is a function of unemployment $u_t$ and aggregate job vacancies $v_t$. It is concave and linearly homogeneous with respect to both of its inputs. Thus, the job finding rate is $p^w_t = m_t / u_t$ and the vacancy filling rate is $p^f_t = m_t / v_t$. Apparently, $p^w_t = \theta_t p^f_t$, where $\theta_t = v_t / u_t$ measures the labor market tightness.

The timing of the model is as follows. In the beginning of each period, after observing her idiosyncratic productivity and the aggregate labor productivity, the nonemployed workers make the labor market participation choice. Then, the unemployed workers and firms are randomly matched, and bargain over the aggregate surplus to determine the wage. However, new hires take effect only in the next period. Production takes place after matching. After production, each firm is separated from a fraction $s$ of its workers in the end of the period.

3.2.1 Workers

There are three groups of workers: the house workers, the unemployed workers and the employed workers.

**House workers.** The house workers stay outside of the labor force and are engaged in
home production. The home production function is $y(x) = x$. In the beginning of the next period, each house worker either stays outside of the labor force or enters the labor market and becomes unemployed, depending on the relative productivity, namely, the difference between the home productivity and the market productivity.

Thus, the value function of a house worker with idiosyncratic productivity $x$ in period $t$ reads

$$H_t(x) = x + \beta E_t \int \max [U_{t+1}, H_{t+1}(x')] dG(x'),$$  

where $U_{t+1}$ denotes the utility value of being unemployed at time $t+1$, $H_{t+1}(x')$ denotes the utility value of being a house worker at time $t+1$ when the idiosyncratic productivity is $x'$, and $\beta$ is the subjective discount factor.

**Unemployed workers.** The unemployed workers search for jobs and each receives unemployment benefits $b$. The job finding rate of an unemployed worker is $p^u_t$. She becomes employed if she finds a job, otherwise, she stays unemployed. In the beginning of next period, she either continues to search for a job or quits the labor force, depending on the relative productivity.

Thus, the value function of an unemployed worker in period $t$ reads

$$U_t = b + \beta E_t \left[ p^u_t W_{t+1} + (1 - p^u_t) \int \max [U_{t+1}, H_{t+1}(x')] dG(x') \right],$$  

where $W_{t+1}$ denotes the utility value of being employed and I assume that new hires take effect one period later.

**Employed workers.** The employed workers are engaged in the production of the market good. Workers and firms bargain over the joint surplus of a match to determine the wage. In the end of the period, a firm is separated from a fraction $s$ of its workers. A worker become unemployed if she is separated. Then, in the beginning of the next period, she either searches for a job or quits the labor force, depending on the relative productivity.

Thus, the value function of an employed worker in period $t$ reads

$$W_t = w_t + \beta E_t \left[ (1 - s)W_{t+1} + s \int \max [U_{t+1}, H_{t+1}(x')] dG(x') \right],$$  

where $w_t$ is the wage and $W_{t+1}$ denotes the utility value of being employed in period $t+1$. 

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3.2.2 Firms

Firms produce the market good using labor as the only input and post costly vacancies to hire new workers. The flow cost of keeping a vacancy is $c$. With probability $p_t^f$, a vacancy is filled. However, the new hires take effect only one period later. Thus, the workforce of each firm evolves as

$$n_{t+1} = p_t^f v_t + (1 - s)n_t,$$

(3.2.4)

and the value of a vacancy reads

$$V_t = -c + \beta p_t^f E_t J_{t+1},$$

(3.2.5)

where $v_t$ denotes the vacancies, $n_t$ denotes the current workforce and $J_{t+1}$ denotes the value of a job in period $t+1$. In the equilibrium, the new hires come exactly from those who find jobs, thus equation (3.2.4) is equivalent to

$$n_{t+1} = p_t^u u_t + (1 - s)n_t.$$  

(3.2.6)

Assuming that there is free entry in the labor market, then the value of a vacancy $V_t = 0$. Namely,

$$0 = -c + \beta p_t^f E_t J_{t+1}. $$

(3.2.7)

Each firm has a profit per job $(z_t - w_t)$, where $z_t$ is the labor productivity. In the end of each period, the firm is separated from a fraction $s$ of its workers. Thus, the value of a job at time $t$ reads

$$J_t = z_t - w_t + \beta (1 - s)E_t J_{t+1}. $$

(3.2.8)

Combining equations (3.2.7) and (3.2.8) yields

$$\frac{c}{p_t^f} = \beta E_t \left[ z_{t+1} - w_{t+1} + (1 - s) \frac{c}{p_{t+1}^f} \right], $$

(3.2.9)

which is the job creation condition of firms.

