Development of a Risk-based Decision Making Framework
for Hydrosystems Engineering Project Design

by

Hsin-Ting, SU

A Thesis Submitted to
The Hong Kong University of Science and Technology
in Partial Fulfillment of the Requirements for
the Degree of Doctor of Philosophy
in Civil Engineering

February, 2013, Hong Kong
Authorization

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Hsin-Ting, SU

February, 2013
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Hsin-Ting, SU

This is to certify that I have examined the above PhD thesis
and have found that it is complete and satisfactory in all respects,
and that any and all revisions required by
the thesis examination committee have been made.

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Supervisor: Prof. Yeou-Koung TUNG

[Signature]
Department Head: Prof. Christopher K.Y. LEUNG

Department of Civil and Environmental Engineering
8, February, 2013
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Development of a Risk-based Decision Making Framework for Hydrosystems Engineering Project Design

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Department of Civil and Environmental Engineering
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Abstract

Problems in hydrosystems engineering management and design face two main challenges: (1) the presence of uncertainties and (2) multiple criteria/objectives in nature. Uncertainties in project design, planning, and operation induce failures rendering undesirable consequences. Involvement of several non-commensurable and often conflicting criteria creates difficulty in reaching an optimal decision. This dissertation addresses these two challenges by proposing a decision making framework according to a new decision rule based on the concept of expected opportunity loss (EOL) that can utilize pertinent uncertainty information of outcomes of alternatives to assist decision making in a highly variable environment with multiple criteria.

The EOL-based decision rule quantifies the potential risk when the chosen design/decision is incorrect. It possesses several important features that are relevant for water resources management and hydrosystem design problems: (1) circumvents the limitations of existing risk-based decision rules of using incomplete uncertain information; (2) accounts for the inter-correlations among the uncertain outcomes of different design alternatives through the mechanism of pair-wise comparison; and (3) allows feasibility test of a design alternative
through comparison with decision maker’s acceptable risk. By coupling with the
minimax rule, the new EOL-based risk measure expands the applicability of
Savage’s minimax regret principle to decision making in problems involving
continuous and unbounded random outcomes. Examples of evaluating flood
inundation reduction projects are used to demonstrate the application of the
proposed decision framework. The effect of the uncertainty level, correlation
between random outcomes of alternatives, epistemic uncertainty due to sampling
error, and decision maker’s acceptable risk are investigated. Numerical results
clearly show that the uncertainty level and correlation associated with the
economic merit of hydrosystem project designs are very significant that should
be not ignored.

In addition, a multi-criteria decision making (MCDM) framework is
developed that explicitly considers the uncertainty features of the performance
values of management alternatives under different conflicting criteria. The
PROMETHEE (Preference Ranking Organization Method of Enrichment
Evaluation) technique is adopted for its compatibility in decision logic and
mathematical treatment with the EOL-based decision criterion involving
pair-wise comparison. Numerical applications demonstrate that the EOL-based
risk criterion can be conveniently incorporated in the framework of the
PROMETHEE for dealing with MCDM problems under uncertainty. It opens up
a new prospect to expand the prevalent practice of deterministic MCDM to
probabilistic MCDM so that the uncertainty features of the performance values
under different criteria are incorporated and utilized in evaluating decision
alternatives in a multi-criteria problem.
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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$ATEC$</td>
<td>Annual total expected cost</td>
</tr>
<tr>
<td>$a_m, b_m, c_m$</td>
<td>Values of model coefficients in Eqs.(3-23a) and (3-23b)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Significant level</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Scale parameter of PDF</td>
</tr>
<tr>
<td>$\hat{\beta}$</td>
<td>Estimated scale parameter of PDF</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Location parameter of PDF</td>
</tr>
<tr>
<td>$\hat{\xi}$</td>
<td>Estimated location parameter of PDF</td>
</tr>
<tr>
<td>$A_i$</td>
<td>$i$-th alternative</td>
</tr>
<tr>
<td>$AR$</td>
<td>Acceptable risk</td>
</tr>
<tr>
<td>$B_{IR}$</td>
<td>Annual inundation-reduction benefit</td>
</tr>
<tr>
<td>$B_{IR,A_i}$</td>
<td>Annual inundation-reduction benefit associated with alternative $A_i$</td>
</tr>
<tr>
<td>$C$</td>
<td>Annual cost</td>
</tr>
<tr>
<td>$C_j$</td>
<td>$j$-th criterion</td>
</tr>
<tr>
<td>$Cov$</td>
<td>Covariance</td>
</tr>
<tr>
<td>$c.o.v$</td>
<td>Coefficient of variation</td>
</tr>
<tr>
<td>$CRF$</td>
<td>Capital recovery factor</td>
</tr>
<tr>
<td>$D$</td>
<td>Damage</td>
</tr>
<tr>
<td>$D_{w/o}(q)$</td>
<td>Flood damage under the condition of “without” project alternative-$A_i$ under a given discharge $q$.</td>
</tr>
<tr>
<td>$D_{w/A_i}(q)$</td>
<td>Flood damage under the condition of “with” project alternative-$A_i$ under a given discharge $q$.</td>
</tr>
</tbody>
</table>
Flood damage under the conditions of “without” and “with” project alternative-$A_i$ under a given discharge $q$

$D^m$ $m^{th}$-order derivatives

$d_k$ Deviation between the performance values of two alternatives for criterion $C_k$

$\Delta_{i*j}$ The outcome difference for choosing alternatives $A_i$ in comparison with alternative $A_j$

$E(*)$ Expectation

$EOL(A_i^*,A_j)$ The $EOL$ for the chosen alternative $A_i$ with reference to $A_j$.

$EOL_m(A_i^*,A_j)$ The $EOL$ associated with the $m^{th}$-order loss function for the chosen alternative $A_i$ with reference to $A_j$.

$EOG(A_i^*,A_j)$ The $EOG$ for the chosen alternative $A_i$ with reference to $A_j$.

$F$ Non-exceedance Probability

$FC$ First cost or total installation cost

$FX$ CDF of $X$

$f_X$ PDF of $X$

$f_{i,j}(•)$ Joint PDF of two random variables $X_i$ and $X_j$

$H_m$ Chebyshev-Hermite polynomials

$h$ Target level of return

$h_{i,j}$ The constraints representing the design specifications that must be satisfied

$K$ Number of criteria

$L$ Loss function

$LB$ Lower bound

$LPM_m$ The $m^{th}$-order lower partial moment

$M$ Partial moment generating function

$m_1'$ Sample mean

$m_2$ Sample variance

$\mu$ Mean

$NB$ Annual net benefit

$N$ Number of alternatives

$n$ Sample size
\( \Omega \) Coefficient of variation

\( p \) Indifference threshold of the reference function

\( P_r \) Probability

\( P_k \) Preference function of

\( \Pi(A_i) \) Net outranking index (net flow) of alternative \( A_i \)

\( \pi^+(A_i) \) Positive outranking index of alternative \( A_i \)

\( \pi^-(A_i) \) Negative outranking index of alternative \( A_i \)

\( \phi(*) \) PDF of the standard normal distribution

\( \Phi(*) \) CDF of the standard normal distribution

\( Q \) Flood magnitude

\( q \) Preference threshold of the reference function

\( q_{c:(A_i)} \) Flow carrying capacity of the system associated with the design alternative \( A_i \)

\( R_A \) Arrow-Pratt coefficient of absolute risk aversion

\( R_{\mu} \) Mean ratio of \( X_i \) and \( X_j \)

\( R_{\sigma} \) Standard deviation ratio of \( X_i \) and \( X_j \)

\( r_{ik} \) Regret associated with a chosen alternative \( A_i \) under scenario \( S_k \)

\( \rho \) Correlation coefficient

\( s \) Intermediate value between \( p \) and \( q \) parameter of the reference function

\( s_e \) Sampling error

\( \sigma \) Standard deviation

\( T \) Return period

\( UB \) Upper bound

\( u \) Utility

\( X, R, \) Random variables

\( x, r \) Realization of \( X \) and \( R \), respectively

\( \hat{X}_T \) quantile estimator of the \( T \)-yr event

\( Y \) Reduced variable to return period \( T, Y = -\ln(-\ln(1-1/T)) \)

\( z \) Standardized normal random variable

\( I \) Preference between two alternatives based on PROMETHEE I method
Indifference between two alternatives based on PROMETHEE I method
Incomparability between two alternatives based on PROMETHEE I method
Preference between two alternatives based on PROMETHEE II method
Indifference between two alternatives based on PROMETHEE II method
Preference between two alternatives based on PROMETHEE II method under uncertainty
Indifference between two alternatives based on PROMETHEE II method under uncertainty

The following abbreviations are used in this thesis:

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>AHP</td>
<td>Analytic Hierarchy Process</td>
</tr>
<tr>
<td>ANP</td>
<td>Analytic Network Process</td>
</tr>
<tr>
<td>ATEC</td>
<td>Annual Total Expected Cost</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Density Function</td>
</tr>
<tr>
<td>CE</td>
<td>Certainty Equivalent</td>
</tr>
<tr>
<td>CP</td>
<td>Compromise Programming</td>
</tr>
<tr>
<td>CRF</td>
<td>Capital Recovery Factor</td>
</tr>
<tr>
<td>CRM</td>
<td>Conditional Risk Measure</td>
</tr>
<tr>
<td>DEA</td>
<td>Data Envelopment Analysis</td>
</tr>
<tr>
<td>DSS</td>
<td>Decision Support System</td>
</tr>
<tr>
<td>Acronym</td>
<td>Full Form</td>
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<tr>
<td>ELECTRE</td>
<td>Elimination Et Choix Traduisant La Réalité</td>
</tr>
<tr>
<td>EMV</td>
<td>Expected Monetary Value</td>
</tr>
<tr>
<td>EOL</td>
<td>Expected Opportunity Loss</td>
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<td>EOG</td>
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<td>Expected Utility</td>
</tr>
<tr>
<td>FC</td>
<td>First Cost</td>
</tr>
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<tr>
<td>HARA</td>
<td>Hyperbolic Absolute Risk Aversion</td>
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<tr>
<td>MADM</td>
<td>Multi-Attribute Decision Making</td>
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<tr>
<td>MCDM</td>
<td>Multi-Criteria Decision Making</td>
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<tr>
<td>MGF</td>
<td>Moment generating function</td>
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<td>MML</td>
<td>The Method of Maximum Likelihood</td>
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<td>MoM</td>
<td>The Method of Moments</td>
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<td>M-V rule</td>
<td>Mean-Variance Rule</td>
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<td>PDF</td>
<td>Probability Density Function</td>
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<td>POL</td>
<td>Probability-of-loss</td>
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<tr>
<td>PROMETHEE</td>
<td>Preference Ranking Organization Method Of Enrichment Evaluation</td>
</tr>
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<td>PWM</td>
<td>The Probability Weighted Moment Method</td>
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<td>RP</td>
<td>Risk Premium</td>
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<td>RMCDM</td>
<td>Risk-Based Multi-Criteria Decision Making</td>
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<td>SMART</td>
<td>Simple Multi-Attribute Ranking Technique</td>
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<tr>
<td>SD</td>
<td>Stochastic Dominance</td>
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<tr>
<td>FSD</td>
<td>First-Degree Stochastic Dominance</td>
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<tr>
<td>Acronym</td>
<td>Description</td>
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<tr>
<td>SSD</td>
<td>Second-Degree Stochastic Dominance</td>
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<tr>
<td>TSD</td>
<td>Third-Degree Stochastic Dominance</td>
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<tr>
<td>TOPSIS</td>
<td>Technique For The Order Of Preference By Similarity To Ideal Solution</td>
</tr>
<tr>
<td>XRM</td>
<td>Xu’s Risk Measure</td>
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CHAPTER 1
INTRODUCTION

1.1 Background

Like all engineering projects, hydrosystems engineering infrastructures are designed to achieve the desired level of safety and performance, while meeting national or regional economic development objectives. For example, flood protection structures (e.g. dams, levees, and storm drains) are designed to protect a region from the adverse effects of floods, including the avoidance of loss of human life and the reduction of flood damage. However, all flood protection systems could fail to perform up to expectations due to uncontrollable natural variations of flood magnitude, human error, or design deficiency. Continuing future development in the protected area could also increase flood damage. Regardless of all the prudence in the design and analysis, systems can still possibly fail to meet the intended performance.

Failures of these hydro-infrastructural systems result in undesirable consequences, such as economic losses, social disruption, environmental/ecological destruction, or even human fatalities. The cumulative damage of many moderate and less intense flood incidents that have occurred all around the world is as severe as some catastrophic flood events. Berz [1] showed that from 1978 to 1997, floods were to blame for about half of the fatalities and contributed to one-third of the global economic losses from all forms of natural hazards.
In this chapter, the issues that are related to risk-based decision making for hydro-system design are discussed. A state-of-the-art analysis on the progress of hydro-system design considering uncertainties is given. The challenges at the present stage of hydro-systems engineering project design indicate that there is a gap between conventional optimal design approaches and a more risk-informed decision framework that considers the uncertainties on the performance of each design alternative and the multiple conflicting evaluation criteria. A general background on decision analysis is provided. The concepts of decision making incorporating risk and multi-criteria decision making (MCDM) provide a viable way to deal with the challenges presently faced by engineers and decision makers. The general background on these issues and concepts is provided, the thesis scope and objectives are defined, and the outline of the thesis is presented.

1.2 Challenges in Hydro-systems Engineering Project Design

Water resources planning is a complex decision-making problem greatly affecting the public, and is an integral part of water resources engineering, analysis and management. Keeney and Wood [2] pointed out the difficulties involved in water resources planning and management decision making, which include: (1) the uncertainties about the performance of a specific alternative; (2) joint consideration of multiple conflicting factors of economic, environmental, and social performances; and (3) the performances may not be quantified or are difficult-to-be-quantified as a criterion for selecting an alternative.

The uncertainties on the performance of a specific project (design) alternative play an important role in water resources engineering and management decision making. Like most real-life engineering problems, the design, management, and operation of hydro-system infrastructure, including flood-damage-reduction systems, are subject to uncertainties. The uncertainties in water resources decision making problems exist in many aspects including, but not limited to, hydrology, hydraulics, structural, economic, and social aspects. The uncertainties of natural phenomena, human error, or knowledge deficiency can result in over- or under-design that causes undesirable consequences. Ignoring or underestimating uncertainties may hinder rational management decision making and the engineering design.
Tung [3] has shown that the annual expected flood damage in a levee design can be significantly underestimated if hydrological parameter uncertainties are ignored, even with a 75-yr long flood record. Although incorporating uncertainties into decision making process is challenging, the importance of the application of risk-based design in hydro-systems management and engineering is advocated [4-12].

Most real-world decision-making problems are multi-criteria by nature. In addition to the economic efficiency aspect, there are generally several multiple factors involving economic, safety, environmental, social, and operational considerations in the design of hydrosystem infrastructures. These criteria, either quantitative or qualitative (intangible), are non-commensurable and are often in conflict with each other. In such cases, different designs or project alternatives are evaluated according to a set of criteria in the MCDM problem. Furthermore, owing to the existence of uncertainties, it is important to develop a practical MCDM approach, in conjunction with the risk-based procedure, to arrive at a comprehensive, logically convincing and legally defensible design.

1.3 Approaches in Hydro-infrastructural Systems Designs Considering Uncertainty

The evolution of present design approaches in hydrosystem infrastructures can be divided into four stages in regard to the way that uncertainties in the design are handled. They are (1) the historical event-based approach, (2) the frequency-based design approach, (3) the conventional risk-based approach, and (4) the optimal risk-based approach [13].

1.3.1 Historical Event-Based Approach

To avoid damage from previous extreme natural hazards, the historical event-based approach, the earliest stage of hydrosystem design method for flood protection, sets the largest flood in the historical record as the design protection level. However, this method is not adequate considering the varying decision environment. In addition, the available historical record may not contain sufficient data on floods and other natural hazard events for a prudent engineering design.
1.3.2 Frequency-Based Approach

The stopgap method of the historical event-based approach has been replaced by the prevailing design frequency approach since the early part of the twentieth century. The frequency-magnitude information is provided by frequency analysis of the observed events. The design protection level (in terms of design frequency) is a pre-selected design parameter which is “supposed to” represent a societally acceptable hazard frequency according to the frequency-magnitude information. However, the selection of the design return period could be questioned and challenged due to the subjective nature of the process. A crucial disadvantage of the design frequency approach is that there is no explicit and direct connection with the failure consequences and frequency. Water resources managers are not able to evaluate the potential losses of extreme events or prepare sufficient budgetary reserves for contingencies to cope with the failures by the design frequency approach.

1.3.3 Conventional Risk-Based Approach

One step further in hydrosystem design is the risk-based approach that considers the trade-offs between investment cost, reliability, and the expected economic losses due to failure. Hydrosystems are constructed to meet the national or regional economic development objectives along with other environmental and social objectives. Cost-benefit analysis is generally used to compare and rank the economic desirability of the project outcomes, including the economic returns. Cost-benefit analysis is stipulated in the U.S. Flood Control Act of 1936 and the Principles and Guidelines issued by the U.S. Water Resources Council in 1983 [14, 15]. This principle has been the impetus for risk-based design in water resources management. In principle, risk can be measured as the failure probability accounting for various uncertainties [13, 16]. A more widely used definition of “risk” accounts for the potential undesirable consequences in conjunction with the likelihood of failure.

Figure 1.1 shows the variation of the annual investment cost and annual expected damage cost with project size. It can be seen that the annual investment cost increases with the project size while, at the same time, the corresponding annual expected damage cost due to the failure of the system decreases. The annual total expected cost (ATEC) is the sum of the
annual installation cost and the annual expected damage cost due to system failure which can be expressed as

\[
ATEC(A_i) = FC(A_i) \times CRF + E(D | A_i)
\]  

(1-1)

where \( FC(A_i) \) is the first cost or total installation cost (consisting costs of capital, operation and maintenance) associated with the project size \( A_i (i = 1, 2, \ldots, N) \) among a set of \( N \) alternatives defined by the size and configuration of the hydrosystem; \( CRF \) is the capital recovery factor that transfers the present worth of cost components in \( FC \) to an annual basis; and \( E(D | A_i) \) is the annual expected damage cost associated with alternative \( A_i \) in terms of the design return period which can be estimated by risk analysis.

**Figure 1.1 Variation of different cost components with project size in a risk-based hydrosystem design.**
The thrust of the exercise in risk-based designs, after performing the uncertainty and risk analyses, is to evaluate the annual expected damage cost. The annual expected damage cost can be determined by integrating the probability density functions of loading and resistance, damage function, and the types of uncertainty considered. To estimate the expected inundation damage with and without a hydrosystem, the relationships between discharge-frequency, stage-discharge, and damage-stage should be defined to determine the frequency-damage function, as shown in Figure 1.2. The discharge-frequency function is defined depending on the available historical flood discharge data for the frequency analysis. The stage-discharge function, also known as the rating curve, can be determined through field measurements or by a hydraulic modeling. The stage-damage function is usually based on surveys of the property values on the study area. The frequency-damage function can be derived by conjoining these relationships. For the various hydrosystem design alternatives, the annual expected values of the flood damage, \( E[D|A_i] \), which corresponds to the area under the frequency-damage curve in Figure 1.2, can be determined by

\[
E[D|A_i] = \int_{q_{c(A_i)}}^{\infty} D(q) f_0(q) dq
\]

where \( D(q) \) is the inundation damage conditioned on the flood magnitude \( q \); \( f_0(q) dq \) is the occurrence probability; and \( q_{c(A_i)} \) is the flow carrying capacity of the hydrosystem associated with the design alternative \( A_i \).

Note that Eq.(1-2) considers only inherent (or aleatory) uncertainty of the natural randomness of floods or hydrologic extremes in the evaluation of annual expected damage. In reality, uncertainties can exist in the discharge-frequency, stage-discharge, and damage-stage relationships as shown in Figure 1.3. For the discharge-frequency function, there is the uncertainty of the distribution parameters due to sampling error resulting from the lack of perfect knowledge. The uncertainty of the rating relationship comes from the hydraulic uncertainties associated with the factors affecting the stage-discharge relationship, such as the variability of bed forms, hydraulic roughness, and channel scour or deposition, unsteady flow effects, changes in channel shapes, and sediment transport. The damage cost function with good quality is also scarce. The uncertainty of a damage-stage function results from the errors
in identifying and classifying the buildings and structures, errors in investigating the first-floor elevation of structures, errors in estimating the replacement value of structures and contents, and errors in post-flood damage survey [17]. However, information on the damage function that is essential for performing the risk-based approach may not be easily obtainable.

Figure 1.2 Conjoined stage-discharge-frequency-damage relationships in conventional expected annual damage computation
1.3.4 Optimal Risk-Based Approach

In the current practice of the risk-based design of hydrosystems, the optimal project alternative having the minimum $ATEC$ is determined by considering the tradeoffs between the project investment cost and the expected damage cost due to system failure. The framework is conceptually an advancement of today’s prevailing design frequency approach by which the return period is a pre-selected design parameter based on subjective judgment without giving explicit account for failure consequences. In the risk-based approach, the design return period or frequency, along with the associated system configuration, are decision variables.
Referring to Figure 1.1, the annual installation cost increases with project size, and the annual expected damage cost due to system failure decreases simultaneously. The optimum design alternative can be obtained by solving the following model

\[
\begin{align*}
\text{Minimize } & \quad TAE(C(A)) \\
\text{Subject to } & \quad h_j(A_j) = 0, j = 1, 2, ..., k
\end{align*}
\]

where \( h_j(A_j) = 0, j = 1, 2, ..., k \) are constraints representing the design specifications that must be satisfied. The optimum solution can be found by using appropriate optimization algorithms [3].

The concept of risk that considers hydrologic inherent uncertainty has long been applied to the design and planning of various hydrosystems infrastructures, such as roadway crossings (e.g. [18-20]), bridges [21], and dams (e.g. [22-25]). In addition to the randomness of the hydrologic processes, hydraulic, parameter/model or economic uncertainties have been incorporated in the evaluation of the expected damage in the design of various hydrosystem infrastructures. Risk-based designs considering both hydrologic and hydraulic uncertainties in project alternatives evaluation can be found for highway drainage structures [26-29]; storm sewer systems [30]; levee systems [31]; riprap for channel stabilization [32]; and river diversions [33]. Others consider hydrologic uncertainty and parameter/model uncertainties simultaneously in levee systems design [34-37]. Furthermore, consideration of economic uncertainty, hydrologic and hydraulic uncertainties in flood-damage-reduction projects can be found elsewhere [38]. Recently, this concept was also applied to determine the optimal safety level of sea dike systems for South Holland [39]; Japan [40]; and Vietnam [41].

1.3.5 Uncertainties of Failure Damage

It is important to recognize that the damage from the failure of a hydrosystem is in fact a random variable because it is a function of the hydrologic, hydraulic, and economic uncertainties that affect the load and capacity of the hydrosystem. Figure 1.1 shows the distributions of the uncertain annual total costs associated with two different project sizes (or levels of protection). Due to hydrologic, hydraulic, and economic uncertainties, the damage
cost of hydrosystem failure is also subject to uncertainty. Most risk-based approaches in hydrosystem designs mainly consider the mean values of the project costs and benefits without giving an explicit account of their probability distributions resulting from the random factors involved in the flood magnitude and/or rainfall amount. Few studies have made a comparison of these ranking methods that consider outcome uncertainty beyond the expected value in the application of water resources planning [9, 12].

When more information about the uncertainty features of the project costs and benefits is available, the decision maker may not be inclined to make the choice solely on the basis of the highest expected net benefit or lowest total expected cost. For instance, between the two competing alternatives for different project sizes \(A_1, A_2\) as shown in Figure 1.1, the lower annual total expected cost of \(A_1\) is accompanied with a higher uncertainty. Because of this higher uncertainty, the project with the lowest annual total expected cost will have a higher likelihood of incurring higher cost and hence it may not necessarily be the optimum choice for a risk-averse decision maker. In such circumstances, the choice of an optimal project alternative can no longer be made in a straightforward manner because the trade-offs between the lower expected annual total cost and its higher uncertainty have to be established. This gives rise to the problem of decision making under risk which can be viewed as a choice between alternatives of different probability distributions for the total annual cost or benefit. In general, the uncertainties of the annual total costs associated with different project sizes are influenced by the variance of the annual damage costs, which depend on the form of the damage function and the frequency-damage relationships of the system.