The flow cost of maintaining a vacancy is $c$ and the average duration of it is $1/p_t^f$, thus
the LHS of equation (3.2.9) measures the total costs of keeping a vacancy. If the vacancy is filled, it generates a net revenue \((z_{t+1} - w_{t+1})\) for the firm in the next period, and with probability \((1 - s)\) the employment relationship is not separated, so the firm saves \(c/n_{t+1}^f\). Thus, the RHS measures the present value of a vacancy. All in all, equation (3.2.9) says each firm creates vacancies to a point where the expected costs and benefits are equal. It also tells that the incentive of job creation is determined by the wage, given the labor productivity and labor market conditions.

3.2.3 Nash Bargaining

Workers and firms bargain over the joint surplus of a match to determine the wage. Following the literature, I assume that bargaining takes place over wage to maximize the following Nash product

\[
[W_t - U_t]^{\eta} J_t^{1-\eta},
\]

where \(\eta\) measures the worker’s bargaining power, \((W_t - U_t)\) is the surplus of a job for the worker and \(J_t\) is the surplus for the firm. First-order condition gives

\[
\frac{W_t - U_t}{\eta} = S_t = \frac{J_t}{1 - \eta},
\]

where \(S_t = J_t + W_t - U_t\) is the joint surplus. Equation (3.2.11) says the worker gets a proportion \(\eta\) of the joint surplus and the firm gets the rest in the equilibrium.

3.2.4 Wage Dynamics

The Nash bargaining solution implies that we need to solve the joint match surplus first in order to derive the wage equation. The dynamics of the surplus is summarized in Proposition 3.1.

**Proposition 3.1** Under the generalized Nash bargaining, the joint surplus of a match evolves as follows.

\[
S_t = z_t - \left(b + \frac{\eta}{1 - \eta} c\theta_t\right) - \beta(1 - s - p_{t}^w) E_t \tilde{x}_{t+1} + \beta(1 - s) E_t S_{t+1},
\]

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where
\[ \bar{x}_{t+1} \equiv \int_{\bar{x}_{t+1}}^{x_{\text{max}}} (x' - \bar{x}_{t+1}) d\mathcal{G}(x'). \] (3.2.13)

The above proposition comes directly from the definition of the joint surplus. It is an important step toward deriving the wage equation, which is presented Proposition 3.2.

**Proposition 3.2** Under the generalized Nash bargaining, the equilibrium wage is
\[ w_t = \eta(z_t + c\theta_t) + (1 - \eta)\tilde{b}, \] (3.2.14)

where \( \tilde{b} = b + \beta(1 - s - p_t^{\omega})E_t\bar{x}_{t+1} \) denotes the expected outside options of workers.

Equation (3.2.14) is extremely similar to the wage equation in the standard DMP model. Indeed, if there is no heterogeneity among workers, then \( \tilde{b} = b \) and equation (3.2.14) reduces to the standard wage equation, which is \( w_t = \eta(z_t + c\theta_t) + (1 - \eta)b \). Hence, the only difference comes from the outside options of workers. In the standard DMP model, the outside options are exactly given by the exogenous unemployment benefits. It is not the case in my model. Although the unemployed workers are not engaged in home production in the model, they can always choose to quit the labor force if the relative productivity is high enough. As a result, the expected outside options depend on the cut-off \( \bar{x}_{t+1} \), which is endogenously determined by the relative productivity.

It is intuitive that \( \bar{x}_t \) is procyclical, because it is less appealing to stay at home in booms. It is also known that the job finding rate \( p_t^{\omega} \) is procyclical. Thus, \( \tilde{b} \) is countercyclical if the density function of the idiosyncratic productivity is monotonically decreasing. Hence, Proposition 3.2 implies that the wage channel is the key to understanding the effects of endogenous participation on the unemployment fluctuations. The standard model fails because the elasticity of wage is too large.\(^4\) Endogenous participation induces a countercyclical component in the wage equation, which makes the elasticity of wage smaller and the elasticity of the labor market tightness larger, compared to the textbook model. I will show it in Section 3.2.6.

\(^4\)In Shimer’s (2005) model, the elasticity of wage with respect to the labor productivity is 0.964, while in the data it is 0.45, according to Hagedorn and Manovskii (2008).
It is still necessary to pin down the cut-off $\bar{x}_t$. Equation (3.A.1) and (3.A.6) imply that $U_t - \beta E_t U_{t+1} = \bar{x}_t + \beta E_t \bar{x}_{t+1}$. Plugging it into equation (3.A.8) and rearranging terms yields

$$\bar{x}_t = b + \frac{\eta}{1-\eta} c \theta_t - \beta p_t^w E_t \bar{x}_{t+1}, \quad (3.2.15)$$

which governs the dynamics of $\bar{x}_t$.

### 3.2.5 Equilibrium

Finally, the equilibrium system is characterized as follows. The equilibrium dynamics for $(\theta_t, w_t, \bar{x}_t, n_t, u_t, h_t)$ satisfy the system of equations (3.2.9), (3.2.14), (3.2.15), (3.2.4), (3.A.3) and (3.A.4), where $v_t = \theta_t u_t$, $p_t^w = m(1, \theta_t)$ and $\bar{x}_{t+1}$ is given by equation (3.2.13).

### 3.2.6 The Steady State Elasticities

In this section, I show that the steady state elasticity of the labor market tightness is larger in my model, which implies that the model can generate larger unemployment fluctuations. To obtain analytical results, I assume that the matching function takes the following form:

$$m(u_t, v_t) = \frac{u_t v_t}{(u_t^v + v_t^v)^{1/\zeta}}. \quad (3.2.16)$$

Thus, the vacancy filling rate is $p_t^v(\theta) = (1 + \theta_t^v)^{-1/\zeta}$ and the job finding rate is $p_t^w(\theta) = (1 + \theta_t^{-\zeta})^{-1/\zeta}$.

In the steady state, equation (3.2.14) becomes

$$w = \eta (z + c \theta) + (1 - \eta) \{ b + \beta [1 - s - p^w(\theta)] \bar{x} \}. \quad (3.2.17)$$

Taking total differentiation of equation (3.2.17) and rearranging terms gives equation the following steady state elasticity of wage with respect to the labor productivity:

$$\epsilon_w z w = \eta z + [\eta c - (1-\eta)A] \theta \epsilon_\theta z - (1-\eta)B \bar{x} \epsilon_{\bar{x}, z}, \quad (3.2.18)$$

where $A = \beta \mu(0) \bar{x}$, $B = \beta [1 - s - p^w(\theta)] \mathcal{G}(\bar{x})$, $\bar{x} = \int^{x_{\text{max}}}_x (x - \bar{x}) d\mathcal{G}(x)$ and $\epsilon_{\bar{x}, z}$ is the
elasticity of $\bar{x}$ with respect to the labor productivity.\textsuperscript{5} Equation (3.2.16) guarantees that $p^\omega(\theta)$ is smaller than 1. Thus, for small $s$, $B > 0$. It is also intuitive that $A$ and $\varepsilon_{\theta,z}$ are positive. Manipulating the standard wage equation in a similar way gives the elasticity of wage in the standard model, which is $\varepsilon_{w,z}w = \eta(z + c\theta \varepsilon_{\theta,z})$.

The elasticity of the labor market tightness depends on the elasticity of wage. In the steady state, equation (3.2.9) becomes

$$\frac{c}{p^f(\theta)} = \frac{z - w}{r + s},$$

where $r = 1/\beta - 1$ and $p^f(\theta) = (1 + \theta^c)^{-1/c}$. Taking logarithm, then taking total differentiation of equation (3.2.19) and rearranging terms gives

$$\varepsilon_{\theta,z} = \left(1 + \frac{1}{\theta^c}\right) \frac{z - \varepsilon_{w,z}w}{z - w},$$

where $\varepsilon_{\theta,z}$ is the steady state elasticity of the labor market tightness with respect to the labor productivity.