1.4 Decision Analysis

The field of decision analysis was defined by Howard in 1966 [42] as an integrated discipline in investigating the theory, procedures, and methods for describing a problem formally and in helping the decision makers to find the best course of action.

1.4.1 Elements in a Decision Analysis

Miser and Quade [43] state that decision making problems have five elements, objectives,
alternatives, outcomes, decision rules and the model.

- **Decision objectives** – are what the decision makers desire to achieve.

- **Decision alternative** – is a course of action to achieve the objectives which can have various forms (e.g., information-collection, time-buying, hedging, risk-sharing, and insurance alternative).

- **Alternative outcome (or consequence, performance)** – is the result ensuing from the execution of the alternatives. In the realm of economic analysis, the outcome can be regarded as project returns, benefits, or costs. The state of nature dictates the condition that may occur in the future of which the decision maker has little or no control (e.g. rainfall, discharge, or development scenarios). Due to numerous unforeseeable changes of the state of nature or future development, each alternative outcome (e.g., project net benefit or life-cycle cost of a system) can be treated as random variables with known probabilistic features [44-46].

- **Decision rule** – an instruction to compare and rank alternatives with respect to the desirability of outcomes or criteria for MCDM. The criteria reflect the characteristics of the decision goals, and are used to evaluate the achievement for the decision goals. For sustainable water resource planning and management, safety, economic, environmental, social, and technical criteria are often considered concurrently.

- **Model** – an abstraction of the real-world by considering the factors relevant to the problem for investigating the behavior or response of a system. It evaluates the effect of alternatives on system performance under different scenarios. Based on the model outcomes (e.g., total flood damage or benefit), decision makers determine a preferred alternative for implementation. In terms of economic cost-benefit analysis, the preferred alternative may be the one with the largest project benefit or the least life-cycle cost of a system.

### 1.4.2 Types of Decision Analysis

Decision problems in general can be grouped into two categories according to the different
nature of the decision analysis.

- **Decision making under certainty, risk, and uncertainty** – According to the amount of information available to describe the system, the decision problem can be classified into decision making under certainty, decision making under risk, and decision making under uncertainty [47]. Decision making under certainty refers to a decision that is made in the condition that the state of nature (future state) is known by the decision maker. Each alternative can have only one possible outcome. Due to the reality of the lack of certainty or inadequate information, information is usually imprecise or absent. The natural processes and the occurrence of nature hazards are obviously stochastic. Decision making under risk refers to a decision that is made in the condition in each alternative could have different possible outcomes and the probability is known by the decision maker. When a decision maker is faced with the choice among various alternatives with uncertain outcomes, it is difficult to ensure that the choice is really the best one. No matter which alternative is chosen, risk exists along with the choice. Decision making under uncertainty refers to the decision that is made in the condition that the alternative outcome is uncertain and the probability is not known by the decision maker.

- **Mono-criterion and multi-criteria decision making** – According to the number of criteria considered in evaluating the alternatives, the decision problem can be classified into mono-criterion decision making and multi-criteria decision making. Mono-criterion decision making has only one objective/criterion, and MCDM has several objectives that have to be considered simultaneously. Most real-world decision problems are MCDM. Coupled with the consideration of uncertainties about the performance under each criterion, most hydrosystem design problems, in fact, are in the realm of MCDM under uncertainty. Figure 1.4 illustrates three aspects of this category of decision problem.
1.5 Objectives and Scope of the Research

The need to tackle the aforementioned challenges (Section 1.2) in hydrosystems engineering project design and management motivates this study in considering the uncertainty of the consequences and intangible factors into a multi-criteria hydrosystem design and management framework. The proposed framework provides a more practical methodology to arrive at a comprehensive, logically convincing and legally defensible design for hydro-infrastructural systems. Following Knight’s [47] definition of decision making under risk, this study focuses on “risk-based decision making” and “risk-based multi-criteria decision making” by further developing a more risk-informed decision making process. The goal of risk-based decision making is to search for the most desirable choice by considering the frequency and the seriousness of any decision error [48]. Considering the fact that, most managers for public-funded projects, such as hydrosystems infrastructures, are conservative in making a decision, in this thesis the assumption is made that the decision makers are risk averse.

The objectives of this study include:
1. **Developing a new risk-based framework for hydrosystem design** –

The framework of risk-based hydrosystem infrastructure design offers water engineers and managers an approach to improve the quality of a rational decision when the consequences of alternatives are uncertain. This study develops a framework that considers the adverse consequences and risks of implementing a course of action in the decision making process.

2. **Proposing new risk-based decision rule for decision making** –

In constructing such a risk-based design framework, a risk measure for project alternatives evaluation and selection needs to be defined. The quantitative risk measure should be able to provide quantifiable information of the associated inherent uncertainties and can be used to identify critical scenarios for risk management. It will be formulated to facilitate communication among analysts, decision makers, and stakeholders in different groups, to integrate diverse disciplines, and as a result to lead to a risk-informed decision making.

3. **Investigating basic properties of the proposed risk-based decision rule** –

Two existing risk measures developed earlier: Xu’s risk measure ($XRM$) [49] and the conditional risk measure ($CRM$) [50] will be demonstrated and compared with the proposed risk measure.

4. **Investigating the effects of the outcome correlation and the decision maker’s acceptable risk on project ranking** –

In hydrosystems engineering and management, the outcomes of competing alternatives are correlated because the outcomes of various competing alternatives are dependent on some common factors or attributes such as hydrologic and hydraulic parameters. Moreover, in a risk-based design, in addition to quantitative measures of reliability and risk cost, consideration of societal acceptable risk should be included if possible. The acceptable risk is defined as the lower limit of risk that is considered to be tolerable. A feasible and implementable alternative should have the
expected loss that is acceptable to the decision maker. The effects of the outcome correlation and the decision maker’s acceptable risk on the ranking results of a set of project alternatives will be investigated. The acceptable risk of decision makers can be assessed by their feedback to a questionnaire about the tolerable risk threshold according to decision makers’ experiences and opinions [49]. The acceptable risk is highly subjective and its determination is very complicated. Hence, the determination of acceptable risk is beyond the scope of this study.

5.  

*Investigating the effects of epistemic uncertainty on project selection –*

Epistemic uncertainty due to sampling error is a result of the knowledge deficiency about the system. Sampling error exists when limited amounts of hydrologic data are used to establish magnitude-frequency relationships and to estimate a T-yr event quantile. Both the natural randomness of hydrologic data and the sampling error in design quantile estimation contribute to the uncertainty in flood damage estimation. This study will incorporate the inherent hydrologic uncertainties and epistemic uncertainties in the risk-based decision making framework for hydrosystem infrastructures project evaluation and selection.

6.  

*Developing framework for MCDM –*

Most public projects are designed to meet diverse objectives. A wide variety of MCDM techniques have been developed to help decision makers in solving the complex decision making situations of a multi-criteria nature. In general, the alternatives are described, evaluated, ranked and selected on the basis of deterministic evaluation of the consequences of each alternative under each criterion. However, various sources of uncertainties are present in the decision making process, including the external uncertainties (related to the imperfect information of the consequences of each alternative) and internal uncertainties (related to the subjective human judgment). In this study, the external uncertainties of the consequences for each alternative will be considered and incorporated into a risk-based MCDM framework.
As indicated in Sub-section 1.3.3, there are four types of uncertainty involved in the estimation of frequency-damage relationship. All of these uncertainties can influence the estimation of the project performance in achieving the decision objective of economic development. In this thesis, only the natural randomness of hydrologic extremes and sampling error associated with the frequency-magnitude relationship are considered.

1.6 Outline of the Thesis

There are six chapters in this thesis. In the first chapter, the challenges in hydrosystem engineering designs are presented and discussed to set the stage for the research. The need for a new risk-based decision making framework for hydrosystem engineering designs is addressed. The objectives and scope of this research are described.

Chapter 2 presents a detailed review of the literature related to the study. Conventional methods for decision making under uncertainty and risk are described. The needs for a new quantitative risk measure to account for full probability features, including the consideration of the correlation among the outcomes of alternatives, are addressed. The background and techniques of the MCDM are discussed.

Chapters 3~5 present three stages toward the objectives in constructing a new risk-based decision framework for hydrosystem project design. In Chapter 3, a new quantitative risk measure, the Expected Opportunity Loss (EOL), for project evaluation is proposed and incorporated into the decision rule for project ranking. The properties of the proposed risk measure are investigated and compared with various existing risk measures, along with their implications in risk-based decision making. The formulation of the quantitative risk measures that adopt different forms of loss function with respect of the decision maker’s risk attitude are derived. The effects of risk attitude on the alternative ranking result are investigated. The proposed decision rule is demonstrated through an application in river basin management for navigation improvement.
Chapter 4 presents a framework for risk-based hydrosystem project evaluation by considering both the hydrologic randomness and epistemic uncertainty. Different risk-based decision making rules are used to evaluate a project’s merit according to the statistical features of the project net benefits. The proposed decision process for risk-based hydrosystem design is applied to a flood-damage-reduction planning case study extracted from the literature. Through this example, investigation and discussion of the performance of the proposed risk-based design approach are made. The effects of epistemic uncertainty are incorporated into the same flood-damage-reduction planning case study. The influence of the hydrological record length on the value of potential risk and the result of project evaluation are investigated. The effects of the outcome correlation and the decision maker’s acceptable risk on the ranking results of a set of project alternatives are investigated in this case study.

In Chapter 5, the uncertainties of consequences for each alternative are considered and incorporated into the framework of MCDM to deal with the situation of risk-based decision making. The concept of the PROMETHEE (Preference Ranking Organization Method of Enrichment Evaluation) technique for MCDM is adopted for its transparent and logical decision procedure. The original deterministic framework of the PROMETHEE is extended to account for the probabilistic features of the consequences associated with an alternative. An example is given to illustrate the risk-based MCDM process for sustainable water resources planning. The influences of the uncertainties of the consequences on the ranking result are investigated. Sensitivity analysis is also conducted to investigate the influence of the outcome correlations and criteria weights on the final decision.

The key activities of this research are summarized in Chapter 6. Conclusions are drawn about the newly proposed risk-based hydrosystem design process and recommendations are given for future studies.
CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

Decision making is an integral part of hydrosystems engineering, analysis and management. There is growing attention for applying risk-based decision making and multi-criteria decision making in hydrosystems engineering and management. The decision process in choosing the best feasible alternative is plagued with uncertain outcomes, resulting from a multitude of uncertainties in a variety of sources. The complexity of the decision problem is also increased by the presence of different risk attitudes of decision makers and multiple conflicting decision criteria. This calls for a water resources planning framework that considers the uncertain future conditions and non-commensurable factors into the risk-based multi-criteria decision making process. To set the stage for discussion, the existing rules for decision making under risk / uncertainty and techniques for MCDM are reviewed in this chapter.

2.2 Procedures for Decision Making in Water Resources Planning

The basic steps in water resources planning processes have been well stipulated by the U.S. National Water Commission [51]. Decision making processes considering the existence of uncertainty in assessing the merit of management alternatives can be found in other sources [12, 43, 52]. A decision analysis considering the outcome uncertainty can provide decision makers with a comprehensive basis for selecting pro-active management alternatives. The
analysis ideally should include a process that is capable of considering the effects of outcome uncertainties explicitly and in providing sufficient information to aid decision making. Ignoring and underestimating the effects of uncertainty can hinder rational decision making.

The decision framework is schematically outlined in Figure 2.1. The dashed-line box contains six core steps in the decision process: (1) problem identification; (2) generation of alternatives; (3) prediction of project outcomes using information from uncertainty analysis; (4) ranking of alternatives by risk-based decision rules; (5) feasibility test of alternatives; and (6) implementation.

(1) **Problem definition** – Identifying the elements in a decision making problem is the first crucial step. Decision elements include external elements (environmental variations), constraints (cost- and time-limits, laws and regulations), related personnel (decision makers, stakeholders, executives), and other elements in the problem definition domain. Appropriate problem definition clarifies a complicated problem for further analysis.

(2) **Generation of possible alternatives** – Decision alternatives can be generated by group decision techniques such as brain-storming of a team of experts in different fields. Creativity is the most important in this stage. Insufficient information could cause the possible omission of good alternatives and the decision problem thus to finally depart from being conclusive.

(3) **Prediction of alternative outcomes** – The model is an abstraction of the real-world by focusing the factors relevant to the problem for investigating the behavior or response of a system. In order to reduce the model uncertainty, calibration and validation are needed after the formulation of a model. Relevant models are used to predict the outcome before the implementation of an alternative, considering the uncertainty of scenarios. Informed risk-based decision making can be implemented based on the information from uncertainty analysis. Prudent uncertainty analysis is essential to identify and quantify the magnitude of probable hazard damage and the associated frequency. In this step, the limitations of knowledge in modeling and inaccurate input
data, which reduce the accuracy of outcome prediction, are the main concerns.

(4) **Rank alternatives by risk-based decision rules** – In a decision problem under risk, the outcome of each alternative can be treated as random variables with known probabilistic features. The goal of risk-based decision making is to search for the most desirable choice by considering the frequency and the seriousness of the decision error [48]. Many decision rules are developed for risk-based decision making, but there is no single rule which is universally the best. In addition, identifying the only best choice will be challenging if none dominates all the others for the given rule. Descriptions and discussion of these conventional decision rules are given in Sections 2.3.1 and 2.3.2, including the limitations of their applications to engineering management problems.

(5) **Check the feasibility of alternatives** – Once the ranking of alternatives is done, the feasibility of implementing an alternative should be examined. A feasible alternative should be the one having a risk lower than the decision maker’s acceptable risk. After examining the practicability of all alternatives, a set of feasible alternatives that meets the level of acceptable risk can be identified. The best decision among the feasible alternatives can be determined according to the decision rule. The first-ranked feasible choice can then be considered for implementation. If there is no feasible alternative, the decision maker could take one or more of the following actions depending on the circumstances [53]: (1) reducing uncertainty by conducting further research or acquiring more information; (2) modifying the current alternatives or generate new ones and repeat the decision analysis; (3) delaying the decision until more information is obtained; and (4) compromising with the first-ranked alternative and increase the contingency.

In reality, uncertainties may exist in all elements of a model, rendering uncertain outcomes. Those uncertainties can arise from the inherent randomness in nature, and knowledge deficiency due to inadequate data and model, as well as scenario uncertainties. The uncertainty inherent in the outcome prediction with regard to scenarios is difficult to control and evaluate in decision making. The exact effect of a chosen alternative is rarely
under one’s firm grasp when the associated outcome is affected by factors having considerable uncertainties. With the uncertain state of nature, the outcome of an alternative cannot be predicted with absolute certainty so as to allow straightforward decision-making in step (4) of Figure 2.1.

2.3 Rules for Decision Making in an Uncertain Environment

Decision making in an uncertain environment is an important subject in decision analysis. It addresses issues relating to the effect of outcome uncertainty on alternative comparison, ranking, and selection. An analysis considering outcome uncertainty can provide decision makers with a comprehensive basis to select pro-active management strategies. Such analysis should include a process that is capable of explicitly considering the effects of outcome...
uncertainties, and to provide sufficient information to the decision maker. For a decision
maker, the root problem in decision making when considering the outcome uncertainty is the
trade-off between the risk and the benefit of the choice.

2.3.1 Decision Making Under Risk

In the context of decision making under risk, decision makers have information about the
probability profile of the state of nature. Due to the unforeseeable changes in future
development scenarios and/or the unpredictable state of nature, alternative outcomes cannot
be assessed with absolute certainty or represented by a single deterministic value. Identifying
a successful or failure decision is not a trivial task. To evaluate the merit of alternatives in an
uncertain environment, uncertain outcomes of the alternatives can be treated as random
variables with different probabilistic features. The choice can be viewed as ranking and
selecting alternatives having outcomes with different probability distributions. Figure 2.2
shows a schematic diagram of decision making under risk considering two alternatives \( A_i \) and
\( A_j \) having random outcomes \( X_i \) and \( X_j \). For the sake of the following discussion, it is assumed
that a larger value of the outcome (e.g., net benefit) is more desirable to the decision maker.
The expectations are \( \mu_i \) and \( \mu_j \) and the probability density function (PDF) are
\( f_i(x_i) \) and \( f_j(x_j) \), respectively. Many decision rules have been developed to provide decision makers with the
basis for the ranking of alternatives under risk while the information about probabilistic
features of alternative outcomes is required. In this case, the rules such as expected-value,
mean-variance, the probability-of-loss, safety-first, or stochastic dominance rules can be
adopted in statistical decision theory for comparing risky alternatives subject to uncertainty.

Expected-value rule — The expected-value rule is widely used in engineering practice
because of its easy implementation without requiring additional statistical properties other
than the expected outcomes. The expected-value rule prefers an alternative with a maximum
expected monetary value (EMV). It is justified in repetitive situations where the decision is
made over and over again so that the confidence in achieving the calculated expected outcome
is increased. The expected outcome is calculated by weighting the outcomes by their
probabilities of occurrence and summing the weighted outcomes across all states of nature for
each alternative. If direct use of monetary terms in decision analysis is not appropriate, utility
theory [54, 55] can be used to assess risky project alternatives by way of measuring the decision maker’s preferences for each outcome level.

Figure 2.2 Schematic diagram of decision making under risk with two random outcomes

In practice, most risk-based hydrosystems designs are based on the mean values of the project economic indicators without considering other uncertainty features. Few studies have made a comparison of these ranking methods that consider outcome uncertainty beyond the expected value in the application of water resources planning [9, 12].

Mean-variance rule — In some decision problems, not only the expected value of the outcome but also the diverse values of the outcome that could possibly occur are of interest to decision makers. The variance of the outcome is a viable measure for the possible variation of outcome. The mean-variance (M-V) rule, first proposed for investment portfolio selection by Markowitz [56], is for a risk-averse decision maker who prefers an alternative with a higher expected outcome and/or lower variability. An alternative with higher expected outcome is generally preferred. In addition, a risk-averse decision maker would like to minimize the
variability. The M-V rule is frequently used to narrow down the project alternatives by forming an efficiency frontier that comprises projects with the lowest risk for a given level of expected return. The efficiency frontier, which is a curve in a mean-variance space, shows alternatives with the lowest risk for a given level of expected return. Under the notion that the variability of a positive return is not undesirable, the mean-semivariance rule is proposed to consider the trade-offs between expectation and the variability of negative outcomes [57].

**Probability-of-loss decision rule** — The probability-of-loss (POL) rule considers the downside risk, and a well-known variation of this rule is the safety-first rule [58, 59] by which the decision maker favors the project alternative with the smallest probability-of-loss. However, the magnitude of the potential risk is overlooked by this rule. The probability-of-loss represents the likelihood of the alternative yielding a return less than zero. For alternative $A_i$, its probability-of-loss can be calculated by

$$\Pr[X_i < 0] = \int_{-\infty}^0 f_i(x) \, dx$$

(2-1)

**Stochastic dominance** — Stochastic dominance (SD) rules provide more theoretical examination in determining the preference among various competing alternatives with uncertain outcomes. The implementation of the SD rules requires knowledge of the marginal probability distribution, not just the mean and variance of outcomes of the alternatives [60-62]. SD rules assume the utility function, $u(x)$, is a non-decreasing function of the outcome level and the first derivative of the function, $u'(x)$, is greater than zero. The first three degree SD rules are commonly used to determine the relative merits of project alternatives [45, 63, 64]. The first-degree stochastic dominance (FSD) rule states that alternative $A_i$ dominates $A_j$ if

$$F_i(x) \leq F_j(x)$$

(2-2)

for all outcome levels of $x$ and the strict inequality holds for at least one value of $x$, where $F_i(x)$ and $F_j(x)$ are the cumulative density function (CDFs) of the two random outcomes $X_i$ and $X_j$, respectively. However, when two CDFs of the outcomes of the two alternatives intersect, the FSD rule will be indecisive and a higher-order SD rule is needed. The second-degree stochastic dominance (SSD) is based on the assumption that the utility function is increasing.
and the marginal utility, $u'(x)$, is decreasing with respect to the outcome level. In other words, the utility function is concave and its second derivative, $u''(x)$, is negative. This implies that the decision maker is risk-averse. By the SSD rule, alternative $A_i$ dominates $A_j$ if

$$\int_{-\infty}^{x} F_i(t) \, dt \leq \int_{-\infty}^{x} F_j(t) \, dt \quad (2-3)$$

for all outcome levels of $x$ and the strict inequality holds for at least one value of $x$. There are also some variations of the conventional stochastic dominance decision rule [65, 66].

The third-degree stochastic dominance (TSD) is applied when the SSD test fails to reach a conclusion. This rule assumes that the risk-aversion of a decision maker diminishes with increasing return and the risk attitude cannot change drastically from risk-aversion to risk-seeking (e.g., $u''(x) < 0$ and $u'''(x) > 0$). The TSD test states that alternative $A_i$ dominates $A_j$ if

$$\int_{-\infty}^{x} \int_{-\infty}^{w} F_i(w) \, dw \, dt \leq \int_{-\infty}^{x} \int_{-\infty}^{w} F_j(w) \, dw \, dt \quad (2-4)$$

for all outcome levels of $x$ and the strict inequality holds for at least one value of $x$ and $\mu_i > \mu_j$. It is clear that SD rules and other risk-based decision rules are all univariate rules and none explicitly accounts for the possible correlations and interactions among the outcomes of different alternatives under consideration.

### 2.3.2 Decision Making Under Uncertainty

In decision making under uncertainty, the probabilities of the possible project outcomes are unknown. When a decision maker does not know the probabilities of the possible alternative outcomes, several rules for decision making under uncertainty have been used depending on the risk attitude of the decision maker. Some commonly used alternative ranking methods are briefly summarized here. These rules include the Laplace rule, the maximin rule, the maximax rule, the Hurwicz decision rule, the minimax regret rule [67], and the expected utility theory.
To demonstrate the implementation of various decision rules discussed in this section, a simple hypothetical example is used. The problem involves four alternatives, each corresponding to a different protection level of a new levee project to mitigate flood problems. Three different discrete scenarios represent different development scenarios in the concerned area. In decision analysis, the payoff matrix is used to represent the outcomes associated with all combinations of alternatives and the states of nature. Table 2.1 shows the payoff matrix containing the values of the net benefits based on the reduced annual damage of the four alternatives under different development scenarios. Alternative $A_4$ assumes a deterministic outcome, although not realistic, which is used herein for the purpose of illustration. When equal probabilities are assigned to all plausible development scenarios, the expectation, variance, minimum and maximum values for each alternative outcome are as listed in Table 2.1.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Scenarios</th>
<th>Mean</th>
<th>Variance</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Hurwicz ($\alpha = 0.2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$S_1$ 0, $S_2$ 9, $S_3$ 21</td>
<td>10.0</td>
<td>74.0</td>
<td>0</td>
<td>21</td>
<td>4.2</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$S_1$ 4, $S_2$ 10, $S_3$ 15</td>
<td>9.7</td>
<td>20.2</td>
<td>4</td>
<td>15</td>
<td>6.2</td>
</tr>
<tr>
<td>$A_3$</td>
<td>$S_1$ 7, $S_2$ 8, $S_3$ 9</td>
<td>8.0</td>
<td>0.7</td>
<td>7</td>
<td>9</td>
<td>7.4</td>
</tr>
<tr>
<td>$A_4$</td>
<td>$S_1$ 6, $S_2$ 6, $S_3$ 6</td>
<td>6.0</td>
<td>0.0</td>
<td>6</td>
<td>6</td>
<td>6.0</td>
</tr>
</tbody>
</table>

Laplace insufficient information decision rule — The Laplace insufficient information rule uses the Johann Bernoulli’s *Principle of Insufficient Reason*, which says that if the decision maker has no insight of the likelihood of occurrence of each state of nature, then there is insufficient reason any one of them has greater probability than any other state. All scenarios are assigned with equal probabilities. If a decision maker has no insight into the
likelihood of the occurrence of each scenario, he or she can choose the alternative with the best expected outcome. In the above example, the ranking result is the same as in the expected-value decision rule. By the Laplace insufficient information rule, alternative $A_1$ with the largest expected net benefit ($1 \times 10^7$) is the best according to the expected-value rule.