Obviously, $\varepsilon_{\theta,z}$ is decreasing in $\varepsilon_{w,z}$. If the elasticity of wage is large, then the innovations to the labor productivity will have a very small effect on the labor market tightness.\textsuperscript{6} Combining equation (3.2.20) and equation (3.2.18) to cancel the $\varepsilon_{w,z}w$ term gives the following steady state elasticity of the labor market tightness with respect to the labor productivity:

$$\varepsilon_{\theta,z} = \frac{(1 - \eta)(z + B\bar{x}\varepsilon_{\bar{x},z})}{\theta^c + 1(z - w) + \eta c\theta - (1 - \eta)A\theta}.$$  

(3.2.21)

Similarly, I can derive the elasticity of the labor market tightness in the standard model, which is

$$\varepsilon_{\theta,z} = \frac{(1 - \eta)z}{\theta^c + 1(z - w) + \eta c\theta}.$$  

(3.2.22)

Obviously, $\varepsilon_{\theta,z} > \varepsilon_{\theta,z}^\star$, because $B\bar{x}\varepsilon_{\bar{x},z} \geq 0$ and $(1 - \eta)A\theta \geq 0$. Thus, the elasticity of the labor market tightness is larger than that in the standard model.\textsuperscript{7}

\textsuperscript{5}The results are similar if I use the Cobb-Douglas matching function $m(u_t, v_t) = \psi u_t^\alpha v_t^{1 - \alpha}$. The only difference is now $A = \beta(1 - \xi)p^f(\theta)^{\bar{x}}$.

\textsuperscript{6}If I use the Cobb-Douglas matching function, then $\varepsilon_{\theta,z} = \frac{1}{\phi} \frac{\varepsilon_{w,z}w}{z - w}$, which is also decreasing in $\varepsilon_{w,z}$. If $\varepsilon_{w,z} = 1$, then $\varepsilon_{\theta,z} = \frac{1}{\phi} \leq 2$ under the conventional calibration of $\xi \in [0.5, 0.72]$ in the literature. This is the essence of the Shimer critique (Shimer, 2005), because $\varepsilon_{w,z}$ is close to 1 in the standard model.

\textsuperscript{7}To be fair, the two steady state wages are different. Under the same parameters, the steady state wage in my model is larger than that in the standard model. However, this fact actually reinforces the result.
3.2.7 Capital

The presence of capital does not change the equilibrium system essentially. Nevertheless, with capital in the model, I can study the real business cycle implications of endogenous participation as in Veracierto (2008). Veracierto concludes that a RBC model augmented with search and endogenous labor market participation generates counterfactual labor market dynamics. This paper shows that the wage channel highlighted in this paper mitigates the problem to a large extent.

Following Merz (1995) and Andolfatto (1996), I adopt the large family and perfect risk-sharing assumption. The economy is populated by a representative household with unit size of members. The household accumulate capital and consume both the market good and the home good. The two goods are perfect substitutes for each other. In the beginning of each period, after observing the aggregate productivity shocks and the idiosyncratic productivity of household members, the household assign the workplaces for its nonemployed members and choose consumption and new capital to maximize the household utility.

Obviously, the household will also adopt a cut-off strategy to assign its members. For some threshold value $\bar{x}_t$, the household require those with idiosyncratic productivity $x > \bar{x}_t$ to stay at home and send the others out to search for jobs. Thus, the three groups of workers are also given by equation (3.2.6), (3.A.3) and (3.A.4), respectively. Given the cut-off strategy, the value functions for the household members in each of the three states are similar to equations (3.2.1), (3.2.2) and (3.2.3). The only difference lies in the discount factor. It is now stochastic, depending on the expected household consumption growth.

The economy is also populated by a continuum of identical firms with unit measure. Firms produce the homogenous market good using labor and capital as inputs. The production function is now $y_t = z_t f(k_t, n_t)$. Each firm hires capital and post vacancies to maximize the profit $\pi_t = z_t f(k_t, n_t) - w_t n_t - r_t k_t - c v_t$. Thus, the firm’s problem can formulated as

$$V^f(n_t) = \max_{k_t, v_t} \left[ \pi_t + E_t^\beta_t V^f(n_{t+1}) \right],$$

(3.2.23)

subject to

$$n_{t+1} = p^f v_t + (1 - s) n_t.$$  

(3.2.24)
The first order condition with respect to capital gives the standard relationship between the rental rate and the marginal product of capital. Namely, \( r_t = z_t f_k(k_t, n_t) \), where \( f_k \) denotes the partial derivative with respect to \( k \). Denote \( J_t \equiv \partial V^f(n_t)/\partial n_t \), so I have

\[
J_t = z_t f_n(k_t, n_t) - w_t + (1 - s) E_t \tilde{\beta}_{t+1} J_{t+1},
\]