Maximin decision rule — The maximin decision rule, also called the pessimistic rule [68], assumes that a decision maker is totally conservative and pessimistic and anticipates that the worst will occur [69, 70]. This rule selects the alternative with the best of the worst possible outcome. By this rule, the risk of large losses is the main concern. For a positive payoff (e.g., profits or incomes) the pessimist behavior can be modeled by the maximin rule; but for a negative payoff such as costs or losses, the minimax rule is applied. Both of these rules reflect the selection from a pessimist. The first step of this rule is to evaluate each alternative by the minimum possible payoff, which is the worst possible outcome if an alternative is chosen, and then select the alternative with the best of the worst possible outcomes.

In the above example, by the maximin rule, a pessimistic decision maker will choose alternative $A_3$ because the worst payoff of the net benefits of $7 \times 10^6$ is better than the worst payoffs of the other alternatives. This selection will avoid extremely large possible losses or small profits resulting from the channel improvement and the levee system alternatives.

Because this rule uses only partial information, the worst payoff of each alternative, it is thought to not be a realistic decision rule. If there are $n$ alternatives, only $n$ worst values of the payoff matrix are considered. For instance, in this example, most of the information has (2/3) been excluded. This rule is also not applicable when the extreme event has no bound.

Maximax rule — The maximax rule, exactly the opposite to the maximin rule, models the behavior of an optimistic decision maker who believes that the best can happen [69]. This rule is also called the optimist rule. The maximax rule chooses the alternative with the maximum optimistic outcome. By this rule, the decision maker is attracted by a large possible profit and is willing to take the risk of large losses. For a positive payoff, such as profits or
incomes, the optimist behavior can be modeled by the maximax rule; but for a negative payoff such as costs or losses, the optimist rule is minimin. The maximax decision rule is conducted by firstly determining the maximum possible payoff of each alternative, and which is the best possible outcome if a decision is made. Sequentially, from these maxima, select the alternative with the maximum of the best possible outcome.

In the above example, an optimistic decision maker will choose alternative $A_1$ because the best payoff of the net benefits of $21 \times 10^6$ is better than the best payoffs of other alternatives.

The maximax rule is not a realistic decision rule as the maximin rule because this rule uses only partial information: the best payoff of each alternative. Both maximin and maximax rules are not applicable when the extreme event has no bound.

**Hurwicz rule** — The Hurwicz rule makes a compromise between absolute pessimism and optimism by introducing a coefficient of optimism $0 \leq \alpha \leq 1$ [71]. This coefficient is a measure of the decision maker’s degree of optimism with $\alpha = 1$ being absolutely optimistic and $\alpha = 0$ being absolutely pessimistic. $\alpha$ can be interpreted as the probability assigned to the scenario producing the best outcome for each alternative. Once the coefficient of optimism is determined, the weighted outcome for an alternative can be computed by

$$WO(A_i) = \alpha \times \max \{X_i\} + (1 - \alpha) \times \min \{X_i\}$$

(2-5)

where $WO(A_i)$ is the weighted outcome for alternative $A_i$ and $X_i$ represents its possible outcome. Then, the alternative with the maximum weighted outcome is chosen.

In the above example, let the coefficient of optimism to be 0.2, and the weighted outcomes of net benefits for the four alternatives, as shown in Table 2.1, are

$$WP(A_1) = 0.2 \times 21 \times 10^6 = 4.2 \times 10^6$$

$$WP(A_2) = 0.2 \times 15 \times 10^6 + 0.8 \times 4 \times 10^6 = 6.2 \times 10^6$$
\[ WP(A_3) = 0.2 \times 9 \times 10^6 + 0.8 \times 7 \times 10^6 = 7.4 \times 10^6 \]
\[ WP(A_4) = 6 \times 10^6 \]

So with \( \alpha = 0.2 \), alternative \( A_3 \) will be selected as it has the highest weighted net benefit of \$7.4 \times 10^6.

Minimax regret rule — The minimax regret principle, introduced by Savage [72], is based on the concept of opportunity loss under the condition that the outcome of a chosen alternative is affected by a finite number of discrete scenarios [73, 74]. This rule assumes that the preference structure of a decision maker in choosing a risky alternative is based on the negative emotion of making a wrong choice rather than the maximization of the anticipated payoff of an alternative.

The term regret is used to describe a decision maker’s negative emotion as a result of not making the best choice. Regret in a choice is defined as the difference between the ensuing payoff from a course of action and the best possible outcome of a non-chosen (forgone) alternative under a given true state of nature. Regret is analogous to the opportunity loss due to the adoption of a particular course of action. Opportunity loss in a chosen alternative could incur when the outcome from an uncertain state of nature is worse than that in selecting the other alternative. From an economic viewpoint, the opportunity loss consists of two components: the actual monetary loss and the unrealized potential profit. Although regret is often regarded as an emotional state, it is assumed as a quantifiable variable here in direct relation to a payoff matrix.

Under the condition that the alternative outcome is affected by different discrete states of nature, the minimax principle is used to select the alternative having the lowest maximum possible regret (opportunity loss). In other words, a risk-averse decision maker chooses from all the available alternatives to avoid the situation where a non-chosen alternative would yield a better outcome in the pair-wise choice. For positive-valued outcomes (e.g. benefits or gains), the regret \( r_{ik} \) associated with a chosen alternative \( A_i \) is defined as
where $x_i^\text{max}$ ($k = 1, 2, \ldots, K$) is the maximum outcome among all considered alternatives under a state of nature $S_k$ and $x_{ik}$ is the outcome of alternative $A_i$ under $S_k$. Specifically, for a particular state of nature, the opportunity loss of the chosen alternative not being an optimal one is the resulting outcome difference between this chosen alternative and the best (highest-yielding) one. The rule selects the alternative with minimum worst (maximum) regret resulting from making a non-optimal decision.

By Eq.(2-6), one can derive the regret matrix (shown in Table 2.2) from Table 2.1. Referring to Table 2.2, the use of Savage's minimax regret principle allows choosing alternative $A_2$ to achieve the lowest maximum possible regret.

### Table 2.2 Regret matrix ($10^6$) in applying Savage’s minimax regret principle

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Scenarios</th>
<th>Maximum regret</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_1$</td>
<td>$S_2$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>$A_2$</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>$A_4$</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

Bell [75], Fishburn [76], and Loomes and Sugden [77] independently developed the expected regret model of pair-wise comparison for option selection under finite random discrete states of nature. Regret theory hypothesizes that an individual anticipates and takes into account the possibility that the chosen alternative may turn out to perform worse than the
non-chosen (forgone) alternatives, for each possible state of nature. Regret theory can explain the Allais Paradox, the Ellsberg Paradox, the common-ratio effect, the common-consequence effect, and the preference reversal phenomenon, that cannot be explained by the utility function [78]. Regret theory has been extensively applied in the areas of psychology [79, 80], marketing [81, 82], finance [83], and microeconomic analysis [78, 84, 85], but not as much in water resources planning and management. Chorus et al. [86] generalized regret theory and applied it to multi-nominal and multi-attribute travel demand decision making in which the state of nature is described by continuous random variables. In engineering management, it can measure the anticipated forgone value that society must bear in using the limited resources in taking a course of action. However, the application of regret theory for continuous state of nature in water resources planning and management has so far been rarely found and the effects of outcome correlation are ignored in these applications.

However, when alternative outcomes are continuous random variables, the value of MAX and x in Eq.(2-6) cannot be determined. To evaluate the relative performance of alternatives with continuous random outcomes, determining the expected opportunity loss of alternatives can only be made through pair-wise comparison. Through aggregating the opportunity cost over the entire range of possible states of nature, the risk measure of the expected regret for each chosen alternative can be quantified.

*Expected utility theory* — The idea of utility in risk-based decision making has been established since Bernoulli [54] first introduced a mathematical form of risk aversion and applied a logarithmic utility function in the expected utility calculation.

Utility theory sets the basis for integrating the preference of an uncertain outcome into decision analysis. The basic hypothesis is that an individual makes an investment decision in order to maximize the expected utility. In financial and economic decision making under uncertainty, an expected utility that takes into account the decision maker’s attitude towards risk is widely used. In engineering economics, Park and Sharp-Bette [44] gave a detailed instruction of the utility theory. Utility theory is both a prescriptive and a descriptive approach.
in decision making. It is an idealized method to describe the behavior of an individual toward risk, and also provides a normative standard way of how to make choices.

The proposition of using a nonlinear utility function in decision analysis can be traced back to the eighteenth century. Bernoulli in 1738 [54] introduced the mathematical form of risk aversion and applied a logarithmic utility function in replacement of the direct monetary consequences in expected utility calculation. Nearly fifty years later, Benthan [87] suggested that the utility could be measured as a cardinal scale of pleasure and pain. Two centuries later, the idea of cardinal utility in decision making under uncertainty reappeared in the second edition of the book of von Neumann and Morgenstern [55] and the expected utility theory was formed. Savage [88] combined utility theory with subjective probability and introduced the subjective expected utility theory. Schlaifer [89] indicated that while small decision problems could be analyzed by maximization of the expected monetary value, risk aversion must be considered in large-scale problems.

The utility function is used to approximate the preference of wealth for an individual in the face of uncertain outcomes. In principle, the expected utility (EU) is defined as

\[ EU = \int u(x) f_X(x) dx \]  (2-7)

in which \( u(x) \) is the utility function of a outcome level \( x \); \( f_X(x) \) is the PDF of a random outcome \( X \). Integrating the utility function with the associated PDF gives the expected utility for a decision maker under the spectrum of an uncertain scenario.

The shape of the utility function determines the risk attitude. It is generally accepted that the utility increases with an increasing state of wealth. Hence the utility function should be an increasing (or at least non-decreasing) function against wealth. Three different types of risk attitudes are risk-averse, risk-neutral, and risk-seeking, as shown in Figure 2.3. For a risk-averse individual, the utility function is concave. In this situation, the marginal utility decreases with an increasing amount of wealth. This means that the individual prefers receiving a payoff with certainty rather than being involved in a lottery with a mean value of the same payoff amount. Defining the certainty equivalent (CE), it follows that
and the risk premium ($RP$) is defined as $RP = EMV - CE$, with $EMV$ being the expected monetary value of a risky alternative [90]. For the risk-averse individual, $RP$ is positive. For a risk-neutral decision maker, the utility function is a linear function, and $RP$ is zero. For a risk-seeking decision maker, the utility function is a convex function, and $RP$ is negative.

Despite the general increasing trend of utility functions, the change rate of curvature along with increasing level of wealth varies with different forms of utility function. The commonly used utility functions can be classified as negative exponential, quadratic, logarithmic, power, and hyperbolic absolute risk aversion utility functions [44], which are listed in Table 2.3.

**Figure 2.3 Utility function of different risk attitude**
### Table 2.3 Types of utility function

<table>
<thead>
<tr>
<th>Types of utility function</th>
<th>Functional forms</th>
<th>$r_A(x)$</th>
<th>$\frac{dr_A(x)}{dx}$</th>
<th>Types in risk aversion</th>
</tr>
</thead>
</table>
| Negative exponential      | $u(x) = 1 - e^{-rx}$  
$c > 0$            | $c$      | $= 0$                | Constant              |
| Quadratic                 | $u(x) = x - ax^2$  
$a > 0, x \leq 1/(2a)$ | $2a/(1-2ax)$ | $> 0$                | Increasing             |
| Power                     | $u(x) = \frac{x^{1-r}}{1-r}$  
$r < 0$            | $r'/x$   | $< 0$                | Decreasing             |
| Logarithmic               | $u(x) = \ln(x+d)$  
$d \geq 0$          | $\ln((x+d)/(x+d))$ | $< 0$                | Decreasing             |
| Hyperbolic absolute risk aversion (HARA) | $u(x) = \frac{1-r}{r} \left(\frac{ax+b}{1-r}\right)^r$  
$b \geq 0$          | $\frac{a(1-r)}{ax+b(1-r)}$ | $r\to 1$: risk neutral  
$r = 2$: quadratic  
$r\to\infty$ and $b = 0$: negative exponential  
$r < 1$ and $b = 0$: power |

The degree of risk aversion can be measured by the Arrow-Pratt coefficient of absolute risk aversion $R_A$ [91, 92], which is defined as

$$R_A = -\frac{u''(x)}{u'(x)}$$ (2-9)

for some level of $x$. This coefficient is a measure of the curvature of the utility function $u(x)$. It can be interpreted as the percent change in marginal utility at any level of $x$. For a negative exponential utility function, the absolute risk aversion coefficient is a constant $c$, which means this kind of utility function reflects a constant risk-aversion behavior. In a similar way, the
quadratic utility function reflects the behavior of increasing risk-aversion, power and logarithmic utility function of decreasing risk-aversion behavior.

The most popular way of acquiring the utility function is by the certainty equivalent method [93]. It can also be empirically determined by interviewing or designing a questionnaire for decision makers. Although the expected utility will be altered by the form of the utility function, it is worth noting that the expected utility is relatively insensitive to the form of the utility function at a given level of risk-aversion, and that the expected utility does not change significantly over a wide range of coefficients of risk-aversion [94].

2.3.3 Limitations of Conventional Decision Rules in Hydrosystem Design

There is no general agreement on which risk-based decision rule is the best for the ranking of alternatives in decision analysis. The above mentioned conventional decision rules have limitations during their implementation in water resources management and hydrosystem design problems. Utilization of the partial probabilistic information of each risky project outcome without a comprehensive evaluation could lead to irrational decisions in some situations. The maximin and the maximax rules only make judgments based on the pessimistic and optimistic outcomes, respectively. The Hurwicz rule only considers the two extreme outcomes and ignores all other possibilities. Savage’s minimax regret principle considers all of the available information in the payoff matrix. However, the problem is that in real-life applications not all outcomes associated with every possible scenario can be recorded or obtained before a scenario has occurred. The M-V rules cannot account for the effect of a skew-distributed decision outcome, not to mention the limitations of the expected-value decision rule. The probability-of-loss rule, or its known variant of the safety-first rule, does not account for the magnitude of loss incurred by a system failure.

In hydrosystem engineering and management, outcomes for different design alternatives are often correlated by some common hydrologic and hydraulic factors. However, all the previously mentioned conventional decision rules are univariate by nature and, therefore, none is able to consider the inter-correlations among project outcomes in the decision making process which often exist in hydrosystem design and management.
Another limitation of these decision rules is the applicability in project ranking. For instance, the M-V and the SD rules may or may not produce conclusive decisions for the optimal course of action. The first two degrees of stochastic dominance may not be sufficient to identify the dominance among several risky projects. The M-V rule produces a set of non-dominant alternatives which may sometimes fail to identify the best decision[95]. For engineering decision making, a clear ranking that accounts for the information of risk is more useful. In addition, not all the methods mentioned above can provide a quantitative criterion to explicitly measure how good a decision is as compared to other ones. Xu and Tung [12] showed that different decision rules could result in different rankings for the same set of competing alternatives. Even for a single decision rule, like the Hurwicz rule, the ranking can be altered by changing the value of the optimism coefficient, and estimation of the coefficient of optimism may not be a trivial task [96]. Moreover, although the expected utility theory is often applied to problems of decision making under uncertainty, its application usually encounters practical difficulties in decision making involving public affairs because the form of a utility function is not easy to determine, especially when the interests of multiple stakeholders are involved and may be in conflict with one another.

2.3.4 Needs for New Risk Measures for Decision Making

The convention risk-based decision rules mentioned above cannot address three important issues in decision making: (1) How much better is a preferred alternative over its competitor? (2) Is the preferred alternative feasible and implementable from the decision maker’s viewpoint? (3) How does the outcome correlation among the alternatives influence the relative merit of these alternatives and the subsequent decision?

When uncertainties are present, no matter which alternative is chosen over the others, the selection always has a non-zero probability of succeeding or failing, even though the decision makers anticipate the chosen alternative would bring a more desirable outcome than the unchosen ones. A correct decision herein is referred to as the chosen alternative that turns out to yield a better outcome than the others. Apostolakis [97] listed the benefits of the quantitative risk assessment in the process of risk-based decision making as: (a) facilitate communication among analysts, decision makers, and stakeholders in different groups and
disciplines; (b) quantify the uncertainties and provide valuable information toward decision making; (c) identify critical scenarios and facilitate risk management; and (d) lead to risk-informed decision making. Therefore, a quantifiable risk measure is needed to assess the relative merits of a chosen alternative with reference to the others and to quantitatively assess the potential loss.

Two other issues are relevant in risk-based decision making: the decision maker’s acceptable risk and the correlation among uncertain alternative outcomes. From the viewpoint of prudent management, setting an acceptable tolerance for potential loss and preparing a proper contingency in advance are advisable practices for dealing with the adverse impacts of failing to achieve the anticipated outcome. In decision making under uncertainty, risk is usually referred to as the probability weighted adverse consequence. The acceptable risk is a level of risk that the stakeholders feel comfortable to take or the acceptable tolerance for the potential loss or the contingency that should be anticipated. The acceptable risk offers managers a basis to examine the feasibility of implementing an alternative by the tolerable level of risk. In a feasibility test, a quantitative risk measure that is commensurable to the acceptable risk should be developed and used. If the potential loss associated with the decision of selecting one particular alternative is lower than the acceptable risk, this decision is implementable because the associated loss is tolerable. Otherwise, implementing the alternative may not be prudent because the associated potential loss could be beyond the decision maker’s or stakeholder’s capacity to absorb. Hence, the acceptable risk set by the decision maker should be considered and used to examine the feasibility of implementing a chosen alternative as shown in step (5) of Figure 2.1

Stewart et al. [98] applied the concept of acceptable risk in the ranking of alternatives of four different bridge designs and informed the decision maker of the acceptable alternatives. Xu [99] designed a questionnaire to obtain decision makers’ acceptable risk as a feasibility criterion for a river basin management project.

In real-life water resource management and hydrosystem design, it is not uncommon that outcomes of various competing alternatives are dependent on some common factors or
attributes which render the outcomes as correlated. For example, in designing a flood control system for a river basin, the project alternatives might be different protection levels against the same random flood load. In this case, the project cost and benefit can be affected by common factors such as rainfall, flow hydrograph, channel geometry, boundary conditions, and topographical/land use features of the study area. To some degree, the outcomes of each alternative are expected to be correlated. In this situation, ignoring the dependence of alternative outcomes will not truly reflect the uncertainty features of the relative merit among different alternatives and, hence, affect the validity of the decision. Practically, all conventional decision making rules reviewed earlier are univariate and do not have the provision to account for the effect of correlation among alternative outcomes.

In addition, from the ‘damage control’ viewpoint, a decision method should consider the consequences of making a wrong decision. Reducing the cost of over-design and the unexpected loss of under-design is the essence of the risk-based design [32, 100]. Recently, the idea of considering the potential loss and gain as the consequences of disease diagnoses has been applied in the retrieval stage of case-based reasoning [101]. This method aims at choosing a best possible diagnosis that minimizes the potential loss in case the diagnosis turns out to be erroneous. For practical engineering management and design, it is desirable to have a risk measure that quantifies the relative risk of a chosen alternative with respect to the others and to assess the associated potential loss [97].

2.4 Multi-criteria Decision Making and Risk-based MCDM

Problems of water resources management and engineering design typically contain several criteria involving intangible factors in addition to the economic efficiency. Examples of intangible factors in water resources planning and management often involve social, economic development, and environmental preservation factors which are typically measured in different units. The non-commensurable and intangible factors can be as important as the economic factors quantifiable by monetary terms in decision making in public infrastructures and might work against the principle of economic efficiency. In a review of the literature considering MCDM in water resources planning and management from 1973, the most
commonly considered objectives were the project cost (net present value), the economic
development, the technical feasibility, the water quality and supply, and the fairness and
 equity of resources distribution issues [102].

A MCDM procedure that evaluates and ranks the alternatives according to multiple
criteria in which economic efficiency is one of many factors to be considered simultaneously.
In general, use of a multiple-criteria approach enhances the transparency, auditability, analytic
rigor, and conflict resolution in decision making [103, 104]. If the overall merit of a specific
alternative depends on the merits of several individual factors which are subject to uncertainty,
the problem becomes a MCDM under uncertainty, as illustrated by Figure 1.4.

2.4.1 Multiple Criteria Problems

A problem in making a choice involving multiple conflicting criteria is MCDM. In decision
analysis, MCDM is a more general concept, and it can be categorized into multiple-attribute
decision making (MADM) and multiple-objective decision making (MODM), or called
multiple objective mathematical programming [105]. These terms are similar literally, but
have different meaning in practice. Generally, MADM evaluates a discrete set of explicit
alternatives that are generated by the decision maker; and the MODM uses mathematical
programming to generate an infinite number of non-inferior solutions that satisfy a set of
constraints. In MADM, the subjective weight associated with each alternative is required to
determine the optimal choice. In a problem of MODM, as some objectives mutually conflict
with each other, the optimal solution is no longer obtainable and the idea of optimal solution
becomes a compromised or efficient solution.

A MCDM analysis generally proceeds the following steps: (i) identifying the decision
context, including the decision makers, stakeholders and decision constraints; (ii) defining the
decision criteria; (iii) eliciting the relative importance of criteria weights; (iv) generating a set
of candidate decision alternatives; (v) evaluating the performance values of decision
alternatives against the criteria; (vi) applying suitable techniques for MCDM; (vii) performing
sensitivity analysis; and (viii) making the final decision.
2.4.2 Existing MCDM Techniques

To solve MCDM problems, a wide variety of techniques have been developed to help decision makers to describe, evaluate, rank and select alternatives according to multiple criteria. A comprehensive review and detailed description of the decision process for these MCDM techniques can be found in Figueira et al. [70] and Triantaphyllou [106]. Studies evaluating these MCDM techniques reveal that no technique is inherently better, and in only a few cases will different techniques generate different ranking results [102, 107-110]. According to Levy [105] and Hajkowicz and Collins [102], these techniques can be categorized into six groups:

- **Multi-criteria Value Function** – Two commonly applied models for the multi-criteria value function are the weighted sum model and weighted product model. The overall performance score $u_i$ of alternative $A_i$ for the additive value model can be expressed as

$$u_i = \sum_{k=1}^{K} w_k v_{i,k} \tag{2-10}$$

where $v_{i,k}$ is the performance value of $A_i$ with respect to criterion $C_k$, and $w_k$ is the weight assigned to criterion $C_k$ representing the relative importance of the criterion. The overall performance score in the multiplication value model can be determined by replacing the summation operation with a multiplication operation which makes the criteria non-compensatory. For any two alternatives $A_i$ and $A_j, A_i \succeq A_j$ if and only if $u_i \geq u_j$. Multi-attribute utility theory (MAUT), axiomatized by [111-114], is a widely used technique for multi-criteria value functions, which solves MCDM problem on the basis of utility theory. The utility function is constructed based on a set of axiomatically defined assumptions about the preference structure of the decision makers. Assuming the utility function satisfies the additive independence, utility independence, and preference independence, the additive value model can be used to calculate the expected utility [115]. The multiplication value model can be used when the utility independence and preference independence are satisfied.
Simple multi-attribute ranking technique (SMART) is the simplest technique in this category. SMART was first proposed by Edwards [116], and is widely applied to decision problems in which the utility function is of linear form. The overall performance score is simply the weighted algebraic average of the performance value associated with the alternative.