(3.2.25)

and

\[
\frac{\partial V^f(n_t)}{\partial v_t} = 0 = -c + p_t^f E_t \tilde{\beta}_{t+1} J_{t+1}.
\]

(3.2.26)

Equation (3.2.25) and (3.2.26) are almost the same with equation (3.2.7) and (3.2.8), respectively. There are only two minor differences. First, \( \beta \) is replaced by the stochastic discount factor \( \tilde{\beta}_{t+1} \). Second, the flow value of a job becomes \( z_t f_n(k_t, n_t) - w_t \), where \( z_t f_n(k_t, n_t) \) is the marginal product of labor. As a result, the vacancy creation condition is also similar, which reads

\[
\frac{c}{p_t^f} = E_t \tilde{\beta}_{t+1} \left[ z_{t+1} f_n(k_{t+1}, n_{t+1}) - w_{t+1} + (1 - s) \frac{c}{p_{t+1}^f} \right].
\]

(3.2.27)

The bargaining problem is exactly the same as that in Section 3.2.3. Hence, the wage equation can be derived in the same manner, which reads

\[
w_t = \eta \left[ z_t f_n(k_t, n_t) + c \theta_t \right] + (1 - \eta) \left[ b + (1 - s - p_t^w) E_t \tilde{\beta}_{t+1} \tilde{x}_{t+1} \right].
\]

(3.2.28)

3.3 Quantitative Results

3.3.1 Calibration

This section parameterizes the full-fledged model with capital. I calibrate the model to match some key observations about the U.S. labor market and goods market. The results are summarized in Table 3.1.

The labor market parameters are chosen as follows. I use the matching function in equation (3.2.16). It guarantees that both the job finding rate and the vacancy filling rate are less than 1. The monthly job finding rate is around 0.6, which is based on Shimer’s (2005,
The average labor market tightness is around 0.7 (Pissarides, 2009). Thus, the relationship
\[ p_t^e(\theta) = \left(1 + \theta_t^{-\kappa}\right)^{-1/\kappa} \] gives \( \kappa = 2.3 \).

The job separation rate \( s = 0.034 \), which also comes from Shimer (2005, 2007). I choose
\( b = 1.35 \) and \( c = 1.9 \), so that the unemployment benefits amount to 40% of the labor income
and the vacancy costs amount to 1.5% of output. Finally, since there is no direct evidence
on the bargaining power of workers, I simply choose \( \eta = 0.5 \).

There are two categories of productivity in the model. There is no direct evidence on the
idiosyncratic productivity. I assume that it follows a power-law distribution, with cumulative
density function \( G(x) = (x/x_{\text{max}})^{\xi} \) and support over \([0, x_{\text{max}}]\). In the following simulation,
I choose \( x_{\text{max}} \) to satisfy the steady state equilibrium conditions\(^9\) and choose \( \xi \) to match
the relative volatility of employment. As a result, I have \( \xi = 0.9 \) and \( x_{\text{max}} = 6.5 \). These
parameters imply that the density of the idiosyncratic productivity has a long right tail, which
is a property of wage dispersion (Mortensen, 2003) and firm’s idiosyncratic productivity in
the data.

The calibration of the aggregate productivity is standard. I assume that it follows the
following \( AR(1) \) process,
\[ \log(z_t) = \rho \log(z_{t-1}) + \varepsilon_t. \] Then following Gertler and Trigari (2009),
I set \( \rho = 0.983 \) and choose \( \sigma_\varepsilon = 0.0065 \) to match the standard deviation of output in my
sample, which is 0.021.

The remaining parts of the model are parameterized as follows. The discount factor
\( \beta = 0.996 \), which matches a quarterly real interest rate of 1.2%. The production function is
Cobb-Douglas, i.e. \( y_t = z_t k_t^\alpha n_t^{1-\alpha} \) and \( \alpha = 1/3 \). Finally, the monthly depreciation rate of
capital \( \delta = 0.0084 \), so the quarterly rate is 0.025.

3.3.2 Empirical Patterns

Let me describe the actual economy before presenting the model economy. There is nothing
new here. Nevertheless, it provides the benchmark for me to evaluate the model’s performance.

\(^8\)What Shimer estimates is the monthly job finding probability. The number is 0.45. Converting it to the
Poisson arrival rate gives around 0.6.