- **Outranking Methods** – Instead of descriptive characterization of the decision maker’s preferences like MAUT, outranking methods construct the preferences in terms of the outranking relations [117]. Methods such as ELECTRE (ELimination Et Choix Traduisant la Réalité) [118] and PROMETHEE (Preference Ranking Organization Method of Enrichment Evaluation) [119] are the most widely used outranking techniques. The preference values of every pair of alternatives are produced giving $N^2 - N$ pairs in total. The total amount of the preference values is then used to identify the ranking orders between each two alternatives.

- **Distance to Ideal Point Methods** – Technique for the order of preference by similarity to ideal solution (TOPSIS) [120] and compromise programming (CP) [121] are classified under this category of methods. These techniques determine the ideal and anti-ideal points for the criteria, then the best decision is the one that is closest to the ideal point and furthest from the anti-ideal point. The goal programming was conceived by Charnes and Cooper [122], and subsequently developed by Lee [123] and Ignizio [124]. It usually applies to multiple-objective linear programming and can also be used to select the best efficient solution from a set of discrete alternatives by measuring the closeness of different alternatives to the numerically defined goal.

- **Pairwise Comparisons** – The analytic hierarchy process (AHP) approach is one of the most widely applied pairwise comparison techniques. AHP was first developed by Saaty in 1970’s and was further developed as analytic network process (ANP) by continuous application and amendment subsequently [125, 126]. AHP can also be used to determine the weight of criteria and performance values of alternatives [125, 127].

- **Other methods** – Hajkowicz and Collins [102] indicated that the most commonly
applied techniques are \textit{CP}, \textit{AHP}, \textit{ELECTRE}, \textit{PROMETHEE}, and \textit{MAUT} in water resources planning and management. There are many newly developed methods or tailored methods based on the adaptation of existing ones. For example, data envelopment analysis (\textit{DEA}) was proposed by Charnes et al. [128] and Banker et al. [129]. The concept is to categorize the attributes as input terms and yield terms, to maximize the efficiency of an alternative, which is the ratio of total yield and total input. The regret-based approaches select an alternative that minimize the worst consequences resulting from one of the scenarios that could occur in the future. Table 2.4 shows some of the selected literature that uses MCDM techniques for solving water resources planning and management problems.

<table>
<thead>
<tr>
<th>MCDM techniques</th>
<th>Examples of application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weighted Sum</td>
<td>Fleming [130]; Howard [131]; Hyde, et al. [132]</td>
</tr>
<tr>
<td>CP</td>
<td>Abrishamchi et al. [133]; Raju, et al. [134]; Scott [114]</td>
</tr>
<tr>
<td>AHP</td>
<td>Levy [105]; Scott [114]; Willett and Sharda [135]</td>
</tr>
<tr>
<td>ELECTRE</td>
<td>Chung and Lee [136]; Raju, et al. [134]; Scott [114]</td>
</tr>
<tr>
<td>PROMETHEE</td>
<td>Al-Kloub, et al. [137]; Al-Rashdan, et al. [138]; Al-Shemmeri, et al. [139]; Abu-Taleb and Mareschal [140]; Maragoudaki and Tsakiris [141]; Martin, et al. [17]; Ozelkan and Duckstein [107]; Raju, et al. [134]; Scott [114]</td>
</tr>
<tr>
<td>MAUT</td>
<td>Brouwer and van Ek [142]; Gershon and Duckstein [108]; Gomez-Limon, et al.[143]; Hayashi [144]; Keeney and Wood [2]; Raju and Vasan [145]</td>
</tr>
</tbody>
</table>
2.4.3 Application of MCDM in hydrosystems planning and management

Because the water resources related decision problems are rarely guided by a single objective, the MCDM analysis gained particular attention and a considerable number of applications have been produced. Table 2.5 shows some representative analyses from the vast applications of the MCDM techniques for different decision problems in water resources management. Among these applications, water policy and supply planning, and infrastructure selection are the most common decision problems [102].

<table>
<thead>
<tr>
<th>Types of MCDM application</th>
<th>Examples of application</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Catchment management</td>
<td>Chang et al. [146]</td>
<td>The Tweng-Wen reservoir watershed in Taiwan</td>
</tr>
<tr>
<td>Ground water management</td>
<td>Almasri and Kaluarachchi [147]</td>
<td>The Sumas-Blaine aquifer in Washington State, US.</td>
</tr>
<tr>
<td>Water supply infrastructure selection</td>
<td>Eder et al. [110]</td>
<td>Danube River, Austrian</td>
</tr>
<tr>
<td>Project appraisal</td>
<td>Al-Rashdan et al. [138]</td>
<td>The Jordan River</td>
</tr>
<tr>
<td>Water allocation selection</td>
<td>Agrell et al. [148]</td>
<td>The Shellmouth Reservoir in south-west Manitoba, Canada</td>
</tr>
<tr>
<td>Water management policy selection</td>
<td>Joubert et al. [149]</td>
<td>The City of Cape Town, South Africa</td>
</tr>
<tr>
<td>Water quality management</td>
<td>Lee and Chang [150]</td>
<td>The Tou-Chen River Basin in northern Taiwan</td>
</tr>
<tr>
<td>Marine protected area management</td>
<td>Fernandes et al. [151]</td>
<td>The Caribbean</td>
</tr>
<tr>
<td>Flood control policies assessment</td>
<td>Brouwer and van Ek [142]</td>
<td>The Netherlands</td>
</tr>
<tr>
<td>Sustainable Water Resources Planning</td>
<td>Raju et al. [134]</td>
<td>Spain</td>
</tr>
</tbody>
</table>
2.4.4 Dealing with Uncertainties in MCDM

The presence of uncertain information cannot guarantee a correct decision. To describe the design alternatives and evaluate, rank and select a best solution, the most readily available MCDM techniques mentioned above use the deterministic performance values of each alternative against a criterion. The uncertainties in the consequences and weights of the criteria are not considered. However, two types of uncertainties are present in the MCDM process [152].

The information required for a MCDM technique can be also categorized as external and internal information. The internal information is used to characterize the decision maker’s principle for consistent and logical decision making which includes the judgment and preference structure of the decision maker. The external information is used to describe the decision context which includes the consequence of a course of action and the decision constraint.

Internal uncertainties may arise when there are ambiguous meanings or imprecise specification of a criterion, or misinterpretation of an alternative which results in a different evaluation of the consequences under the same criterion. This type of uncertainty relating to the imprecisions about subjective cognitions can be modeled by a fuzzy set [153] or a rough set [36, 154]. Belton and Stewart [155] suggested that appropriate sensitivity analysis or robust analysis can be used to deal with internal uncertainties and reduce the complexity added to the problem, whereas internal uncertainties cannot be resolved in MCDM.

External uncertainty refers to the imperfect information of the consequences of each alternative, resulting from a lack of perfect understanding of the decision environment or randomness that is inherent in the system. The performance of each alternative is uncertain because different possible states of nature associated with different possibilities will result in different payoffs. Similar to the treatment of internal uncertainty, the external uncertainty can be dealt with by using an extensive sensitivity analysis after performing the deterministic MCDM technique. In this study, the external uncertainties are explicitly treated in a probabilistic fashion in MCDM.
Four techniques are currently used that consider the effect of external uncertainties in MCDM. These are multi-attribute utility theory and its variations, pairwise comparisons of alternatives by use of stochastic dominance, incorporation of a surrogate risk measure as additional criterion, and merging scenario analysis into MCDM [152].

Stewart [152] discussed the limitation of applying these approaches in the MCDM under uncertainty. The multi-attribute utility theory is based on a set of assumptions of the preference structure and the utility functions are not easy to define. By using stochastic dominance in pairwise comparisons of alternatives, the correlations between the performance values of different alternatives are neglected. Incorporating a surrogate risk measure as an additional criterion raises another issue about how to measure and define the weights of the risk measures. Some questions remain in the application of integrating MCDM and scenario analysis. For example, how many scenarios are sufficient for analyzing the effect of uncertainties on the results? How to construct the representative scenarios (standard scenarios or ranges of variation that can plausibly occur)? How to define the weight for different scenarios?

2.5 Acceptable Risk

In risk-based design, in addition to a quantitative measure of reliability and risk cost, consideration of intangible factors and societally acceptable risk issues should be included if possible. In the U.S., the societally acceptable frequency of flood damage was formally set to once on average in 100 years. This acceptable hazard frequency was to be applied uniformly throughout the U.S. without regard to the vulnerability of the surrounding land. The selection was not based on a benefit-cost analysis or an evaluation of probable loss of life. Mitigation of natural hazards requires a more rigorous consideration of the risk resulting from the hazard and society's willingness to accept that risk.

In other cases of disaster, societally acceptable hazard levels also have been selected without formal evaluation of benefits and costs. Vrijling [156] showed that a comparison of the worst-case approach to the probabilistic-load approach resulted in a 40-percent reduction
in the design load when the actual, societally acceptable protection failure hazard was considered. This illustrates that when a comprehensive risk assessment is performed, the societally acceptable safety can be maintained (and in some cases improved), and while at the same time effective utilization of scarce financial resources is achieved. Some work on societally acceptable risk and intangible factors can be found elsewhere [157, 158].
CHAPTER 3
RISK-BASED DECISION MAKING
CONSIDERING EXPECTED OPPORTUNITY LOSS

In this chapter, the concept of opportunity loss in game theory is incorporated into the
evaluation of relative merit for choosing an alternative among multiple risky alternatives. A
risk measure, the expected opportunity loss (EOL), is introduced to quantify the potential loss
in making an incorrect choice in risk-based decision making. This is generally consistent with
the risk-averse attitude of decision makers in choosing alternative for which public funds are
used. Differing from Savage’s [72] minimax regret principle, \( EOL \) can account for the
unbounded continuous random outcomes of alternatives and the decision maker’s acceptable
risk. This chapter presents the mathematical formulation of the proposed risk measure of \( EOL \)
for engineering decision making under risk and studies its properties. The proposed minimax
\( EOL \) decision rule for project ranking and evaluation is also described. This chapter describes
the properties of three expected loss-based risk measures and their implications on decision
making, including the proposed \( EOL \) and two related risk measures developed earlier: Xu’s
risk measure (\( XRM \)) [49] and the conditional risk measure (\( CRM \)) [50]. Both risk measures of
\( EOL \) and \( CRM \) share the same notion that a best alternative should have a minimum loss when
the decision turns out to be wrong. The analytical expression of \( EOL \) when the loss function
in power form is derived under normally distributed random outcomes.

In summary, the objectives of this chapter are to (1) investigate the characteristics of
the risk measure; (2) examine the effects of the forms of loss function, correlation among the

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outcomes, and the acceptable risk on the ranking results; and (3) to compare various risk measures by applying them to a simplified river basin management study of the Elbe River in Germany and a hypothetical case study of selecting a flood control project.

3.1 Introduction

In the presence of uncertainty, no matter which alternative is chosen over the other choices, there is always a non-zero probability of yielding a less desirable outcome, even though the decision maker anticipates that the chosen alternative will result in a more desirable outcome than the others. A risk measure is needed to quantify the relative merit of a project alternative with respect to other choices and to assess the potential loss.

The minimax regret decision rule has its advantages in applications to water resources planning and management. It is an appropriate rule for taking the decision maker’s risk attitude into account. Intuitively, for most hydro-infrastructure system designs, the water resource managers appear to be more conservative and reluctant to bear the risk of possible system failure. This risk-aversion behavior can be captured by the minimax rule. The term *regret* in Savage’s minimax regret criterion [72], also called *opportunity loss*, is defined as the difference between the actual payoff one receives for the decision he or she makes and the optimal payoff that would be obtained if the best decision is made [73, 74]. It represents the unrealized part of a payoff and can be used to facilitate decision making among multiple risky projects. Charnes [122] states that in order to articulate the most cost-effective alternative, the opportunity loss in achieving the objective in relation to other alternatives can be used as the basis for expressing the beneficial effects.

When regret of a particular alternative is defined for each discrete state of nature, the minimax regret decision rule can be applied to rank the relative preferences of different decision alternatives. However, while the project outcomes are continuous random variables, the “best possible outcome under a given state of nature” cannot be determined. Savage’s [72] minimax regret criterion will encounter implementation difficulties when the state of nature of one or more random outcomes has unbounded distributions, such as in a normal distribution.
This is because, with unbounded random outcomes, the maximum regret for all alternatives will be infinity and thus make the evaluation of the relative merits of different alternatives difficult. In this case, the minimax regret rule, originally developed for problems with a finite set of predetermined discrete scenarios, will no longer be applicable.

To meet the needs for a quantified risk measure, circumvent the shortcomings of existing rules for risk-based decision making, and expand the applicability of Savage’s [72] minimax regret principle to a situation involving unbounded random outcomes, a modified opportunity loss on the basis of pair-wise comparison is adopted. A new risk measure of the \( EOL \) is formulated for practical engineering decision problems where continuous random economic indicators (project costs or benefits) with known probability distributions are involved. Differing from the conventional definition of regret, the opportunity loss in this thesis is defined as the payoff difference between a pair of chosen and unchosen alternatives, rather than the difference between the payoff of the chosen alternative and the optimal payoff. The modified pair-wise-based opportunity loss can provide more information than the conventional regret with reference to the optimal payoff.

The risk measure of the \( EOL \) of a chosen project alternative was used to quantify the potential loss (or regret) in comparison with other competing alternatives under an uncertain state of nature. The \( EOL \) is obtained by integrating the random outcome difference of a chosen and a non-chosen alternative with the associated probability density function of the outcome when the outcome is inferior to its competitor in a pair-wise comparison manner. Therefore, as long as the probabilistic features of the outcome of a project alternative are available, the \( EOL \) for the chosen alternative can be determined. At the same time, the minimax regret principle works well with the \( EOL \) for ranking and evaluating multiple risky alternatives.

The \( EOL \)-based ranking is based on Savage’s minimax regret decision rule while taking into account the full joint probability distribution of the outcomes of two competing alternatives. Based on the \( EOL \) matrix established from the values of \( EOL \) for each pair of competing alternatives, the minimax regret decision rule can be employed for ranking.
multiple risky alternatives. Moreover, the decision maker can choose an alternative having the minimum potential loss among all competing alternatives in view of controlling the risk of extreme losses.

### 3.2 Mathematical Definition of EOL

In the context of a decision-making problem under risk, several alternatives $A_i$ ($i = 1, 2, \ldots, N$) are considered by the decision maker. Each of these alternatives has a corresponding random outcome $X_i$ due to the presence of uncertainties. To facilitate the discussion, let the random outcomes be the net benefits with their distribution functions and associated statistical properties known. The first two moments (i.e., the mean and standard deviation) of the random outcome of alternative $A_i$ are denoted by $\mu_i$ and $\sigma_i$, respectively. The correlation coefficient between two random alternative outcomes $X_i$ and $X_j$ is $\rho_{ij}$.

For any two alternatives $(A_i, A_j)$ with $\mu_i > \mu_j$ ($i \neq j$), alternative $A_i$ would generally be a logical choice because of its higher long-term expected outcome value according to the expected-value criterion. However, for any chosen alternative with uncertain outcomes, there is a possibility that the realized outcome of alternative $A_i$ is less desirable than the outcome of the unselected alternatives. In other words, it is possible that the decision maker’s anticipated net gain or profit from the chosen alternative $A_i$ may not be realized. The outcome difference for choosing alternatives $A_i$ in comparison with alternative $A_j$ is defined as $A_{*,j} = X_i - X_j$ in which the asterisk (*) signifies the chosen alternative. The probability of the chosen alternative $A_i$ being worse than the competitive alternative $A_j$ is the volume under the joint PDF in the domain of $x_i < x_j$, which can be mathematically expressed as

$$P_r(X_i < X_j) = P_r(A_{*,j} < 0)$$

$$= \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\delta} f_{i,j}(x_i, x_j) \, dx_j \right] \, dx_i$$

$$= \int_{-\infty}^{0} f_{A_{*,j}}(\delta) \, dx_i$$  \hspace{1cm} (3-1)
where \( f_{i,j}(\cdot) \) is the joint PDF of \( X_i \) and \( X_j \). Hence, \( \Delta_{i,j} \) is a random variable having a PDF of \( f_{\Delta_{i,j}}(\cdot) \) with the mean \( \mu_\Delta \) and variance \( \sigma_\Delta^2 \) as

\[
\mu_\Delta = E[\Delta_{i,j}] = \mu_i - \mu_j \tag{3-2a}
\]

\[
\sigma_\Delta^2 = Var[\Delta_{i,j}] = \sigma_i^2 + \sigma_j^2 - 2 \rho_{ij} \sigma_i \sigma_j \tag{3-2b}
\]

The loss is a function of \( \Delta_{i^*,j} \) and the form of the loss function reflects the relative importance of the error committed from a choice under a particular state of nature [48]. A loss function denoted by \( L(\Delta_{i^*,j}) \) in terms of \( \Delta_{i^*,j} \) over the range of negative-valued \( \Delta_{i^*,j} < 0 \) is used to reflect the risk attitude of the decision maker. In general, the form of a performance function depends on the decision maker’s views on loss or gain which can be reflected by a utility function [48]. Let the loss function with respect to the chosen alternative \( A_i \) be defined as

\[
L(\Delta_{i,j}) = -\Delta_{i,j}, \text{ for } j \neq i \tag{3-3}
\]

The negative sign is used to yield a positive-valued loss. An opportunity loss occurs in choosing alternative \( A_i \) over \( A_j \) when the value of \( \Delta_{i,j} \) is negative. Otherwise, the decision maker will have a positive opportunity gain for his or her decision. To evaluate the relative merit of the selected alternative with respect to other unselected alternatives in terms of uncapitalized outcomes, the \( EOL \) can be computed through a pair-wise comparison in terms of the outcome. To reflect the opportunity loss of a particular choice, the proposed risk measure of the \( EOL \) in choosing alternative \( A_i \) over \( A_j \) is evaluated in the domain of \( \Delta_{i^*,j} < 0 \), which can be mathematically defined as

\[
EOL(A_{i^*}, A_j) = -\int_{-\infty}^{\infty} \left[ \int_{-\infty}^{x_i} f_{i,j}(x_i, x_j) dx_j \right] dx_i \tag{3-4a}
\]

\[
= -\int_{-\infty}^{0} \delta f_{\Delta_{i,j}}(\delta) d\delta
\]

where \( EOL(A_{i^*}, A_j) \) is the \( EOL \) for the chosen alternative \( A_i \) with reference to \( A_j \). A chosen alternative with a higher value of \( EOL(A_{i^*}, A_j) \) is less attractive to the decision maker. The values of \( EOL \) for different pairs of competing alternatives are influenced by their expected
outcomes and associated uncertainty features ($\mu_i, \mu_j, \sigma_i, \sigma_j,$ and $\rho_{ij}$). Since the joint PDF is used in the evaluation of potential losses, the effect of correlation is accounted for. The conventional decision rules described previously (e.g., M-V and SD) only utilize information on marginal distributions.

The $EOL$ is a probability-weighted opportunity loss over the range of possible outcomes of a chosen alternative $A_i$ when the induced outcome is inferior to its competitor alternative $A_j$, $\Delta_{i,j} \leq 0$ ($X_i \leq X_j$). Hence, pair-wise comparison in terms of the alternative outcomes is required to determine the $EOL$ associated with a chosen alternative $A_i$ against an unselected alternative $A_j$. In other words, the $EOL$ represents the expected potential loss due to the chosen alternative being wrong when compared with other alternatives. By symmetry, the $EOL$ for choosing alternative $A_j$ with reference to $A_i$ is

$$EOL(A_i, A_j^*) = \int_0^\infty \delta f_{\Delta_{i,j}}(\delta) d\delta, \text{ for } \Delta_{i,j} > 0 \quad (3-4b)$$

The difference between Eqs.(3-4b) and (3-4a) is the long-term expectation of $\Delta_{i,j}$. By comparing a pair of alternatives, it can be proved that

$$E(\Delta_{i,j}) = \mu_i - \mu_j = EOL(A_i, A_j^*) - EOL(A_j^*, A_j) \quad (3-5)$$

which shows that the difference between $EOL$ of choosing one alternative over the other is equal to the difference in their long-term expected outcomes. The equation indicates that once the $EOL$ of selecting alternative $A_i$ over $A_j$ is computed, the $EOL$ for selecting alternative $A_j$ over $A_i$ can be easily obtained. Furthermore, the relationship between $EOL(A_i^*, A_j)$ and $EOL(A_i, A_j^*)$ can be summarized in the following three cases as

1. When $\mu_i = \mu_j$, $EOL(A_i^*, A_j) = EOL(A_i, A_j^*)$
2. When $\mu_i > \mu_j$, $EOL(A_i^*, A_j) < EOL(A_i, A_j^*)$
3. When $\mu_i < \mu_j$, $EOL(A_i^*, A_j) > EOL(A_i, A_j^*)$
Therefore, ranking between two alternatives based on EOL is consistent with the expected-value rule. In other words, if one alternative has a higher expected outcome, the value of EOL by choosing this alternative will be less than that by choosing the other. This implies that when there are only two competing alternatives under consideration, the best choice based on the expected-value rule would be identical to that based on the EOL.

When three or more alternatives are involved, the ranking order by the proposed decision rule might be different from that using the expected-value rule. One has to consider the trade-off between the mean values and the degree of uncertainties (variances) of the alternative outcomes. It is expected that the value of $EOL(A_i^*, A_j)$ would be affected by the relative magnitudes of the means and standard deviations of the two alternatives. The degree of uncertainties (variances) of the project outcomes will affect the relative magnitude as well as the expected values of the project outcomes. For example, consider any pair of alternatives, $A_i$ and $A_j$ within multiple alternatives, with independent and normal outcomes. The relative mean and standard deviation of $X_i$ and $X_j$ are denoted by the mean ratio $R_{\mu} = \mu_i/\mu_j$ and the standard deviation ratio $R_{\sigma} = \sigma_i/\sigma_j$, respectively. The variation of $EOL(A_i^*, A_j)$ with respect to $R_{\mu}$ and $R_{\sigma}$ under $\mu_j = \sigma_j = 1$ is shown in Figure 3.1 which shows that the EOL value is sensitive not only to the change in mean ratio ($\mu_i/\mu_j$), but also in the standard deviation ratio ($\sigma_i/\sigma_j$). One can observe that the value of EOL increases as $R_{\sigma}$ gets larger at a fixed level of $R_{\mu}$ whereas the value of EOL decreases monotonically with increasing $R_{\mu}$ when $R_{\sigma}$ is fixed. The relative merit for different pairs of alternatives will be dependent on the relative growth and decline of their $R_{\mu}$ and $R_{\sigma}$ values. This behavior agrees intuitively that choosing an alternative with a relatively higher outcome variance will lead to a higher potential risk and expected loss.

The complement of EOL is the expected opportunity gain (EOG). The EOG for choosing alternative $A_i$ with reference to $A_j$ can be similarly expressed as

$$EOG(A_i^*, A_j) = \int_{-\infty}^{\infty} \left[ \int_{x_j}^{\infty} (x_i - x_j) f_{\mu_i, \sigma_i} (x_i, x_j) \, dx_j \right] dx_i$$

$$= \int_{0}^{\infty} \delta f_{\mu_i, \sigma_i} (\delta) \, d\delta, \quad \text{for } \Delta_{i,j} > 0$$

(3-6)
Eqs.(3-4a) and (3-6) represent the potential loss and gain for a chosen alternative over the other. They can be used as figures of merit, along with the decision maker’s risk attitude, to evaluate the relative preference among the two competitive alternatives under consideration.

Figure 3.1 Effect of \( R_\mu \) and \( R_\sigma \) on the \( EOL \)

It can be readily shown that the long-term expected return associated with choosing alternative \( A_i \), \( E[X_i - X_j] \), can be partitioned into two parts as

\[
E[X_i - X_j] = EOG(A_i^*, A_j) - EOL(A_i^*, A_j)
\]  

(3-7)

Furthermore, it can also be proved that \( EOL(A_i^*, A_j) = EOG(A_i, A_j^*) \) and \( EOL(A_i, A_j^*) = EOG(A_i^*, A_j) \) by changing the order and the boundary of the integration in Eqs.(3-4a) and (3-6). Then, Eq.(3-5) can be expanded as
Chapter 3 Risk-based decision making using EOL

The properties and relationships of EOL and EOG are summarized in Appendix-A.