\(^9\)The steady state equilibrium conditions used in the calibration are equations (3.2.9), (3.2.14) and (3.2.15).
Table 3.1: Baseline calibration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta )</td>
<td>Matching elasticity</td>
<td>2.3</td>
<td>( p^\alpha(\theta) = 0.6, \theta = 0.7 )</td>
</tr>
<tr>
<td>( \beta )</td>
<td>The discount factor</td>
<td>0.996</td>
<td>Quarterly ( r = 1.2% )</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Bargaining power of workers</td>
<td>0.5</td>
<td>Arbitrary but standard in the literature</td>
</tr>
<tr>
<td>( s )</td>
<td>Separation rate</td>
<td>0.034</td>
<td>Shimer (2005, 2007)</td>
</tr>
<tr>
<td>( b )</td>
<td>Unemployment benefits</td>
<td>1.35</td>
<td>40% of labor income</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Productivity persistence</td>
<td>0.983</td>
<td>Gertler and Trigari (2009)</td>
</tr>
<tr>
<td>( \sigma_z )</td>
<td>Productivity shocks</td>
<td>0.0065</td>
<td>Output volatility</td>
</tr>
<tr>
<td>( \xi )</td>
<td>Shape parameter</td>
<td>0.9</td>
<td>( \sigma_n/\sigma_y = 0.62 ), steady state relationship</td>
</tr>
<tr>
<td>( \kappa_{\text{max}} )</td>
<td>Scale parameter</td>
<td>6.5</td>
<td>( \sigma_n/\sigma_y = 0.62 ), steady state relationship</td>
</tr>
<tr>
<td>( c )</td>
<td>Flow vacancy cost</td>
<td>1.9</td>
<td>( cv/\dot{y} = 1.5% )</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Capital share</td>
<td>1/3</td>
<td>Standard</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Capital depreciation rate</td>
<td>0.0084</td>
<td>Quarterly rate 0.025</td>
</tr>
</tbody>
</table>

The sample draws from the NIPA and CPS tables. Because the calibration of labor market parameters relies heavily on Shimer’s (2005) estimation, I choose the same 1953Q1 – 2003Q4 sample as in his paper. I work with quarterly observations. If a variable is only available at monthly frequency, then I take the three-month average to obtain the quarterly data. All of the variables are log-detrended using the HP filter, with the standard smoothing parameter 1600.

There are eight variables in the sample, which are named after their model counterparts. \( y \) is the output of the non-farm business sector. Its standard deviation is 0.021. The other variables are defined as follows. \( \dot{u} \) and \( v \) are the civilian unemployment rate and job openings, respectively. \( \bar{lp} \) is the average labor productivity, i.e. \( y/n \) and \( \bar{w} \) is the average hourly earnings of production workers in the private sector.\(^{10}\) \( n \) is the civilian employment. Finally, \( i \) denotes the non-residential fixed investment and changes in private inventories and \( c \) denotes the non-durable goods consumption and service.

Table 3.2 lays out the business cycle properties of the above-mentioned variables. First, both the unemployment rate and job openings are highly volatile and persistent. The unemployment rate is highly countercyclical, while the job openings are highly procyclical. Second, Wage and the labor productivity move closely. Both are less volatile than the output and are moderately procyclical. Last, both employment and consumption are less volatile while investment is more volatile than output. All of them are highly persistent and procyclical.

\(^{10}\)The wage data begin from 1964 Q1, so I use the sample between 1964 Q1 and 2003 Q4.
3.3.3 Simulated Results

In this section, I evaluate how well the model can explain the patterns of Table 3.2. I simulate the model for 636 periods, take the three-period average and then detrend them using the HP filter with parameter 1600. So I have 212 observations, which mimic the 212 quarters in the sample. Table 3.3 reports the results. All of the numbers in the table are the averages over 1000 simulations.

<table>
<thead>
<tr>
<th></th>
<th>y</th>
<th>( \ddot{u} )</th>
<th>v</th>
<th>lp</th>
<th>w</th>
<th>n</th>
<th>h</th>
<th>i</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative standard deviation</td>
<td>1.00</td>
<td>5.87</td>
<td>6.61</td>
<td>0.63</td>
<td>0.52</td>
<td>0.62</td>
<td>4.1</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.83</td>
<td>0.86</td>
<td>0.90</td>
<td>0.76</td>
<td>0.90</td>
<td>0.89</td>
<td>0.85</td>
<td>0.86</td>
<td></td>
</tr>
<tr>
<td>Correlation with ( y )</td>
<td>1.00</td>
<td>-0.81</td>
<td>0.89</td>
<td>0.72</td>
<td>0.59</td>
<td>0.78</td>
<td>0.83</td>
<td>0.81</td>
<td></td>
</tr>
</tbody>
</table>

**Wage dynamics.** The wage channel is the key to understanding the effects of endogenous participation. The standard model fails because the wage is too elastic and hence the elasticity of the labor market tightness is small. As a result, the model can hardly generate the highly volatile unemployment rate and job openings. Section 3.2.6 shows that the model with endogenous participation has a potential to solve this problem. However, it all depends on how the model can explain the wage dynamics.

It turns out to be quite impressive. The model captures the wage dynamics almost as well as Gertler and Trigari’s (2009) model with staggered Nash wage bargaining. Notably, the relative volatility of wage in the model is 0.47, which is close to 0.52 in the data. It confirms the explanation in Section 3.2.3. Endogenous participation makes the expected outside option of workers countercyclical and hence makes wage less responsive to the productivity shocks.
Figure 3.1 shows the impulse responses of \( w \) and \( \tilde{b} \), where \( bt \) denotes \( \tilde{b} \). Not surprisingly, \( \tilde{b} \) is decreasing on impact of the aggregate productivity shocks. Hence, the wage only increases a little. This result matters much for the dynamics of other labor market variables, which I will show soon.

Wage and the labor productivity move closely in the data. The model here also captures this point. It generates a labor productivity that is moderately procyclical and less volatile than output.

\[
\begin{align*}
\text{Figure 3.1: The impulse responses of wage and the outside options of workers} \\
\text{Unemployment rate and job openings.} \quad \\
\end{align*}
\]

\textbf{Unemployment rate and job openings.} Table 3.2 shows that the relative standard deviations of unemployment rate and job openings are 5.87 and 6.6, respectively. The standard model explains less than 10\% of the volatility, according to Shimer (2005).

Consistent with its ability to capture the wage dynamics, Table 3.3 shows that my model performs much better in this respect. It explains 62\% of the job openings volatility and 45\% of the unemployment volatility. It also generates a countercyclical unemployment volatility, which is notable, because the previous models with endogenous participation, including the models of Tripier (2003) and Veracierto (2008), typically generate a procyclical unemployment rate.

However, the model does have a problem to capture the persistency of labor market variables. The autocorrelation coefficients of the unemployment rate, job openings, wage and
the labor productivity are all smaller than their empirical counterparts. This is mainly due to
the assumption of i.i.d. idiosyncratic productivity. Introducing more persistent idiosyncratic
productivity will amend this problem. However, it makes the model complicated without
making any further point. Thus, I keep the original assumption, which makes the mechanism
of the model more explicit and intuitive.

In a word, the presence of endogenous participation significantly enhances the perfor-
mance of search model in explaining the labor market fluctuations.

**Employment and nonparticipation rate.** The model is calibrated to match the
employment volatility. Table 3.3 shows that it captures the persistency quite well, too.
The model also generates a highly volatile and countercyclical nonparticipation rate, which
is different from the data if we look at the gross nonparticipation rate. However, among
the nonparticipated workers, which amounts to nearly 35% of the population, only a small
proportion of them are active at the margin and hence are relevant for participation choice.
The transition rate confirms this assertion. As mentioned in the introduction of this paper,
the transition rate from the overall nonparticipation force to labor force is only around 10%,
while the transition rate from the marginally attached workers alone is 40 – 60%. Hence,
it makes more sense to take the majority of nonparticipation workers in the model as those
"marginally attached workers".

BLS starts collecting data on this group from 1994, the sample shows that the mar-
ginally attachment rate is almost as volatile as the unemployment rate and is also highly
countercyclical. The relative standard deviation is 6.01, the autocorrelation is 0.71 and the
correlation with output is −0.72. So if we look at the marginally attached workers, then the
model performs quite satisfactorily in the respect of nonparticipation rate.