Like most risk-based decision making rules, the main difficulty in the proposed approach is the specification of the joint probability distribution of the random outcomes of different alternatives. Mathematical complexity could hinder practical implementation of the proposed approach if the probability distributions of the outcomes of the alternatives are not as simple as the normal distribution considered herein. In water resources planning and management, normal distribution for random net benefit has been adopted by the US Army Corps of Engineers (USACE) [38] and justified by Tung [159] on the basis of the central limit theorem because the net benefit, in general, is the sum of several random benefits and cost components.

Note that in the above discussion, the utility of $X_i$ and $X_j$ are assumed to be monotonically increasing with the values of $X_i$ and $X_j$, i.e., the alternative is more desirable with a higher outcome value. On the other hand, if the outcome has a decreasing utility as its value increases, such as cost, a negative sign should be attached to the outcome. In doing so, the calculations of the EOL by Eqs. (3-4a) and (3-6) and its properties summarized in Appendix-A would be applicable and valid.

3.3 Minimax EOL Rule for Ranking of Alternatives and Feasibility Testing

Referring to the decision making framework, as shown in Figure 2.1, the procedure for the ranking of alternatives (i.e., step (4)) based on EOL is described in this section.

After the values of EOL for all alternative pairs are calculated, an EOL matrix can be established by the EOL values for each pair of competing alternatives in a pair-wise comparison. From the EOL matrix, Savage’s minimax regret principle can be generalized and used for the ranking and selection of alternatives. The maximum value for each alternative,
\[ EOL_{\text{max}}(A^*_i) = \max_{j<l} \{ EOL(A^*_i, A_j) \} \]  \hspace{1cm} (3-9)

is used to represent the merit of the chosen alternative \( A^*_i \). Then, by the minimax \( EOL \) rule, the optimal alternative among all alternatives considered, \( A_{\text{opt}} \), is the one with the lowest maximum \( EOL \), that is,

\[ A_{\text{opt}} \in \arg \min_{j<l} \{ EOL_{\text{max}}(A^*_i) \}, \]  \hspace{1cm} (3-10)

According to the value of the maximum \( EOL \) for each of the \( M \) alternatives — that is, \( EOL_{\text{max}}(A^*_i) \), for \( i = 1, 2, \ldots, N \) — one can rank the relative preference of all alternatives as

\[ A_{(1)} \succeq A_{(2)} \succeq \ldots \succeq A_{(N)} \]  \hspace{1cm} (3-11)

where \( A_{(i)} \succ A_{(j)} \) means the \( i \)-th ranked alternative is preferred over or is indifferent with the \( j \)-th ranked alternative since \( EOL_{\text{max}}(A_{(i)}^*) \leq EOL_{\text{max}}(A_{(j)}^*) \) with subscript (1) representing the best alternative. The indifference of two alternatives holds when \( EOL_{\text{max}}(A_{(i)}^*) = EOL_{\text{max}}(A_{(j)}^*) \).

Once the alternative ranking is done, the feasibility of implementing an alternative should be examined. As \( EOL \) is a quantitative indicator of the potential loss for a decision when its outcome turns out to be not as expected, its meaning is commensurable with the notion of the acceptable risk (\( AR \)) of a decision maker. Therefore, the \( AR \) can be regarded as a threshold level of \( EOL \) for the feasibility test of step (5) in Figure 2.1. If the maximum \( EOL \) associated with a particular alternative is lower than the \( AR \), this alternative is considered implementable since the associated potential loss due to making the wrong decision is tolerable. Otherwise, implementing the alternative may not be prudent because the associated potential loss could be beyond the decision maker’s or stakeholder’s capacity to absorb. After examining the practicability of all alternatives, a set of feasible alternatives that meets the level of \( AR \) can be identified along with their rankings. The first-ranked feasible alternative can be chosen for implementation. If there is no feasible alternative with respect to \( AR \), the decision maker can take one or more of the following actions [53]: (1) delay the decision until more information is obtained; (2) reduce uncertainty by conducting further research or buying more information; (3) compromise with the first-ranked alternative and increase the budgetary
reserves for the contingency; (4) modify current alternatives or generate a new alternative set and repeat the decision analysis; and (5) be conservative in selecting the alternative with the best outcome in the worst situation; that is, the maximin rule.

Consider the previous hypothetical example presented in Section 2.3.2 in which discrete scenarios are assumed to be mutually exclusive with an equal probability of occurrence. The calculated $EOL$ matrix is shown in Table 3.1, in which each element is based on a pair-wise comparison in terms of their outcomes. The value of $EOL$ for a chosen alternative $A_i$ compared with $A_j$ is computed by determining the summation of the product of the random outcome difference ($\delta_{i,j}$) and the probability for each possible scenario. The diagonal elements have no loss because the chosen alternative is compared to itself. The maximum $EOL$ value of each alternative is listed in the last column of Table 3.1, which indicates that alternative $A_2$ has the lowest maximum $EOL$ and hence is the optimum alternative.

<table>
<thead>
<tr>
<th>Chosen alternatives $A_i$</th>
<th>Competing alternatives $A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
<th>Maximum $EOL$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0</td>
<td>5/3</td>
<td>7/3</td>
<td>2</td>
<td>2.33</td>
</tr>
<tr>
<td>$A_2$</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>2/3</td>
<td>$2.00$</td>
</tr>
<tr>
<td>$A_3$</td>
<td>13/3</td>
<td>8/3</td>
<td>0</td>
<td>0</td>
<td>4.33</td>
</tr>
<tr>
<td>$A_4$</td>
<td>6</td>
<td>13/3</td>
<td>2</td>
<td>0</td>
<td>6.00</td>
</tr>
</tbody>
</table>

### 3.4 Comparison of Risk Measures

In this section, the properties of two related risk measures developed earlier: Xu’s risk
Chapter 3  Risk-based decision making using EOL

measure (XRM) [49] and the conditional risk measure (CRM) [50], along with the EOL, are investigated.

The XRM corresponding to the decision of selecting alternative $A_i$ instead of $A_j$ is defined as [49]

$$XRM \left( A_i^*, A_j \right) = (\mu_i - \mu_j) \times P_r \left( X_i < X_j \right)$$

(3-12)

Therefore, $XRM(A_i^*, A_j)$ can be viewed as “the risk of obtaining an unacceptable ranking for the pair of management alternatives under consideration” [49].

Anticipating the outcome of the unselected alternative $A_j$ being better than the chosen alternative $A_i$, the potential loss that would be incurred can be expressed as a conditional expected loss. The CRM associated with a chosen alternative $A_i$ over $A_j$ is defined as [50]

$$CRM \left( A_i^*, A_j \right) = E \left[ X_j - X_i \mid X_i < X_j \right]$$

$$= - \frac{\int_{-\infty}^{\mu_j} \left[ \int_{x_i}^{\infty} (x - x_i) f_{i,j} (x_i, x_j) \, dx_j \right] dx_i}{\int_{-\infty}^{\mu_j} \left[ \int_{x_i}^{\infty} f_{i,j} (x_i, x_j) \, dx_i \right] dx_j}$$

(3-13)

The negative sign introduced on the right-hand side is to make the loss positive-valued. The CRM represents the expected loss anticipating that the decision turns out to be wrong.

For clarity, a simplified case of a decision making problem involving only one out of two alternative outcomes is random. Then, the analysis is made in the case where both alternatives produce uncertain outcomes.

3.4.1  1-D Risk-based Decision Making

Consider two alternatives $A_i$ and $A_j$ in which the chosen alternative $A_i$ has a deterministic outcome $x_i^*$, whereas the other alternative $A_j$ has an uncertain outcome $X_j$ having a probability distribution $f_j(x_j)$ with a mean $\mu_j$ and standard deviation $\sigma_j$ (see Figure 3.2).
In this investigation, the values of the three risk measures are calculated by changing the ratio between the constants $x_i^*$ and $\mu_j$. Suppose that a random outcome $X_j$ of alternative $A_j$ has a normal distribution with $\mu_j = 307$ and $\sigma_j = 25$. The curves of the three risk measures over the range of $0.8 < x_i^*/\mu_j < 1.2$ are shown in Figure 3.3. The value of $x_i^*/\mu_j$ is used to define the upper horizontal axis. To the left of the center line ($x_i^*/\mu_j < 1$), alternative $A_j$ is chosen due to its higher expected outcome. On the right-hand side of the center line, alternative $A_i$, with a deterministic outcome, is preferred and chosen.

- **EOL** – Referring to Figure 3.2, the mathematical expression of EOL in selecting alternative $A_i$ can be reduced to a single integration as

$$EOL(A_i^*, A_j) = -\int_{x_{i^*}}^{\infty} (x_{i^*} - x_j) f_j(x_j) dx_j$$  \hspace{1cm} (3-14)

As $x_i^*/\mu_j$ increases, the value of $EOL(A_i^*, A_j)$ continues to decrease with a diminishing slope. In other words, the value of $EOL(A_i^*, A_j)$ decreases with increasing $x_i^*$ when
choosing $A_i$, just the same as $EOL(A_i, A_j^*)$ decreases with increasing $\mu_j$ when choosing $A_j$. This behavior agrees intuitively that choosing an alternative with relatively larger expected outcome will lead to a decrease in potential risk and expected loss.

**Figure 3.3 Comparison of XRM, CRM, and EOL**

- **XRM** – As shown in Figure 3.2, the XRM associated with selecting alternative $A_i$ for $x_i*/\mu_j > 1$ can be computed by

$$XRM \left( A_i^*, A_j \right) = (x_i* - \mu_j) \times P_r \left( x_i* \leq X_j \right)$$

(3-15)

It is interesting to note that the XRM value, shown in Figure 3.3, does not change monotonically with $x_i*/\mu_j$ on either side of the center line. It consists of two
components: the mean difference of the two alternative outcomes and the probability of having an undesirable outcome corresponding to the chosen alternative. Figure 3.4(a) shows the variation of these two components with respect to \( x_i^*/\mu_j \). When the mean outcomes of the two alternatives are identical \((x_i^*/\mu_j = 1)\), the probability of \( x_i^*<X_j \) is at its highest level of 50%. However, the XRM has a minimum value of zero due to the zero mean difference in this case. In this sense, the influence of the probability of a wrong decision on the potential loss cannot be properly accounted for because it is masked by the zero difference in the long-term mean outcomes. As there is a 50-50 chance that one alternative can outperform the other when \( x_i^*/\mu_j = 1 \), making a correct selection would be the most difficult, so that the value of the risk associated with making the wrong decision should be the highest rather than zero.

- **XRM vs EOL** – Referring to Figure 3.3, there are two intersections of the XRM and EOL curves on either side of \( x_i^*/\mu_j = 1 \). EOL will result in a more conservative loss figure than XRM when \( x_i^*/\mu_j \) is in the range of these two points (i.e. \( r_1 < x_i^*/\mu_j < r_2 \)). The two intersection points can be computed by using the lower partial mean \[160\). The two intersection points of XRM and EOL curves on either side of \( x_i^*/\mu_j = 1 \) satisfy the following equations

(i) For \( x_i^*/\mu_j \geq 1 \)

\[
(2x_i^* - \mu_j) \cdot [1 - F_j(x_i^*)] = \mu_j - \mu_{d(X_j,x_r)}
\]

(ii) For \( x_i^*/\mu_j \leq 1 \)

\[
(2x_i^* - \mu_j) \cdot F_j(x_i^*) = \mu_{d(X_j,x_r)}
\]

where \( F_j(*) \) is the cumulative probability distribution (CDF); and \( \mu_{d(X_j,x_r)} \) is the lower partial mean of \( X_j \) over the range of \([\infty, x_i^*] \) defined by Buck and Askin \[160\]

\[
\mu_{d(X_j,x_r)} = \int_{-\infty}^{x_i^*} x_j f_j(x_j) dx_j
\]
When the probability distribution of $X_j$ is normal, the intersection point $r_1$ for $x_{i^*}/\mu_j \geq 1$ can be determined by solving

$$\frac{\phi(z_1)}{1 - \Phi(z_1)} - 2(z_1) = 0 \quad (3-18a)$$

where $z_1 = (r_1 - 1)/\Omega_j$ with $\Omega_j = \sigma_j/\mu_j$ and $r_1$ being the magnitudes of $x_{i^*}/\mu_j$: $\phi(\cdot)$ and $\Phi(\cdot)$ represent the standard normal PDF and CDF, respectively. Due to the symmetry of normal distribution, the intersection point $r_2$ for $x_{i^*}/\mu_j \leq 1$ can be obtained from solving

$$\frac{\phi(z_2)}{\Phi(z_2)} + 2(z_2) = 0 \quad (3-18b)$$

Detailed derivations of Eqs.(3-18a) and (3-18b) can be found in Appendix-B.

When $r_1 < x_{i^*}/\mu_j < r_2$, the value of EOL is higher than XRM. This is because the value of XRM is primarily influenced by a relative smaller difference in the mean outcomes, $x_{i^*} - \mu_j$, with little effect of a higher likelihood that the chosen alternative could be incorrect. For example, when the outcomes of the two alternatives are equal, the decision in choosing any alternative would have a probability of 50% being wrong. In this situation, the influence of this probability on the potential loss cannot be properly accounted for because it is masked by the zero difference in long-term mean outcomes. When the decision is likely to affect the well-being of the public, the decision maker generally tends to be more cautious in trying to avoid mistakes, but XRM will undervalue the potential risk in this situation.

On the other hand, when $x_{i^*}/\mu_j < r_1$ or $x_{i^*}/\mu_j > r_2$, the dominance of the difference in long-term mean outcomes by the XRM results in higher values than the EOL. Hence, the latter reflects more accurately the influence of the small probability of getting an undesirable loss. It means that, in this situation, a decision maker who adopts XRM to evaluate potential risk might have to prepare for more contingency to cover any potential loss in case the decision turns out to be wrong.
Figure 3.4 Decomposition graph of XRM

(a) The values of $x_i^* - \mu_j$ and $Pr(X_j > X_i)$ against $x_i^*/\mu_j$

(b) The XRM values against $x_i^*/\mu_j$
- **CRM** – The CRM associated with selecting alternative \( A_i \) in this 1-D decision making problem can be computed by

\[
CRM\left( A^*, A_j \right) = \frac{x_{i^*} - \int_{x_j}^{\infty} x_j f_j(x_j) \, dx_j}{\int_{x_j}^{\infty} f_j(x_j) \, dx_j}
\] 

(3-19)

In Figure 3.3, the two corresponding curves have similar shapes but the value of CRM is higher than that of EOL. This is because the CRM is the EOL rescaled by a factor representing the probability that the outcome of the unselected alternative \( X_j \) exceeds that of the chosen \( x_{i^*} \). Hence, there is an inherent pessimism built in CRM because the decision maker anticipates that the decision would turn out to be erroneous.

### 3.4.2 2-D Risk-based Decision Making

Consider the situation where the outcomes of both alternatives are uncertain. The effects of the relative mean and uncertainty in outcome on the risk measures are investigated in this section. The outcomes \( X_i \) and \( X_j \) are assumed to be independently and normally distributed. The relative mean and standard deviation of \( X_i \) and \( X_j \) are denoted by \( R_\mu = \mu_i / \mu_j \) and \( R_\sigma = \sigma_i / \sigma_j \), respectively. In the following discussion, the mean and standard deviation of \( X_j \) are kept fixed with \( \mu_j = 307 \) and \( \sigma_j = 25 \). \( \mu_i \) and \( \sigma_i \) are changed according to \( \mu_j \) and \( \sigma_j \), respectively.

Figures 3.5(a) and 3.6(a) show the contours of XRM and EOL, respectively, for different combinations of \( R_\mu \) and \( R_\sigma \). Figures 3.5(b) and 3.6(b) show the variation of the two risk measures with increasing \( R_\mu \) under various \( R_\sigma \). Figure 3.5(b) reveals a similar shape of the XRM curve as Figure 3.4(b) under various \( R_\sigma \). As shown in Figure 3.6(b), the value of the EOL associated with choosing alternative \( A_i \) decreases monotonically and approaches zero with an increase in \( R_\mu \) in a fixed \( R_\sigma \). These figures show that the level of outcome uncertainty has a great influence on the values of XRM and EOL. Generally, for a fixed \( R_\mu \), the values of XRM and EOL increase with an increase in standard ratio \( R_\sigma \).
Figure 3.5 Effect of $R_{\mu}$ and $R_{\sigma}$ on XRM

(a) Top. The XRM contour
(b) Bottom. The XRM curves of different standard deviation ratio

Mean ratio ($R_{\mu} = \mu_i / \mu_j$)

Standard deviation ratio ($R_{\sigma} = \sigma_i / \sigma_j$)

$R_{\sigma} = 0.5$
$R_{\sigma} = 1.0$
$R_{\sigma} = 1.5$
$R_{\sigma} = 2.0$
$R_{\sigma} = 2.5$
$R_{\sigma} = 3.0$
Figure 3.6 Effect of $R_\mu$ and $R_\sigma$ on $EOL$

(a) **Top.** The $EOL$ contour

(b) **Bottom** The $EOL$ curves of different standard deviation ratio
With the general relations of $EOL$ and $(R_\mu, R_\sigma)$ as shown in Figure 3.6, it can be used to facilitate alternative evaluation in the form of pair-wise comparisons. For illustration, let’s consider three alternatives whose outcomes are random, with $\mu_1 > \mu_2 > \mu_3 > 0$ and $\sigma_1 < \sigma_2 < \sigma_3$. Schematically, the positions of $R_\mu$ and $R_\sigma$ in Figure 3.7 corresponding to decisions $[A_1^*, A_3]$ and $[A_2^*, A_3]$ are shown at points I and II, respectively. The value of $EOL$ in choosing $A_1$ over $A_3$, $EOL(A_1^*, A_3)$, is less than that of choosing $A_2$ over $A_3$, $EOL(A_2^*, A_3)$. It is the case when comparing the $EOL$ values of two different chosen alternatives (i.e., $A_1^*$ and $A_2^*$) with respect to the same competitor ($A_3$), where the expected loss associated with choosing an alternative with greater uncertainty ($A_2$) will be higher. On the other hand, when considering two pair-wise comparisons for $[A_1^*, A_3]$ and $[A_1^*, A_2]$, the positions of $R_\mu$ and $R_\sigma$ corresponding to the two pairs again are indicated by I and II, respectively. Hence, the value of $EOL(A_1^*, A_2)$, will be higher than $EOL(A_1^*, A_3)$.

Figure 3.7 Schematic diagram showing the use of $R_\mu$ and $R_\sigma$ in assessing relative magnitude of $EOL$ considering different alternative pairs

![Figure 3.7](image-url)
3.5 Effect of Correlations on EOL

As mentioned earlier, in water resources planning and management, the outcomes of different management alternatives are often correlated through the sharing of some common hydrologic and hydraulic parameters. For example, in a flood control system design problem, the project cost and benefit of different alternative designs can be affected by common factors such as rainfall, flow hydrograph, channel geometry, boundary conditions, and topographical and land use features of the susceptible inundation area. Hence, the outcomes of these alternative designs, to some degree, are correlated. Such correlation might affect the value of the risk measure adopted which, in turn, affects the relative merits of the two alternatives and the decision maker’s preference.

For clarity, consider two correlated net benefits $X_i$ and $X_j$ having a bivariate normal distribution $f_{i,j}(x_i, x_j)$. Figure 3.8(a)-(b) reveal that the correlations among the alternative outcomes can have a great influence on the EOL, especially when the selected alternative has an expected outcome similar to its competitor. For the decision of choosing alternative $A_i$, defining the means ratio $R_{\mu} = \mu_i/\mu_j$ and the standard deviation ratio $R_\sigma = \sigma_i/\sigma_j$, Figure 3.8(a) shows the effect of the correlation on the EOL($A_i^*, A_j$) under a fixed value of standard deviation ratio $R_\sigma$. In this Figure, each point represents the relative merit of choosing alternative $A_i$ from a pair of alternatives [$A_i, A_j$]. The mean ($\mu_j$) and standard deviation ($\sigma_j$) of alternative $A_j$ are both set to be one. The expected outcome in choosing alternative $A_i$ is $\mu_i = R_\mu \mu_j$ and ranges from 1.0 to 8.0. Set $R_\sigma = 2$ as a constant, and $\sigma_i$ will be equal to 2. It shows that when comparing two correlated alternatives, a higher positive correlation results in a lower value of the EOL but a higher negative correlation results in a larger EOL (i.e. $\text{EOL}(\rho > 0) < \text{EOL}(\rho < 0)$). This can be explained by the fact that the EOL represents the probability weighted outcome differences between two alternatives in the joint PDF region within the domain of $x_i < x_j$. Figure 3.9 shows the contours of the standardized bivariate normal distribution under two situations for $\rho_{i,j} = \pm0.6$ in which the diagonal line represents the two standardized random variables $x'_i$ and $x'_j$ that are identical with the upper half of the domain for $x'_i < x'_j$. For different correlations and fixed $\mu_i$ and $\mu_i$, the volumes in the standardized domain of $x'_i \leq x'_j$ under the joint PDF are equivalent. However, under the condition of $\rho_{i,j}$
=−0.6, there are higher probabilities for larger values of \(|x'_i−x'_j|\) than the probabilities for the differences in \(\rho_{ij} = +0.6\). This observation in the standardized domain is also valid in the original variable scale and explains why a higher positive correlation would result in a lower value of \(EOL\).

Figure 3.8(b) shows that once \(R_\mu\) becomes larger, the effect of correlation on the \(EOL(A_i^*, A_j)\) decreases and diminishes. Thus, considering the decision maker’s acceptable risk, the rankings between multiple alternatives could vary with the existence of correlated outcomes. The percentage difference of \(EOL\) increases with \(R_\mu\) is shown in Figure 3.8(c). Figure 3.9 can be also used to explain the behavior of the diminishing \(EOL\) value with increasing mean ratio \(R_\mu\). When \(R_\mu\) increases, the joint PDF contours will move to the right and/or downward as \((\mu_i−\mu_j)\) increases. This will result in a decrease in the probability volume in the \(x'_i < x'_j\) domain and consequent a decrease in the \(EOL(A_i^*, A_j)\) values.

### 3.6 Formulation of \(m\)th-order \(EOL\)

#### 3.6.1 Generalized \(EOL\) in \(m\)-th Order Loss Function (\(EOL_m\))

A loss function describes the perception of the outcome loss to the stakeholders under a given state of nature and for a particular decision. In general, the form of the loss function reflects the relative importance of the error committed from a choice under a particular state of nature [48]. If the utility function is specified, the loss function can also be defined by the difference in utility values to reflect the decision maker’s risk attitude toward the loss. Herein, the loss of concern is a function of the random outcome difference \(L(A_i^*,A_j)\) for a chosen alternative \(A_i^*\) with reference to an unselected \(A_j\) under a given state of nature. The expectation of opportunity loss and gain can be expressed, respectively, in a general form as

\[
EOL(A_i^*, A_j) = \int_{-\infty}^{0} L(\delta) \cdot f_{A_i^*, A_j}(\delta) \ d\delta
\]

\[
EOG(A_i^*, A_j) = \int_{0}^{\infty} L(\delta) \cdot f_{A_i^*, A_j}(\delta) \ d\delta
\]

(3-20a)

(3-20b)
where $L(\delta)$ is the loss function for a given value of $\delta = x_i - x_j$. Eq.(3-4a) is a special case of Eq.(3-20) in which $L(A_{i*,j}) = -A_{i*,j}$.

Figure 3.8 Effect of correlation on the EOL under $R_\sigma = 2$

(a) Top. Illustrative diagram of EOL of the choice between two correlated or uncorrelated competing alternatives.
(b) Bottom left. Difference of EOL of the choice between two correlated or uncorrelated competing alternatives.
(c) Bottom right. Percentage difference of EOL of the choice between two correlated or uncorrelated competing alternatives.