### 3.4 Conclusion

This paper shows that the presence of endogenous labor market participation choice greatly
enhances the performance of the search theoretic model in explaining the unemployment
fluctuations. It also shows that the wage channel is the key to understanding how it happens.
Endogenous participation makes the expected outside options of workers countercyclical.
Under Nash bargaining, it induces a countercyclical component in the wage equation, which
makes the elasticity of wage with respect to productivity smaller. As a result, the incentives of job openings are enhanced in booms and hence the model can generate larger unemployment fluctuations, compared to the textbook model.
3.A Proofs

**Proof of Proposition 3.1** It is obvious that there exists a unique cut-off value $\bar{x}_t$ in each period $t$, such that $H(\bar{x}_t) = U_t$ and $H(x) \gtrless U_t$ if and only if $x \gtrless \bar{x}_t$, since equation (3.2.1) tells that $H(x)$ is monotonically increasing in $x$ and equation (3.2.2) tells that $U_t$ does not depend on the current realization of $x$. It’s easy to verify the assertion. Evaluating equation (3.2.1) at $\bar{x}_t$ gives

$$
\beta E_t \int \max [U_{t+1}, H_{t+1}(x')] dG(x') = U_t - \bar{x}_t.
$$

(3.A.1)

Plugging it back into equation (3.2.1) yields

$$
H_t(x) = U_t + x - \bar{x}_t.
$$

(3.A.2)

Thus, if $x = \bar{x}_t$, then $H_t(x) = U_t$ and $H(x) \gtrless U_t$ if and only if $x \gtrless \bar{x}_t$.

Thus, nonemployed workers follow a cut-off strategy. At the beginning of each period $t$, a nonemployed worker chooses to enter or stay in the labor force if her idiosyncratic productivity $x \leq \bar{x}_t$. Otherwise, she chooses to stay outside of the labor force. As a result, the group of unemployed workers evolves as

$$
u_{t+1} = \mathcal{G}(\bar{x}_{t+1}) [(1 - p_t^{\nu})u_t + sn_t + h_t],
$$

(3.A.3)

and the group of house workers evolves as

$$
h_{t+1} = [1 - \mathcal{G}(\bar{x}_{t+1})] [(1 - p_t^{\nu})u_t + sn_t + h_t],
$$

(3.A.4)

where $(1 - p_t^{\nu})u_t + sn_t + h_t$ denotes the group of nonemployed workers in the beginning of period $t + 1$, before the idiosyncratic productivity realizes.

The cut-off strategy also implies that

$$
E_t \int \max [U_{t+1}, H_{t+1}(x')] dG(x') = E_t \left[ \int_{0}^{x_{t+1}} U_{t+1} dG(x') + \int_{x_{t+1}}^{x_{\max}} H_{t+1}(x') dG(x') \right].
$$

(3.A.5)
Replacing $H_{t+1}(x')$ by equation (3.A.2) gives

$$E_t \int \max [U_{t+1}, H_{t+1}(x')] \, dG(x') = E_t U_{t+1} + E_t \tilde{x}_{t+1}. \quad (3.A.6)$$

Because $\tilde{x}_{t+1} \geq 0$, equation (3.A.6) implies that the value of being unemployed is larger than that in the standard model. This is an intuitive result. The presence of home production makes the expected outside options of workers larger than the unemployment benefits $b$. I will show it in detail later.

By definition, $S_t = J_t + W_t - U_t$. Substituting $J_t$ and $W_t$ by equations (3.2.8) and (3.2.3), respectively, and using equation (3.A.6) to replace the integral part gives

$$S_t = z_t - (U_t - \beta E_t U_{t+1}) + \beta s E_t \tilde{x}_{t+1} + \beta (1 - s) E_t S_{t+1}. \quad (3.A.7)$$

Then combining equations (3.A.6), (3.2.2) and (3.2.7) yields

$$U_t - \beta E_t U_{t+1} = b + \frac{\eta}{1 - \eta} \theta_t + \beta (1 - p_t^n) E_t \tilde{x}_{t+1}, \quad (3.A.8)$$

where I use the fact that $W_{t+1} - U_{t+1} = \eta J_{t+1}$. Replacing $(U_t - \beta E_t U_{t+1})$ in equation (3.A.7) by equation (3.A.8) gives equation (3.2.12).

**Proof of Proposition 3.2** Replacing $J_t$ and $J_{t+1}$ in equation (3.2.8) by the Nash bargaining solutions $J_t = (1 - \eta) S_t$ and $J_{t+1} = (1 - \eta) S_{t+1}$, respectively, and re-arranging terms gives

$$z_t - w_t = (1 - \eta) [S_t - \beta (1 - s) E_t S_{t+1}] \quad (3.A.9)$$

Then, combining equations (3.2.12) and (3.A.9) gives equation (3.2.14).
Bibliography and References