Note: $R_{\mu} = \mu_i / \mu_j$; $R_{\sigma} = \sigma_i / \sigma_j$; $\Delta EOL = (EOL_\rho - EOL_{\rho=0.0})$;
$\Delta EOL\% = (EOL_\rho - EOL_{\rho=0.0}) / EOL_{\rho=0.0}$
Figure 3.9 Bivariate standard normal distribution

(a) Top left. Contour when $\rho = +0.6$
(b) Top right. Contour when $\rho = -0.6$
(c) Bottom left. Density for $X_i < X_j$ when $\rho = +0.6$
(d) Bottom right. Density for $X_i < X_j$ when $\rho = -0.6$
The value of $EOL(A_i^*, A_j)$ in Eq.(3-20) can be calculated numerically with the specified loss function $L(A_i^*, j)$ and the probability distribution of the random outcome difference $A_i^*, j$. This section focuses on a special type of loss function in power form in $A_l^*, j$, namely, the $m^{th}$-order loss function of $A_i^*, j$ as $L(A_i^*, j) = (A_i^*, j)^m$. The $EOL$ associated with the $m^{th}$-order loss function ($EOL_m$) in choosing alternative $A_i$ over $A_j$ can be defined as

$$EOL_m(A_i^*, A_j) = \int_{-\infty}^{0} (-\delta)^m \cdot f_{A_i^*, j} (\delta) d\delta$$

(3-21)

where $EOL_m(A_i^*, A_j)$ is the $m^{th}$-order $EOL$. It is analogous to the $m^{th}$-order lower partial moment ($LPM_m$) of $A_i^*, j$, which has been used by Bawa and Lindenberg [161] as a risk measure defined only on a return smaller than the target return as

$$LPM_m(h, R) = \int_{-\infty}^{h} (h-r)^m f_R (r) dr$$

(3-22)

where $R$ is the random return; $h$ is the target return; and $f_R(r)$ represents the PDF of $R$. When $m = 0$, Eq.(3-21) reduces to the safety-first criterion; and when $m = 2$, it is analogous to the semi-variance of Markowitz [162]. The first and second lower partial moments for several commonly used probability distribution functions were derived by Buck and Askin [160].

3.6.2 Analytical Expressions of $EOL_m$

To evaluate the effect of loss functions with different orders of $A_i^*, j$ on the value of $EOL_m$, as well as on the ranking of several competing alternatives, the analytical expression of $EOL_m$ is derived under a normally distributed $A_i^*, j$ for $m = 0, 1, \ldots, 4$. A normally distributed $A_i^*, j$ is used because the sum of two normally distributed random variables remains normal. Furthermore, to facilitate the evaluation of $EOL_m$ for non-integer $m$, the results of $EOL_m$ from integer-valued $m$ are used to establish empirical relations with $m$ through curve-fitting, and are described later in this section.

The analytical expressions for $EOL_m$ in Eq.(3-21), if they can be derived, are able to provide efficient calculation of the risk measure. It is the $m^{th}$-order lower partial moment of a
random variable $\Delta_{i,j}$ for $-\infty \leq \Delta_{i,j} \leq 0$. Under the condition of normality, random $\Delta_{i,j}$ values with mean $\mu_{\Delta}$ and variance $\sigma_{\Delta}^2$ can be obtained by Eqs.(3-2a) and (3-2b).

The expressions for the first four orders of $EOL_m$, $0 \leq m \leq 4$, are derived and shown in Table 3.2. Analytical derivation of the normal-based $EOL_m$ is summarized in Appendix-C. Table 3.2 can be used to compute the EOL of two competing risky alternatives with normally distributed random outcomes. It shows that the $EOL_m$ are functions of $\mu_{\Delta}$ and $\sigma_{\Delta}$, which are related to $\mu_i$, $\mu_j$, $\sigma_i$, $\sigma_j$, and $\rho_{ij}$.

<table>
<thead>
<tr>
<th>$m$</th>
<th>$EOL_m = E\left[\Delta_{i,j}, \Delta_{i,j} &lt; 0\right]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\mu_{\Delta} \Phi(\beta) - \sigma_{\Delta} \phi(\beta)$</td>
</tr>
<tr>
<td>2</td>
<td>$(\mu_{\Delta}^2 + \sigma_{\Delta}^2) \Phi(\beta) - 2\mu_{\Delta} \sigma_{\Delta} \phi(\beta) - \beta \sigma_{\Delta}^2 \phi(\beta)$</td>
</tr>
<tr>
<td>3</td>
<td>$(\mu_{\Delta}^3 + 3\mu_{\Delta} \sigma_{\Delta}^2) \Phi(\beta) - 3\sigma_{\Delta} (\mu_{\Delta}^2 + \sigma_{\Delta}^2) \phi(\beta) - 3\beta \mu_{\Delta} \sigma_{\Delta} \phi(\beta) - (\beta^2 - 1) \sigma_{\Delta}^3 \phi(\beta)$</td>
</tr>
<tr>
<td>4</td>
<td>$(\mu_{\Delta}^4 + 6\sigma_{\Delta}^2 \mu_{\Delta}^2 + 3\sigma_{\Delta}^4) \Phi(\beta) - 4\sigma_{\Delta} (\mu_{\Delta}^3 + 3\sigma_{\Delta}^2 \mu_{\Delta}) \phi(\beta)$ $- 6\beta \sigma_{\Delta}^2 \left(\mu_{\Delta}^2 + \sigma_{\Delta}^2\right) \phi(\beta) - 4\left(\beta^2 - 1\right) \mu_{\Delta} \sigma_{\Delta} \phi(\beta) - (\beta^3 - 3\beta) \sigma_{\Delta}^4 \phi(\beta)$</td>
</tr>
<tr>
<td>5</td>
<td>$(\mu_{\Delta}^5 + 10\mu_{\Delta}^3 \sigma_{\Delta}^2 + 15\mu_{\Delta} \sigma_{\Delta}^4) \Phi(\beta) - 5\sigma_{\Delta} (\mu_{\Delta}^4 + 6\mu_{\Delta}^2 \sigma_{\Delta}^2 + 3\sigma_{\Delta}^4) \phi(\beta)$ $- 10\beta \sigma_{\Delta}^2 \left(\mu_{\Delta}^3 + 3\mu_{\Delta} \sigma_{\Delta}^2\right) \phi(\beta) - 10\left(\beta^2 - 1\right) \sigma_{\Delta}^3 \left(\mu_{\Delta}^2 + \sigma_{\Delta}^2\right) \phi(\beta) - 5 \left(\beta^3 - 3\beta\right) \mu_{\Delta} \sigma_{\Delta}^3 \phi(\beta) - \left(\beta^4 - 6\beta^2 + 3\right) \sigma_{\Delta}^5 \phi(\beta)$</td>
</tr>
</tbody>
</table>

Note: $\beta = -\mu_{\Delta} / \sigma_{\Delta}$
3.6.3 Normalized $EOL_m$

Analytical expressions for $EOL_m$ in Table 3.2 can be normalized by defining $EOL_m' = EOL_m / \sigma^K_m$ as shown in Table 3.3. The normalized $EOL_m$ is a function of $\beta (= -\mu_{\Delta} / \sigma_{\Delta})$ and the relations between $EOL_m'$ and $\beta$ are shown in Figure 3.10. Figure 3.10 illustrates the variation in magnitude of $|EOL_m'|$ as the order of loss function $m$ changes from zero to 4. It reveals that the values of $|EOL_m'|$ increase monotonically with an increase in $\beta$. In other words, the relative merit in terms of $EOL_m'$ for a particular chosen alternative depends on the rates of growth and decline of $\mu_{\Delta}$ and $\sigma_{\Delta}$, by way of pair-wise comparison. It is expected to influence the ranking of alternatives on the basis of the minimax $EOL$ criterion. The sensitivity of $EOL_m'$ with respect to $\beta$ gets larger when $m$ increases.

3.6.4 Approximated $EOL_m$

To facilitate the evaluation of $EOL_m'$ for a noninteger $m$, empirical relationships for $EOL_m'$ with $m$ and $\beta$ are established from the results of an integer-valued $m$. Through the curve-fitting process, appropriate models approximating the $EOL_m' - \beta$ relationship for a fixed order $m$, $0 \leq m \leq 4$, are established as follows:

- For $-3 \leq \beta \leq 0$,
  \[ \ln(EOL_m') = a_m + b_m \beta + c_m \beta^2 + d_m \beta^3 \]  \hspace{1cm} (3-23a)

- For $0 \leq \beta \leq 3$,
  \[ \ln(EOL_m') = a_m + b_m \beta + c_m \beta^2 + d_m \beta^3 + e_m \beta^4 \]  \hspace{1cm} (3-23b)

where the values of the model coefficients in Eqs. (3-23a) and (3-23b) are listed in Table 3.4(a) and 3.4(b), respectively. The range of $\beta$, $-3 \leq \beta \leq 3$, corresponds to the range of the coefficient of variation for $\Delta_{i,j}$ from $-0.3$ (for $\mu_i < \mu_j$) to $0.3$ (for $\mu_i > \mu_j$). Figure 3.11 and Figure 3.12 show that the results of curve fitting are sufficiently accurate. The last columns in Table 3.4(a) and 3.4(b) show that the maximum percentage errors were less than 7.3 and 1.9% for Eqs. (3-23a) and (3-23b), respectively, and occur when $m = 4$. When $m$ is not an
integer in $0 \leq m \leq 4$, the value of $EOL_m$ can be determined by using Eq.(3-23a) or (3-23b) by interpolating the coefficients listed in Table 3.4(a) or Table 3.4(b).

<table>
<thead>
<tr>
<th>$m$</th>
<th>$EOL_m' = EOL_m / \sigma^m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-\beta \Phi(\beta) - \phi(\beta)$</td>
</tr>
<tr>
<td>2</td>
<td>$(\beta^2 + 1)\Phi(\beta) + \beta\phi(\beta)$</td>
</tr>
<tr>
<td>3</td>
<td>$-(\beta^3 + 3\beta)\Phi(\beta) - (\beta^2 + 2)\phi(\beta)$</td>
</tr>
<tr>
<td>4</td>
<td>$(\beta^4 + 6\beta^2 + 3)\Phi(\beta) + (\beta^3 + 5\beta)\phi(\beta)$</td>
</tr>
</tbody>
</table>

Note: $\beta = -\mu_\Delta / \sigma_\Delta$

Figure 3.10 Effect of $\beta$ and $m$ on dimensionless $EOL_m'$

Note: $\mu^*_i > \mu^*_j \iff \mu^*_i < \mu$
Table 3.4 Fitted coefficients, adjusted $R^2$, and maximum absolute errors for $|EOL_m|$

(a) By Eq.(3-24a) for $-3 \leq \beta \leq 0$

<table>
<thead>
<tr>
<th>$m$</th>
<th>$a_m$</th>
<th>$b_m$</th>
<th>$c_m$</th>
<th>$d_m$</th>
<th>$Adj R^2$</th>
<th>Max. abs. errors</th>
<th>Max. error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.6937</td>
<td>0.7932</td>
<td>-0.3312</td>
<td>0.0223</td>
<td>0.9999986</td>
<td>0.00045</td>
<td>4.81</td>
</tr>
<tr>
<td>1</td>
<td>-0.9195</td>
<td>1.2494</td>
<td>-0.2951</td>
<td>0.0213</td>
<td>0.9999979</td>
<td>0.00041</td>
<td>2.98</td>
</tr>
<tr>
<td>2</td>
<td>-0.6935</td>
<td>1.5956</td>
<td>-0.2736</td>
<td>0.0226</td>
<td>0.9999989</td>
<td>0.00040</td>
<td>4.90</td>
</tr>
<tr>
<td>3</td>
<td>-0.2259</td>
<td>1.8790</td>
<td>-0.2679</td>
<td>0.0206</td>
<td>0.9999994</td>
<td>0.00047</td>
<td>4.96</td>
</tr>
<tr>
<td>4</td>
<td>0.4060</td>
<td>2.1333</td>
<td>-0.2537</td>
<td>0.0228</td>
<td>0.9999975</td>
<td>0.00349</td>
<td>7.29</td>
</tr>
</tbody>
</table>

(b) By Eq.(3-24b) for $0 \leq \beta \leq 3$

<table>
<thead>
<tr>
<th>$m$</th>
<th>$a_m$</th>
<th>$b_m$</th>
<th>$c_m$</th>
<th>$d_m$</th>
<th>$e_m$</th>
<th>$Adj R^2$</th>
<th>Max. abs. errors</th>
<th>Max. error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.6948</td>
<td>0.8150</td>
<td>-0.3570</td>
<td>0.0679</td>
<td>-0.0046</td>
<td>0.999992</td>
<td>0.0009</td>
<td>0.17</td>
</tr>
<tr>
<td>1</td>
<td>-0.9201</td>
<td>1.2594</td>
<td>-0.2964</td>
<td>0.0393</td>
<td>-0.0019</td>
<td>0.999993</td>
<td>0.0051</td>
<td>0.35</td>
</tr>
<tr>
<td>2</td>
<td>-0.6934</td>
<td>1.5995</td>
<td>-0.2812</td>
<td>0.0314</td>
<td>-0.0015</td>
<td>0.999999</td>
<td>0.0050</td>
<td>0.32</td>
</tr>
<tr>
<td>3</td>
<td>-0.2253</td>
<td>1.8798</td>
<td>-0.2700</td>
<td>0.0252</td>
<td>-0.0010</td>
<td>0.999996</td>
<td>0.0507</td>
<td>0.30</td>
</tr>
<tr>
<td>4</td>
<td>0.3866</td>
<td>2.1899</td>
<td>-0.3288</td>
<td>0.0470</td>
<td>-0.0042</td>
<td>0.999995</td>
<td>0.3584</td>
<td>1.87</td>
</tr>
</tbody>
</table>
Figure 3.11 Curves for fitting $|EOL_m'|$ with respect to $-3 \leq \beta \leq 0$
by Eq.(3-23a) under different orders.

Note: The dashed and solid lines are analytical and fitted values of $|EOLm'|$, respectively.

(a) Top left. $m = 0$  
(b) Top right. $m = 1$

(c) Middle left. $m = 2$  
(d) Middle right. $m = 3$

(e) Bottom left. $m = 4$
Figure 3.12 Curves for fitting $|EOL_m'|$ with respect to $0 \leq \beta \leq 3$ by Eq.(3-24b) under different orders.

(a) $m = 0$  
(b) $m = 1$
(c) $m = 2$  
(d) $m = 3$
(e) $m = 4$

Note: The dashed and solid lines are analytical and fitted values of $|EOL_m'|$, respectively.
3.7 Illustrative Example: River Basin Management Decision Making Using the Proposed Minimax EOL Decision Rule

In this section, an example of a simplified river basin management study of the Elbe River in Germany from Xu and Tung [49] is used to illustrate the use and performance of the three risk measures: EOL, XRM [49], and CRM [50] whose properties are demonstrated and compared through this case study. Both risk measures of EOL and CRM share the same notion that a best alternative should have a minimum loss when the decision turns out to be wrong.

The two management objectives considered by the decision maker are: (1) to improve the navigability along the Elbe main channel; and (2) to preserve the ecological state of the floodplains. The outcomes associated with the management objectives, respectively, are the annual number of shipping days for navigation, and the diversity of biotypes in the Elbe River floodplains. The system outcomes were determined by the Elbe Decision Support System (DSS) developed by de Kok [163]. The inherent problem is a multiple criteria decision making considering the trade-off between two management objectives. For the purpose of demonstrating the decision rules, only the navigation aspect is considered in this example and the annual number of shipping days is the management outcome. An alternative with more shipping days in a year is more desirable.

The decision is to evaluate the relative merit among three alternatives with regard to the navigability. However, the complexity of this decision making problem results from the model output uncertainties, induced by the model inputs uncertainties, such as errors in the water levels and rating curves in the DSS. Hence, the annual number of shipping days is not certain and is assumed to follow a normal distribution. The means and standard deviations of the annual number of shipping days associated with the three management alternatives in the Elbe River example are shown in Table 3.5.

The values of the three risk measures (XRM, CRM, and EOL) with respect to the three different chosen alternatives are shown in Table 3.6 in the manner of pair-wise comparison. The values of CRM range from 23.49 to 44.35 days and that of EOL from 5.66 to 33.66 days.
Since the CRM is computed with the anticipation that the chosen alternative would be a failure, the CRM expectedly yields larger values of potential loss than the EOL. Thus, it is a more conservative risk measure and the decision maker needs to allow more contingency and be more tolerant to offset the potential loss of an anticipated failure that may not occur.

Table 3.5 Management alternatives and corresponding statistics of the annual number of shipping days in the Elbe River example (Adapted from Xu and Tung [49])

<table>
<thead>
<tr>
<th>Management alternatives</th>
<th>Descriptions of management alternatives</th>
<th>Mean (days)</th>
<th>Stdev (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>Original situation (Status-quo)</td>
<td>307</td>
<td>25</td>
</tr>
<tr>
<td>$A_2$</td>
<td>50% groyne removal</td>
<td>279</td>
<td>31</td>
</tr>
<tr>
<td>$A_3$</td>
<td>Re-naturalization, changing meadow grass and agriculture in the left bank to broad-leaved forest</td>
<td>306</td>
<td>26</td>
</tr>
</tbody>
</table>

The basic idea for alternative selection using EOL is to choose an alternative with lowest maximum EOL compared with all the other alternatives. In this case, $A_1$ is preferred to $A_3$ and $A_3$ is preferred to $A_2$. However, the implementability of the chosen alternative can only be determined with reference to the decision maker’s AR. A feasible alternative should have a maximum EOL value less than the acceptable risk when comparing all the other alternatives. After examining the implementability of all alternatives, a set of feasible alternatives that meet the decision maker’s acceptable risk can be identified. The best decision is alternative $A_1$ with lowest maximum EOL among the feasible alternatives.

According to XRM, if the decision maker chooses not to follow the ranking of $A_1 > A_3$ between the alternative pair [$A_1, A_3$], $XRM(A_1^*, A_3) = 0.49$ which is lower than that for the alternative pair [$A_1, A_2$] of $XRM(A_1^*, A_2) = 6.75$. In other words, the expected loss of not choosing $A_1$ over $A_3$ is higher than that of not choosing $A_1$ over $A_2$. Examining the situation
closely, the expected annual navigable days of alternatives \( A_1 \) and \( A_3 \) are very close and the \( XRM(A_1^*, A_3) \) cannot be regarded as a risk in choosing \( A_1 \). This scenario should present the most difficult situation in making a clear choice from one alternative to another, so that the value of a risk measure for making a wrong decision should be the highest rather than the lowest. Hence, the \( EOL \) is a plausible risk measure that reflects a more accurate picture of the merit associated with the decision of choosing a particular alternative.

Table 3.6 Values of three risk measures associated with pair-wise alternatives

\( (\rho = 0.0) \)

<table>
<thead>
<tr>
<th>Pair-wise comparison</th>
<th>Risk measures for navigability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( XRM ) (days)</td>
</tr>
<tr>
<td>( A_1 ) ( A_2 )</td>
<td>6.75</td>
</tr>
<tr>
<td>( A_1 ) ( A_3 )</td>
<td>0.49</td>
</tr>
<tr>
<td>( A_2 ) ( A_1 )</td>
<td>-</td>
</tr>
<tr>
<td>( A_2 ) ( A_3 )</td>
<td>-</td>
</tr>
<tr>
<td>( A_3 ) ( A_1 )</td>
<td>-</td>
</tr>
<tr>
<td>( A_3 ) ( A_2 )</td>
<td>6.81</td>
</tr>
</tbody>
</table>

3.7.1 Effect of Acceptable Risk

Based on the \( EOL \) values shown in Table 3.6, the results of alternative selection corresponding to different levels of acceptable risk \( (R_a) \) are listed in Table 3.7. As can be seen, \( R_a \) has great influence on the number of feasible alternative set. For example, if \( R_a = 14 \text{ days} \), the values of \( EOL(A_1^*, A_2) \) and \( EOL(A_1^*, A_3) \) are low enough to be acceptable and implementable. Consequently, alternative \( A_1 \) could eliminate both alternatives \( A_2 \) and \( A_3 \) and
choosing it would lead to a minimum and acceptable loss. In this condition, the status-quo \((A_1)\) is the most preferable choice in terms of the navigability. If the decision maker has a higher tolerance, say, \(R_A = 16\) days, then, both \(A_1\) and \(A_3\) are implementable because the values of \(EOL\) corresponding to their selection are smaller than the stipulated \(R_A\) of 16 days. In this case, alternative \(A_2\) can be discarded. With an even higher tolerance, say, \(R_A = 34\) days, all three alternatives are acceptable and implementable with respect to the decision maker’s \(R_A\). However, if \(R_A = 12\) days, none of the three alternatives can be considered implementable. In this case, the decision maker could take one of the following three courses of action [53]: (1) make a new list of alternatives and examine them; (2) conduct further investigations to reduce the outcome uncertainty; and (3) increase the budgetary reserves for the contingency to have a higher \(R_A\).

### Table 3.7 Effect of acceptable risk on the alternative selection based on \(EOL\) \((\rho = 0.0)\)

<table>
<thead>
<tr>
<th>Assumed (R_A)</th>
<th>Feasible alternatives</th>
<th>Alternatives cannot be discarded, nor implementable</th>
<th>Discarded alternatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_A = 12)</td>
<td>-</td>
<td>(A_1, A_3)</td>
<td>(A_2)</td>
</tr>
<tr>
<td>(R_A = 14)</td>
<td>(A_1)</td>
<td>-</td>
<td>(A_2, A_3)</td>
</tr>
<tr>
<td>(R_A = 16)</td>
<td>(A_1, A_3)</td>
<td>-</td>
<td>(A_2)</td>
</tr>
<tr>
<td>(R_A = 34)</td>
<td>(A_1, A_2, A_3)</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

#### 3.7.2 Effect of Outcome Uncertainty

When a large model uncertainty and/or low \(R_A\) hinder the decision maker from clearly identifying feasible alternatives, one might be interested in knowing how much reduction in uncertainty is needed to yield at least one feasible alternative among those under consideration.
Table 3.8 shows the effect of uncertainty reduction on the values of $EOL$. The standard deviations of the three alternatives are reduced by 30% individually, except the last row which involves simultaneous uncertainty reductions in two alternative outcomes. The mean and baseline standard deviation values remain unchanged, as in Table 3.5, and the alternative correlations are zero. The percentages of reduction in $EOL$ are shown in parentheses. It shows that when the standard deviation of $A_i$ is reduced, both $EOL(A_i^*, A_j)$ and $EOL(A_j^*, A_i)$ decrease. When $R_A = 5$ days, the decision maker’s tolerance to failure is too low to produce an acceptable alternative for implementation, no matter how small the uncertainty is reduced to. When the $R_A$ is increased to 10 days, $A_1$ can be selected as an ultimate alternative if $\sigma_1$ and $\sigma_3$ are both reduced by 30%. However, the degree of reduction in expected loss has its limits. For example, even if the uncertainty of $A_1$ and $A_3$ are reduced by 30%, $A_2$ and $A_3$ cannot be regarded as feasible with reference to $R_A = 10$ days. This indicates that there is a trade-off between the effort and effect in reducing the uncertainty.

### 3.7.3 Effect of Outcome Correlation

Table 3.9 shows the effect of correlation on the $EOL$ values of the three alternatives. The correlation of each pair of alternative outcomes ranges from $-0.6$ to $+0.6$ and the other statistics are identical to those in Table 3.5. Table 3.9 shows that the values of $EOL$ decrease with higher positive correlations. The reason why a higher positive correlation would result in a lower value of $EOL$ has been explained in Section 3.5. It is clear that the outcome correlation has a considerable effect on the value of $EOL$ which, in turn, would determine if a chosen alternative is feasible or not.
### Table 3.8 Effect of outcome uncertainty on the *EOL* ($\rho = 0.0$)

<table>
<thead>
<tr>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>$\sigma_3$</th>
<th>Changes in uncertainty</th>
<th>Values of <em>EOL</em> in pair-wise comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \sigma_1$</td>
<td>$\Delta \sigma_2$</td>
<td>$\Delta \sigma_3$</td>
<td>($A_1^*, A_2$)</td>
<td>($A_3^*, A_2$)</td>
</tr>
<tr>
<td>25</td>
<td>31</td>
<td>26</td>
<td>5.66</td>
<td>6.11</td>
</tr>
<tr>
<td>17.5</td>
<td>31</td>
<td>26</td>
<td>-30%</td>
<td>0%</td>
</tr>
<tr>
<td>25</td>
<td>21.7</td>
<td>26</td>
<td>0%</td>
<td>-30%</td>
</tr>
<tr>
<td>25</td>
<td>31</td>
<td>18.2</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>17.5</td>
<td>31</td>
<td>18.2</td>
<td>-30%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Note: The changes in uncertainty are expressed as percentages.
Table 3.9 Effect of correlation on EOL value

<table>
<thead>
<tr>
<th>Pair-wise comparison</th>
<th>Values of EOL</th>
<th>( \rho = -0.6 )</th>
<th>( \rho = -0.3 )</th>
<th>( \rho = 0.0 )</th>
<th>( \rho = +0.3 )</th>
<th>( \rho = +0.6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chosen alternatives</td>
<td>Competing</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>alternatives</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A_1 )</td>
<td>( A_2 )</td>
<td>9.05</td>
<td>7.41</td>
<td>5.66</td>
<td>3.77</td>
<td>1.78</td>
</tr>
<tr>
<td>( A_1 )</td>
<td>( A_3 )</td>
<td>17.70</td>
<td>15.91</td>
<td>13.90</td>
<td>11.55</td>
<td>8.61</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>( A_1 )</td>
<td>37.05</td>
<td>35.41</td>
<td>33.66</td>
<td>31.77</td>
<td>29.78</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>( A_3 )</td>
<td>36.64</td>
<td>34.94</td>
<td>33.11</td>
<td>31.12</td>
<td>28.98</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>( A_1 )</td>
<td>18.70</td>
<td>16.91</td>
<td>14.90</td>
<td>12.55</td>
<td>9.61</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>( A_2 )</td>
<td>9.64</td>
<td>7.94</td>
<td>6.11</td>
<td>4.12</td>
<td>1.98</td>
</tr>
</tbody>
</table>

3.8 Illustrative Example: Flood Control Decision Making

In this section, the proposed minimax EOL decision rule for continuous random alternative outcomes is compared with the conventional decision rules by a simplified decision problem extracted from a flood control study [9].

The study area is in a community of Tonsking of Maiden County known as Heck Valley that situated along the Heck River in northeastern Midstate. The existing flood control system was designed and completed in 1943 to protect the study area from flood magnitude of 232,000 ft³/s, which is estimated to be a 50-year flow. However, following the devastating flood in 1982, a post-flood study pointed out that it would be needed to increase the existing protection level. Three alternatives (reservoir, dredging and levee-raising) out of seven that passed the benefit-cost ratio test in the report of USACE [9] are considered herein. The data used involve the actual data from the U.S. Army Corps of Engineers project, and some fabricated data to facilitate uncertainty analysis. Considering the uncertainty of first costs and
annual benefits, the net benefit for each alternative is treated as continuous random variable. Their statistics associated with the three alternatives are listed in Table 3.10. The normal probability distribution is used for these random net benefits, and the corresponding PDFs are shown in Figure 3.13. The decision analysis is to evaluate the relative merits among the three alternatives with regard to the net benefits.

Under the condition of the continuous random net benefit, the conventional decision rules of the maximin, the maximax, Hurwicz criterion, and Savage’s minimax regret are not practicable without further assumptions. From a long-term perspective, the ranking order would be \( A_2, A_1, \) and then \( A_3 \) by the expected-value rule. From the M-V rule and the second-degree of stochastic dominance rules, \( A_2 \) is preferred over both \( A_1 \) and \( A_3 \) due to its highest average return and the smallest variability. However, the decision maker cannot make a clear distinction between the remaining two alternatives. When setting the threshold level of the net benefit as zero, the probabilities of having a negative net benefit are 0.50, 0.0 (rounded number), and 0.47 for alternative \( A_1 \), \( A_2 \), and \( A_3 \), respectively. Hence, by the probability-of-loss rule, alternative \( A_2 \) is preferred to \( A_3 \), and \( A_3 \) is preferred to \( A_1 \).

Table 3.10 Alternatives and corresponding statistics of the net benefits in Heck Valley flood control case study (after USACE [9])

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Descriptions of alternatives</th>
<th>Mean ($10^3$)</th>
<th>Stdev ($10^3$)</th>
<th>Ranking orders</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>Lake Floyd reservoir</td>
<td>201.4</td>
<td>27179.3</td>
<td>2 2 2 3</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>Channel dredging</td>
<td>662.0</td>
<td>156.1</td>
<td>1 1 1 1</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>Levee-raising</td>
<td>159.1</td>
<td>1981.1</td>
<td>3 2 2 2</td>
</tr>
</tbody>
</table>

\( E-V \) \( M-V \) \( SSD \) \( POL \)

\( E-V \): Expected value decision rule; \( M-V \): Mean-variance decision rule; \( SSD \): Second-order stochastic dominance decision rule; \( POL \): Probability-of-loss decision rule
Applying the proposed minimax $EOL$ decision rule, a random opportunity loss could incur when each of the alternative $A_i$ ($i = 1, 2, 3$) is chosen. The best feasible alternative is the one having the minimum largest $EOL$ while, at the same time, satisfying the decision maker’s acceptable risk. To evaluate the potential loss incurred from selecting one alternative, pair-wise comparisons with other possible competing alternatives are conducted. The values of $EOL$ are calculated for each comparison (assuming independence of alternative net benefits) and the results are presented in Table 3.11 by a matrix form with values ranging from $567,000$ to $11,100,000$. The table shows that the values of $EOL$ of choosing alternative $A_2$ comparing with the other two competitive alternatives are relatively smaller. The second column from the last shows the maximum $EOL$ of selecting each alternative listed in the first column. Observed from the last column of Table 3.11, alternative $A_2$ having the smallest maximum expected loss is the best, followed by alternative $A_3$. Alternative $A_1$ is ranked last due to its largest maximum $EOL$ even its expected net benefit is higher than $A_3$. Notice that
this ranking is not the same as the expected-value rule but the same as the probability-of-loss rule.

### Table 3.11 Expected opportunity loss matrix ($10^3$) and ranking of uncorrelated alternative outcomes by the minimax EOL decision rule

<table>
<thead>
<tr>
<th>Chosen alternatives</th>
<th>Competing alternatives</th>
<th>Max. ( EOL )</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>0</td>
<td>11100</td>
<td>11100</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>10600</td>
<td>0</td>
<td>567</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>10900</td>
<td>1070</td>
<td>0</td>
</tr>
</tbody>
</table>

Utilizing the data in the example, the effects of alternative outcomes correlation and the influence of decision maker’s acceptable risk on the final decision are investigated in this section.

**Effects of acceptable risk (AR)**

A feasible alternative should have the value of \( EOL \) lower than the decision maker’s acceptable risk (AR) when comparing with all the other alternatives in each pair-wise comparison. The basic idea for alternative selection using the \( EOL \) is to choose those with low potential losses in case the chosen alternative turns out to be wrong. However, the feasibility of the chosen alternative can only be determined with reference to the decision maker’s AR.

The \( AR \) can be used to check if the \( EOL \) for selecting one alternative meets the decision maker’s tolerance of risk. Referring to the Heck Valley example shown in Table 3.11, when \( AR \) is under $106,000, none of the alternatives are acceptable (or implementable) to the decision maker. In this case, the decision maker could take one of the plans of Madansky [53]...
that outlined previously. When \(106,000 < AR < 109,000\), alternative \(A_2\) would be a best feasible plan. However, the reality is that decision maker’s \(AR\) could also be uncertain and its implications on the EOL-based decision making should be further studied.

Effects of correlated outcomes

As stated previously, in water resources planning and management, the outcomes of different management alternatives are often correlated from sharing some common hydrologic and hydraulic parameters. Hence, the outcomes of the management alternatives, to some degrees, are correlated. Such correlation might affect the relative merits of two alternatives in a pair-wise comparison and also their acceptability by the decision makers.

Consider that the outcomes \(X_i\) and \(X_j\) are bi-variate normal random variables with a joint PDF. The correlations \((\rho)\) among the alternative outcomes can have a significant influence on the value of \(EOL\), especially when the chosen alternative has an expected outcome close to its competitor. As \(R_\mu\) gets larger, the effect of the correlation on the \(EOL\) diminishes. Figure 3.14 shows the similar pattern with different alternative pairs of the Heck Valley case study. It shows that the \(EOL\) values are smaller for a positive correlation than the same alternatives pair with a negative correlation. Table 3.12 shows the maximum values of \(EOL\) when the three alternatives are considered individually to be the chosen one under different correlation among the annual net benefits of the three alternatives. The final rankings of the three alternatives are shown in the parentheses next to the values of \(EOL\). It shows that the rankings can be also changed by the strength of the correlations.

<table>
<thead>
<tr>
<th>Chosen alternative</th>
<th>(EOL_{\text{max}}(A_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\rho = -0.6)</td>
</tr>
<tr>
<td>(A_1)</td>
<td>11314 (2)</td>
</tr>
<tr>
<td>(A_2)</td>
<td>10652 (1)</td>
</tr>
<tr>
<td>(A_3)</td>
<td>11356 (3)</td>
</tr>
</tbody>
</table>
3.9 Illustrative Example: Flood Levee Design

To demonstrate the effect of the loss function on the ranking of competing risky alternatives, and the final decision, a levee design problem consisting of five mutually exclusive alternatives of different protection levels, is considered herein. The basic statistical properties of the annual net benefits of the five alternatives are listed in Table 3.13 and the corresponding PDFs under the normal distribution condition are shown in Figure 3.15.

As can be seen in Table 3.13, the five alternatives are sorted in descending order of expected net benefits, which coincides with the ranking according to the expected-value rule. The uncertainty characteristics are set purposely to make all five alternatives lie on the efficiency frontier. In this circumstance, the M-V rule and the first two SD rules would not be able to yield a decisive ranking. The coefficients of variation are 0.5, 0.4, 0.3, 0.2, and 0.1 for the five alternatives $A_1$, $A_2$, $A_3$, $A_4$, and $A_5$, respectively.
Table 3.13 Alternatives and corresponding statistics of net benefits in hypothetical example

<table>
<thead>
<tr>
<th>Chosen Alternative $A_i$</th>
<th>Mean ($10^6$)</th>
<th>Standard deviation ($10^6$)</th>
<th>Coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>131.90</td>
<td>65.95</td>
<td>0.5</td>
</tr>
<tr>
<td>$A_2$</td>
<td>127.71</td>
<td>51.08</td>
<td>0.4</td>
</tr>
<tr>
<td>$A_3$</td>
<td>124.06</td>
<td>37.22</td>
<td>0.3</td>
</tr>
<tr>
<td>$A_4$</td>
<td>120.65</td>
<td>24.13</td>
<td>0.2</td>
</tr>
<tr>
<td>$A_5$</td>
<td>109.99</td>
<td>11.00</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Figure 3.15 Probability PDFs of five net benefits of alternatives in the hypothetical example
3.9.1 Effect of Loss Function Model

Tables 3.11(a)-(c) present the maximum values of $EOL_m$ when each of the five alternatives is considered for selection under different conditions of $m = 0, 1, \ldots, 4$, and with different correlation among the annual net benefits of the five alternatives. The final ranking of the five alternatives under various $m$ are shown in the parentheses next to the value of $EOL_m$. When $m = 0$, $EOL_0$ is the probability of $\Delta_{i,j}$ that is less than zero; i.e., $P_r[\Delta_{i,j} < 0]$, which corresponds to the probability-of-loss rule. It can be observed that the ranking under $m = 0$ is dominated by the long-term expected net benefits of the alternatives. However, as $m$ increases, the rankings are influenced by the uncertainty features of the net benefit of the alternatives. For example, when $\rho_{i,j} = 0$, alternative $A_5$, which has the least uncertainty and also the least expected net benefit, is ranked fifth when $m = 0$ and becomes the second best choice when $m = 4$. The same situation occurs with alternative $A_4$, the fourth ranked alternative under $m = 0$, which becomes the optimal choice under $m \geq 2$. This indicates that as $m$ gets larger, the degree of uncertainty starts to exert more influence on the overall merit of risky alternatives and leads to a reversal of the ranking of the five alternatives.

This example shows that, with the risk measure of the $EOL_m$ and the rule of minimax $EOL$, the potential loss in choosing a risky alternative can be evaluated and elucidated in a quantitative manner. The uncertainties can be incorporated into the decision-making process explicitly and expressed by the risk measure of the $EOL_m$. From this example, the influence of outcome uncertainties becomes more significant when $m$ gets larger and the decision maker tends to become more conservative and reluctant to take risks. Namely, the attractiveness of alternatives with high expected net benefits is decreased due to high uncertainty.
Table 3.14 Maximum $|EOL_m|$ and alternative rankings in the hypothetical example

(a) Uncorrelated alternative outcomes ($\rho_{ij} = 0$)

| Chosen alternative | $EOL_0$ | $|EOL_1|$ | $EOL_2$ | $|EOL_3|$ | $EOL_4$ |
|--------------------|---------|-----------|---------|-----------|---------|
| $A_1$              | 0.48 (1)| 31.2 (1)  | 3,209 (2)| 421\times10^3 (3)| 652\times10^5 (4) |
| $A_2$              | 0.52 (2)| 35.4 (4)  | 3,767 (5)| 509\times10^3 (5)| 808\times10^5 (5) |
| $A_3$              | 0.54 (3)| 34.3 (3)  | 3,373 (3)| 420\times10^3 (2)| 613\times10^5 (3) |
| $A_4$              | 0.56 (4)| 34.0 (2)  | 3,162 (1)| 371\times10^3 (1)| 510\times10^5 (1) |
| $A_5$              | 0.66 (5)| 39.0 (5)  | 3,665 (2)| 429\times10^3 (2)| 586\times10^5 (2) |

(b) Negative correlated alternative outcomes ($\rho_{ij} = -0.6$)

| Chosen alternative | $EOL_0$ | $|EOL_1|$ | $EOL_2$ | $|EOL_3|$ | $EOL_4$ |
|--------------------|---------|-----------|---------|-----------|---------|
| $A_1$              | 0.48 (1)| 39.8 (2)  | 5,160 (4)| 854\times10^3 (4)| 1670\times10^5 (4) |
| $A_2$              | 0.52 (2)| 44.0 (5)  | 5,860 (5)| 992\times10^3 (5)| 1980\times10^5 (5) |
| $A_3$              | 0.53 (3)| 41.2 (3)  | 4,950 (3)| 754\times10^3 (3)| 1350\times10^5 (3) |
| $A_4$              | 0.55 (4)| 38.9 (1)  | 4,230 (2)| 580\times10^3 (2)| 933\times10^5 (2) |
| $A_5$              | 0.63 (5)| 41.4 (4)  | 4,210 (1)| 535\times10^3 (1)| 791\times10^5 (1) |

(c) Positive correlated alternative outcomes ($\rho_{ij} = +0.6$)

| Chosen alternative | $EOL_0$ | $|EOL_1|$ | $EOL_2$ | $|EOL_3|$ | $EOL_4$ |
|--------------------|---------|-----------|---------|-----------|---------|
| $A_1$              | 0.47 (1)| 19.5 (1)  | 1,290 (1)| 108\times10^3 (1)| 108\times10^5 (1) |
| $A_2$              | 0.53 (2)| 23.7 (2)  | 1,650 (2)| 145\times10^3 (2)| 150\times10^5 (2) |
| $A_3$              | 0.56 (3)| 25.2 (3)  | 1,760 (3)| 154\times10^3 (3)| 159\times10^5 (3) |
| $A_4$              | 0.58 (4)| 28.0 (4)  | 2,070 (4)| 193\times10^3 (4)| 209\times10^5 (4) |
| $A_5$              | 0.71 (5)| 36.5 (5)  | 3,110 (5)| 331\times10^3 (5)| 409\times10^5 (5) |
When the loss function $L(\Delta_{i,j}) = (-\Delta_{i,j})^m$ is treated as a utility function $u$, the phenomenon of the risk aversion increasing as $m$ gets larger can be explained by the absolute risk aversion coefficient $R_A$ [91, 164], defined as

$$R_A = \frac{-u''(\delta)}{u'(\delta)}$$

for some level of $\Delta_{i,j}$. This coefficient is a measure of the curvature of the utility function $u(\delta)$. It can be interpreted as the percentage change in the marginal utility at any level of $\Delta_{i,j}$. The absolute risk aversion coefficient can be derived as $R_A = (m-1)/\delta$. Hence, when $m$ increases, the degree of risk aversion also increases.

Figure 3.16 shows the contours of $EOL_m$ with respect to the mean ratio $R_{\mu} = \mu_i/\mu_j$ and $R_{\sigma} = \sigma_i/\sigma_j$ under $\mu_j = \sigma_j = 1$ when $\rho_{i,j} = -0.6, 0.0, 0.6$. It demonstrates the increasing trend of risk aversion as $m$ gets larger. As $m$ increases, the value of $dR_\mu / dR_\sigma$ also increases. By $\frac{dR_\mu}{dR_\sigma} = \frac{dEOL_m}{dR_\sigma}$, the relative sensitivity of $EOL_m$ with respect to the uncertainty of outcome compared to the mean outcome becomes larger so that the decision maker becomes more risk-averse.

### 3.9.2 Effects of Correlated Outcomes

Considering a decision maker’s acceptable risk, preference in selecting one particular alternative can also be affected by the strength of the correlation among different outcomes of alternative. In this example, the effects of correlations between the net benefits of the alternatives are examined. The correlation coefficients among the net benefits of the alternatives have a considerable influence on the value of $EOL_m$. Table 3.14(b) and (c) show the maximum values of $EOL_m$ for each chosen alternative under the conditions $m = 0, 1, \ldots, 4$, when $\rho_{i,j} = -0.6$ and $+0.6$, respectively. The two tables show that a more positive correlation yields a lower value of $EOL_m$, whereas a more negative correlation results in a larger $EOL_m$. Figure 3.16 shows the influence of the correlation on the $EOL_m$ values and reveals that the correlations between the alternative outcomes can have a great influence on the $EOL$. 
especially when the expected outcome of a selected alternative is close to that of its competitor.

**Figure 3.16 Contours of $EOL_m$ with respect to $R_\mu$ and $R_\sigma$**

![Figure 3.16](image)

### 3.9.3 Effect of Acceptable Risk

For prudent management, setting an acceptable tolerance for potential loss and preparing a proper contingency fund in advance are advisable practices for dealing with the adverse impacts of failing to achieve the anticipated outcome. The acceptable risk of decision makers can be assessed by their feedback on a questionnaire about the tolerable risk threshold according to decision makers’ experiences and opinions [49]. This acceptable tolerance for the potential loss is compatible with the meaning of acceptable risk. If the maximum $EOL$ associated with the decision of selecting one particular alternative is lower than the acceptable
risk, this decision is implementable because the associated potential loss is tolerable. Otherwise, when the maximum $EOL$ associated with an alternative is higher than the acceptable risk, implementing the alternative may not be prudent because the associated potential loss could be beyond the decision maker’s or stakeholder’s capacity to absorb. Hence, the acceptable risk set by the decision maker should be considered and used to examine the feasibility of implementing an alternative as stage-(5) in Figure 2.1 of the decision-making process. Stewart et al. [98] applied the concept of acceptable risk in the ranking of alternatives of four different bridge designs and informed the decision maker of the acceptable alternatives. Xu and Tung [49] designed a questionnaire to determine the decision makers’ acceptable risk as a feasibility criterion for a river basin management project.

Referring to Table 3.14(a), when $AR = 31 \times 10^6$ and net benefits of the alternatives are uncorrelated, none of the alternatives is acceptable (or implementable) with reference to $EOL_1$. In this case, the decision maker could consider formulating a new list of alternatives and repeating the decision analysis; reduce the outcome uncertainty or delay the decision until more information is obtained; or increase the budgetary reserves for the contingency by having higher acceptable risk. When $AR > 31.2 \times 10^6$ and $\rho_{ij} = 0$, $A_1$ is better than the other three feasible alternatives (i.e., $A_1$, $A_3$, $A_4$). In reality, the decision maker’s acceptable risk could be uncertain and its effect on the final decision can be further studied.

By comparing the magnitude of the maximum $EOL_1$ among the five alternatives under different correlations conditions (Table 3.14(a)~(c)), one can clearly see the influence of the correlation of alternative outcomes on the feasibility of the alternative. When $AR = 31 \times 10^6$ or less, Table 3.14(a) and (b) show that none of the alternatives is feasible if alternative outcomes are uncorrelated or negatively correlated, whereas the first four alternatives (i.e., $A_1$, $A_2$, $A_3$, $A_4$) are feasible if alternative outcomes are positively correlated.

### 3.10 Summary and Conclusions

Considering the presence of uncertainty in each stage of the engineering design and decision-making process, neglecting uncertainties would underestimate the potential risk, so
risk-based decision making in engineering project design and analysis is receiving increased attention. For decision making in an uncertain environment, zero risk for any decision is not attainable because any chosen alternative has a certain possibility of being inferior to its competing alternatives. As stated by Castro et al. [101], “Being wrong about alternatives with a lower expected loss is preferable to being wrong about alternatives with a higher expected loss”.

In current decision-making practice, the decision maker’s acceptable risk is rarely considered in an explicit manner, and the effect of the outcome correlation is often overlooked. Although conventional decision rules (e.g., the M-V method and SD criterion) are used to rank alternatives according to a decision maker’s risk attitude (e.g., risk-averse), these rules can neither account for the effect of correlation between alternatives nor explicitly incorporate the decision maker’s acceptable risk in testing an alternative preference or acceptability.

This research study is concerned with the process of alternative ranking, and proposes $EOL$ as a risk measure that reflects the potential loss in case the chosen alternative turns out to be inferior to its competitor. In conjunction with the evaluation of $EOL$, Savage’s minimax principle was employed for the ranking of alternatives in this study. This decision-making model permits joint consideration of the acceptable risk and the correlation among alternative outcomes.

The $EOL$ reflects more accurately the relative merit of two competing alternatives without suffering the pessimism of the CRM and the counter-intuition of XRM. The values of $EOL$ of the two competing alternatives under consideration are dependent on the magnitudes of their means, standard deviations, and correlations. The choice between two alternatives with smaller mean outcome, higher uncertainty, and/or large negative correlation could result in a larger value of $EOL$.

Various forms of loss function are considered in a general manner. The analytical expressions of $EOL_m$ for normal random outcomes are derived when $m$ is an integer. For
Chapter 3  Risk-based decision making using EOL

non-integer \( m \) values, empirical relations of non-dimensional \( EOL_{m'} \) with \( m \) and 
\[ \beta = -\frac{\mu_\lambda}{\sigma_\lambda} \] are provided.

The proposed risk measure and some conventional ranking rules are demonstrated through examples in river basin management, flood control decision making, and a hypothetical example that involves a continuous random state of nature. Based on the \( EOL \), a decision maker can select an alternative with the lowest maximum \( EOL \) compared to all the others. The results showed the practical advantages of the proposed decision criterion in alternative selection under uncertainty and proved that the uncertainty degree of the outcomes, and the outcomes correlations, would affect the ranking of alternatives.

The effects of loss functions on the ranking of alternatives and the final decision are demonstrated through the hypothetical example involving the continuous random state of nature. The results show that the form of loss functions can greatly influence the ranking thus the optimal choice can be altered. As \( m \) gets larger, the ranking is more influenced by the degree of uncertainty. In other words, the decision maker tends to be more conservative and risk-averse. Thus, the loss function can be used to reflect the decision maker’s risk attitude and incorporated into the risk-based decision-making model.
CHAPTER 4
INCORPORATING ALEATORY AND EPISTEMIC UNCERTAINTIES IN FLOOD-DAMAGE-REDUCTION PROJECT EVALUATION

Epistemic uncertainty is a result of knowledge deficiency about the system. Sampling error exists when limited amounts of hydrologic data are used to estimate a $T$-yr event quantile. Both the natural randomness of the hydrologic data and the sampling error in the design quantile estimation contribute to the uncertainty in flood damage estimation. This chapter presents a framework for evaluating a flood-damage-mitigation project in which both the hydrologic randomness and epistemic uncertainty due to sampling error are considered in flood damage estimation. Different risk-based decision making rules are used to evaluate project merits based on the mean, standard deviation, and probability distribution of the project net benefits. The results show that the uncertainty of the project net benefits is quite significant. Ignoring the data sampling error will underestimate the potential risk of each project. It can be clearly shown that adding data to the existing sample observations leads to improved quality of information, enhanced reliability of the estimators, and reduced sampling error and uncertainty in the project net benefits. Through the proposed framework, the proper length of the extended record for risk reduction can be determined to achieve the required level of acceptable risk.
4.1 Introduction

Risk is defined as the probability weighted undesirable consequences of a project or management decision. As the net benefits from a flood-damage-reduction project are uncertain, there will be a risk. Like most engineering problems, the design, management, and operation of decision making of hydrosystem infrastructure, including flood-damage-reduction systems, are subject to uncertainties. These uncertainties could arise from, but are not limited to, hydraulic, hydrologic, structural, environmental, and socio-economic aspects. When uncertainties are present, the ranking and selection of projects involving risk are no longer trivial tasks. When engineering designs involve uncertainties, sensitivity analysis is commonly used to assess the influence of the variation in the factors subject to uncertainty in regard to system responses. However, sensitivity information is ineffective for design, management, and operation since it does not provide a full range of variations for the system responses. A more effective approach for engineering design under uncertainty is to conduct a comprehensive risk analysis based on the results of uncertainty analysis.

Uncertainty analysis aims at enhancing risk-informed decision making under the situation that not all information is reliable or sufficient. It enables better water resources project designs to achieve the target outcome. Uncertainties, in general, can be classified into two types: aleatory uncertainty and epistemic uncertainty [165]. The aleatory uncertainty (also referred to as the variability or the irreducible uncertainty) is primarily due to the natural randomness in a physical phenomenon (e.g., variation in rainfall amount, flood magnitude, etc.), which is inherently unpredictable in time and/or space. This type of uncertainty cannot be reduced by obtaining more information, but can be estimated on the basis of available data or information (e.g., estimating the annual exceedance probability from historical data through frequency analysis). The epistemic uncertainty (also called the reducible uncertainty) is due to knowledge deficiency regarding system behavior on the part of the observer (e.g., annual exceedance probability is not firmly assured because of the use of a finite number of observations). The use of a finite sample could lead to uncertainty in estimating the model and parameters. Epistemic uncertainty can be reduced, although knowledge improvement can be
difficult or expensive to achieve. Consideration of the effect of epistemic uncertainty in structural design, based on the upper limit for the confidence interval of the life-cycle cost, as described elsewhere [166, 167], in which the confidence level relies on professional judgment. In water resources planning and management, several studies have considered the epistemic uncertainty along with the inherent hydrologic uncertainty in evaluating annual expected flood damage [13, 28, 31, 34-37]. The main thrust of this chapter is to incorporate epistemic uncertainty, in particular parameter uncertainty, and to examine its effect on the results of risk-based flood-damage-reduction project evaluation.

To conduct a typical design of a flood-damage-reduction project, information is gathered to estimate the frequency of flooding and the corresponding damage in the areas of concern. Relationships between frequency-discharge, discharge-stage, and stage-damage can be developed based on observed data. From the list of candidate flood-damage-reduction alternatives under consideration, the corresponding frequency-damage relationship can be established to compute the statistical features of the damage reduction for each alternative. As demonstrated in Chapter 3, information such as the expected net benefits and residual risk that incorporate the randomness of the flood magnitude can then be used to evaluate the economic merits of the project alternatives. However, the discharge quantile-frequency relationship is developed on the basis of a finite amount of flood data. For example, the flood magnitude corresponding to a 100-year return period may be estimated from 30 years of historical flood data. The use of the data from a finite sample introduces sampling errors in estimating the probability distribution parameters, which further induces uncertainty to the estimated frequency-quantile relationship (see Figure 4.1) and the resulting flood damage. The sampling distribution in Figure 4.1 shows the uncertainty of the estimated $T$-yr quantile estimator $\hat{X}_T$ which has incorporated the uncertainties in distribution parameters.

Due to the sampling error, the $T$-yr quantile estimator $\hat{X}_T$ can be treated as a random variable. In practice, two methods are used to describe the sampling error associated with an estimator, i.e., the standard error of estimation $s_e(\hat{X}_T)$ and the confidence limit of a specific significance level $\alpha$ $(\hat{X}_{T,\alpha}^{UB}, \hat{X}_{T,\alpha}^{LB})$, where $UB$ and $LB$ indicate the upper and lower bounds,
respectively. The former is a measure indicating the standard deviation of the magnitude of a $T$-yr event estimated from a limited sample, whereas the latter is an interval that has a specified likelihood of capturing the true but unknown magnitude of the $T$-yr event. In general, longer data records can yield a more reliable estimation of the quantile-frequency relationship. Hence, the degree of uncertainty of the quantile estimator is a function of the sample size and the underlying probability distribution, from which the sample data are generated. In this paper, the standard error of the quantile estimator of a Gumbel distribution model is used for demonstration purposes. The idea is equally applicable to other distribution models.

**Figure 4.1 Uncertainty of estimated T-yr discharge quantile due to sampling error**

When the uncertainties of the project alternatives are too high, or the potential losses associated with making a decision are higher than the acceptable risk, a decision maker may be uncomfortable about making a choice among the candidate alternatives. He/she can opt to postpone the decision and wait for more information. As new observations are added to the existing sample data, the reliability of the quantile estimator is enhanced because the associated uncertainties are reduced. Increased information obtained by extending the sample
size would yield a reduced potential risk due to the reduction in the sampling error. The influence of the hydrological record length on the value of potential risk and the result of project evaluation is investigated here. The chapter focuses on two aspects: (1) propose and demonstrate a risk-based decision making framework that considers both the hydrologic randomness and epistemic uncertainty in flood damage estimation, and (2) analyze the effects of sample size on the potential risk of alternatives and the ranking result. Section 4.2 introduces the framework composed of the methods of evaluating the sampling error of the frequency-discharge quantile estimator, estimating the flood damage that considers aleatory uncertainty or both aleatory and epistemic uncertainty, and the minimax expected opportunity loss (EOL) ranking rule. Section 4.3 demonstrates the proposed risk-based decision making framework through selecting a flood control project by utilizing the conventional and newly developed ranking rule. Lastly, the summary and conclusions are presented in Section 4.4.

4.2 Methodology

Flood protection infrastructures (e.g., dams, levees, and storm drains) are designed to protect a region from the threat of floods. Cost-benefit analysis is often applied to compare and rank the economic merits of project design alternatives based on the corresponding benefits or costs. The flood damage estimation can be undertaken through the use of frequency-discharge-damage relationships. Due to the unforeseeable future state of nature or development, the outcome of a project alternative (e.g., project net benefit or life-cycle cost of a system) cannot be predicted with absolute certainty. It is important to recognize that the damage resulting from the failure of a hydrosystem is in fact a random variable. The presence of uncertainties in a decision process induces an uncertain project outcome which makes decisions and judgments between project alternatives a non-trivial task.

Most risk-based design approaches in hydrosystems mainly consider the mean values of the project costs and benefits without giving an explicit account for their probability distributions resulting from natural randomness and sampling error. When information about the uncertainty features of the project costs and benefits becomes available, the decision
maker may not necessarily be inclined to make the choice solely on the basis of the highest expected benefit or the lowest expected cost.

The process of risk-based decision making can be found elsewhere [12, 43, 52], and it generally involves (1) identification of the problem; (2) generation of project alternatives; (3) prediction of project outcomes using information from uncertainty analysis; (4) ranking of project alternatives by the risk-based decision rule; (5) feasibility test of project alternatives; and (6) implementation.

In this section, a methodological framework for novel risk-based project evaluation is introduced. The framework integrates the randomness of the flood magnitude and epistemic uncertainties resulting from the sampling errors into the evaluation of the economic merits of the project alternatives. Methods for calculating the sampling error of a Gumbel-based quantile estimator for different parameter estimation procedures are described first. Then the methods of assessing the mean, standard deviation, and correlation coefficient between the net benefits resulting from different project alternatives by considering only the natural randomness of floods and by integrating both aleatory and epistemic uncertainties in the flood damage estimation are formulated. Based on the statistics of the net benefits of different project alternatives, several rules for project evaluation under uncertainty can be adopted to assess the merit of a project.

### 4.2.1 Sampling Error of Gumbel Distribution Quantiles

The cumulative distribution function (CDF) of the Gumbel (extreme value type I) distribution is

\[
F_X(x) = \exp \left[ -\exp \left( -\frac{x - \xi}{\beta} \right) \right]
\]  

(4-1)

where \( \xi \) is the location parameter and \( \beta \) is the scale parameter (\( \beta > 0 \)). Several methods have been developed to determine the distribution parameters of the Gumbel distribution [168, 169]. In this chapter, three parameter estimation methods (i.e., the method of moments (MoM), the method of maximum likelihood (MML), and the probability weighted moment
method (PWM) are considered to calculate the standard errors associated with the Gumbel T-yr quantile estimator.

The quantile estimator of the T-yr event $\tilde{X}_T$ can be obtained from Eq.(4-1) with the non-exceedance probability $F=1-(1/T)$ as

$$\tilde{X}_T = \xi - \tilde{\beta} \ln \left[ -\ln \left( 1 - \frac{1}{T} \right) \right] \quad (4-2)$$

where $T$ is the return period. The sample mean $m'$ and variance $m_2$ are used to determine the corresponding Gumbel estimated parameters $\tilde{\beta}$ and $\xi$, respectively, by

$$\tilde{\beta} = \frac{\sqrt{6m_2}}{\pi} \quad (4-3a)$$

$$\xi = m' - \frac{0.5772}{\pi} \sqrt{6m_2} \quad (4-3b)$$

The standard error, $s_e(\tilde{X}_T)$, of the quantile estimator $\tilde{X}_T$ is a function of the sample size, $n$, and $T$. Table 4.1 summarizes the variances for the Gumbel T-yr quantile estimator based on the MoM, MML, and PWM estimation methods. Assuming the sampling distribution of the T-yr quantile estimator to be normal [170], the 95% upper and lower confidence limits can be determined by $\tilde{x}_T \pm 1.96 s_e(\tilde{x}_T)$.

4.2.2 Inundation-Reduction Benefit Statistics without Considering Sampling Errors

In the assessment of a flood-damage-reduction plan, the economic performance indicator can be the annual net benefit $NB$, which is computed as the annual inundation-reduction benefit, $B_{IR}$, minus the annual cost, $C$, of implementing the plan

$$NB = B_{IR} - C \quad (4-4)$$

When the water surface level exceeds the levee crown elevation or the flood magnitude $Q$ exceeds the design capacity of the system, flood damage will occur. Each damage-reduction
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A project will have its corresponding damage-frequency relationship. The damage function under the “without-project” condition serves as the baseline in the inundation-reduction benefit calculation. The inundation-reduction benefit, $B_{IR}$, of each project alternative is equal to the difference in inundation damage under the conditions of “without” and “with” implementing the flood-damage-reduction project as

$$B_{IR,A}(q) = D_{w/o,A}(q) - D_{w,A}(q)$$

(4-5)

where $D_{w/o,A}(q) = D_{w/o}(q) - D_{w,A}(q)$ with $D_{w/o}(q)$ and $D_{w,A}(q)$, respectively, being the flood damage under the conditions of “without” and “with” project alternative-$A_i$ under a given discharge $q$.

Because of the random occurrence of the flood magnitude, the inundation-reduction benefit, $B_{IR,A}$, is uncertain and can be treated as a random variable. Based on the random nature of floods, one can calculate the expectation and variance of the annual inundation damage of each project alternative; as well as the covariance of the inundation damage between two different projects.

Table 4.1 Variances of Gumbel quantile $\bar{x}_T$ based on the MoM, MML, and PWM estimation methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Variances of Gumbel quantile $\bar{x}_T$</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>MoM</td>
<td>$s^2_{\bar{x}_T}(\bar{x}_T) = \frac{\beta^2}{n}(1.15894 + 0.19187Y + 1.1Y^2)$</td>
<td>Kite [168]</td>
</tr>
<tr>
<td>MML</td>
<td>$s^2_{\bar{x}_T}(\bar{x}_T) = \frac{\beta^2}{n}(1.1087 + 0.5140Y + 0.6079Y^2)$</td>
<td>Kite [168]</td>
</tr>
<tr>
<td>PWM</td>
<td>$s^2_{\bar{x}_T}(\bar{x}_T) = \frac{\beta^2}{n}(1.1128 + 0.4574Y + 0.8046Y^2)$</td>
<td>Rao and Hamed [169]</td>
</tr>
</tbody>
</table>

Note: $n =$ sample size, $Y = -\ln(-\ln(1-1/T))$
To implement a risk-based decision making analysis, the statistical features (e.g., mean, variance, correlation coefficient, and distribution) of the economic indicators are needed. Such information can be derived from the inundation damage-frequency relationships under the conditions of “without” and “with” projects. The statistical features of $B_{IR}$ for each project alternative can be used to compute $\mu_{\Delta}$ and $\sigma^2_{\Delta}$ in Eqs. (3-2a), and (3-2b), that are then used to compute the $EOL$ in Eq. (3-4a). For each project alternative, the expected value of the annual net benefit due to reduced damage, $B_{IR}$, under project alternative $A_i$ can be computed by

$$E\left[B_{IR,A_i}\right] = \int_0^{-} D_{w/o-A_i} (q) f_Q(q) dq$$

where $f_Q(q)$ is the PDF of the floods which can be established through a flood frequency analysis. To assess the uncertainty features of the annual inundation-reduction benefit associated with alternative $A_i (B_{IR,A_i})$, its variance can be computed by

$$Var\left[B_{IR,A_i}\right] = E\left(B^2_{IR,A_i}\right) - E^2\left(B_{IR,A_i}\right)$$

$$= \int_0^{\infty} D^2_{w/o-A_i} (q) f_Q(q) dq - E^2\left(B_{IR,A_i}\right)$$

In practice, numerical integration is implemented to calculate the mean and variance in Eqs. (4-6) and (4-7).

Usually, in the same river basin, the economic merits for different flood-damage-reduction projects are correlated because they are affected by some common hydrologic and hydraulic factors. For example, in designing a levee system in a river basin, the inundation-reduction benefits associated with different alternatives can be affected by common factors such as rainfall, flow hydrograph, channel geometry/boundary conditions, and topographical/land use features in the river basin. Hence, the covariance ($Cov$) of two alternative benefits, $B_{IR,A_i}$ and $B_{IR,A_j}$, and their correlation coefficient ($\rho$) can be calculated by
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\[ \text{Cov}\left[B_{IR,A}, B_{IR,A_j}\right] = E\left(B_{IR,A} B_{IR,A_j}\right) - E\left(B_{IR,A}\right) E\left(B_{IR,A_j}\right) \]
\[ = \int_{0}^{\infty} \left[ D_{w/o-w/A} (q) \times D_{w/o-w/A_j} (q) \right] f_{Q}(q) \, dq \]
\[ - E\left(B_{IR,A}\right) E\left(B_{IR,A_j}\right) \]  (4-8)

\[ \rho\left[B_{IR,A}, B_{IR,A_j}\right] = \text{Cov}\left[B_{IR,A}, B_{IR,A_j}\right] \left/ \sqrt{\text{Var}\left(B_{IR,A}\right) \times \text{Var}\left(B_{IR,A_j}\right)} \right. \]  (4-9)

In practice, numerical integration is carried out in the calculation of Eqs.(4-6)–(4-8).

4.2.3 Inundation-Reduction Benefit Estimation Considering Sampling Errors

Due to the presence of sampling error, the uncertainty of the quantile estimator (in terms of its standard error) can be incorporated into the estimation of the statistical moments of the inundation-reduction benefit. For a specific return period, the flood magnitude estimator \( \tilde{Q}_r \) is a random variable with mean \( \tilde{q}_r \), variance, \( s_e^2(\tilde{q}_r) \), and sampling distribution, \( h_{\tilde{Q}_r}(q_r|n) \). The mean and variance of the annual inundation-reduction benefit, considering the quantile sampling errors, of an individual alternative as well as the covariance between two projects alternatives, can be calculated by

\[ E\left[B_{IR,A} | n\right] = E_{\tilde{Q}_r} \left[ E_{\tilde{Q}_r} \left[ D_{w/o-w/A} (Q_r) | n\right], Q_r = \tilde{Q}_r \right] \]
\[ = \int_{0}^{\infty} \left[ \int_{0}^{\tilde{Q}_r} \left[ D_{w/o-w/A} (q_r) \right] h_{\tilde{Q}_r}(q_r|n) \, dq_r \right] f_{\tilde{Q}_r}(q) \, dq \]  (4-10)

where \( h_{\tilde{Q}_r}(q_r|n) \) is the sampling distribution of \( T \)-yr quantile estimator, \( \tilde{Q}_r \), based on sample size \( n \).

\[ \text{Var}\left[B_{IR,A} | n\right] = E\left[B_{IR,A}^2 | n\right] - E^2\left[B_{IR,A} | n\right] \]
\[ = E_{\tilde{Q}_r} \left[ E_{\tilde{Q}_r} \left[ D_{w/o-w/A} (Q_r)^2 | n\right], Q_r = \tilde{Q}_r \right] - E^2\left[B_{IR,A} | n\right] \]
\[ = \int_{0}^{\tilde{Q}_r} \left[ \int_{0}^{\infty} \left[ D_{w/o-w/A} (q_r)^2 \right] h_{\tilde{Q}_r}(q_r|n) \, dq_r \right] f_{\tilde{Q}_r}(q) \, dq \]
\[ - E^2\left[B_{IR,A} | n\right] \]  (4-11)
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\[
\text{Cov}\left[B_{IR,A_i}, B_{IR,A_j} \mid n\right] = E\left[B_{IR,A_i} B_{IR,A_j} \mid n\right] - E\left[B_{IR,A_i} \mid n\right] E\left[B_{IR,A_j} \mid n\right] \\
= E_Q\left[E_{Q_T} \left[D_{w/o-wl/A_i}(Q_T) \times D_{w/o-wl/A_j}(Q_T) \mid n\right], Q_T = Q\right] - E\left[B_{IR,A_i} \mid n\right] E\left[B_{IR,A_j} \mid n\right] \\
\text{(4-12)}
\]

Eqs.(4-10)–(4-12) involve double integration in computing the statistics of the inundation-reduction benefit in that \( f_Q(q) \) accounts for the aleatory uncertainty of flood magnitude, while \( h_{Q_T}(q_T \mid n) \) deals with the epistemic uncertainty associated with the sampling error of the \( T \)-yr flood quantile estimator.

4.2.4 Rules for Project Evaluation

Consider a decision problem in selecting among a set of \( m \) project alternatives, \( A_i (i = 1, 2, \ldots, N) \), with random outcomes, \( X_i \), for which a larger value of the outcome is more desirable to the decision maker. In decision making under uncertainty, whereby the probabilities of the possible project outcomes are unknown, several decision rules can be used depending on the risk attitude of the decision maker. These rules include the maximin rule, the maximax rule, the Hurwicz rule, the minimax regret rule, and the Laplace rule [67]. When the probability profiles of the random project outcomes are available, the expected-value rule, expected utility theory, M-V rule, or stochastic dominance rules can be adopted, as discussed in Chapter 2.

The previously mentioned conventional decision rules have limitations during their implementation in flood-damage-reduction project evaluation. To circumvent the shortcomings of the existing rules for decision making in an uncertain environment, the expected opportunity loss (EOL) calculated from Eq.(3-4a) can be used under the condition that the project outcomes are continuous random variables with known probability distributions. To rank alternatives, the EOL can be considered as an indicator representative of the merit of each alternative. In order to control the worst potential risk, the maximum value of \( EOL(A_i^*, A_j) \) for each pair of a chosen alternative \( A_i \) with all the other alternatives can be used as a viable figure of merit, that is \( EOL_{\text{max}}(A_i) \) in Eq.(3-9). \( EOL_{\text{max}}(A_i) \) is the worst regret value a decision maker could anticipate in choosing alternative \( A_i \). The minimax
principle is to choose the alternative with a minimum value of $EOL_{\text{max}}(A_i)$ among all the alternatives. Then, according to the minimax regret principle described above, alternative $A_i$ is preferred over alternative $A_j$ if $EOL_{\text{max}}(A_i) < EOL_{\text{max}}(A_j)$. In this way, alternative ranking can be conducted in the following steps: (1) calculate $EOL(A_i^*, A_j)$ for each alternative pair $[A_i, A_j]$ with $i \neq j$; (2) identify the maximum value of $EOL$ for each candidate alternative $A_i$ ($EOL_{\text{max}}(A_i)$); and (3) rank the desirability of the alternatives by the $EOL_{\text{max}}(A_i)$ values from the smallest (the best) to the largest (the worst). It should be noted that there is a substantial difference between Savage’s minimax regret principle and the proposed minimax $EOL$ rule. The former depends only on the outcomes of a set of predetermined extreme scenarios, whereas the latter integrates the probability of all possible outcome scenarios in quantifying the regret associated with two alternatives to compute the $EOL$ for further ranking.

A feasible alternative should have its maximum $EOL$ value lower than the decision maker’s acceptable risk. After examining the feasibility of all alternatives (as shown in step (5) of Figure 2.1), a set of implementable alternatives that meet the decision maker’s acceptable risk can be identified. The best implementable alternative is the one with the lowest maximum $EOL$ in the feasible set of alternatives. When the maximum $EOL$ value of the best alternative exceeds the decision maker’s acceptable risk, all considered alternatives are not feasible. Under such a circumstance, the decision maker could take one or more further actions depending on the circumstances [53]: (1) reduce the uncertainty by conducting further research or acquiring more information; (2) modify the current alternatives or generate new ones and repeat the decision analysis; (3) delay the decision until more information is obtained; and (4) compromise with the first-ranked alternative and increase the budgetary reserves for the contingency.

4.3 Case Study

In this section, a flood-damage-reduction design project that considers the uncertainty from sampling error and the inherent random nature of floods is demonstrated through a case study extracted and modified from a report from the USACE [38]. The uncertainty associated with the economic efficiency of each flood-damage-reduction project is accounted for in order to
evaluate the expected opportunity cost in pair-wise alternative comparisons. Furthermore, the economic efficiency is measured by the net benefits based on which the EOL is computed. In addition, this example also examines how the correlations of the project net benefits and the decision maker’s AR influence the project ranking. In addition, the influence of sample size, in terms of the record length, on project evaluation and ranking is also examined.

4.3.1 Descriptions of Study Area

The Chester Creek basin in Philadelphia, Pennsylvania had long been subjected to serious flood damage. The most serious flood was in 1971, which caused 17.6 million estimated damage (in 1978 dollars) and the loss of eight lives [38]. A variety of levee and floodwall systems had been constructed previously to reduce local flooding, but during the 1971 flood event, the Eyre Park levee project, one of the existing levee systems in the Chester Creek basin, was overtopped and subsequently breached. In demonstrating the proposed ranking and decision making process, new levee systems of four different heights (6.68 m, 7.32 m, 7.77 m, and 8.23 m) to reduce the potential inundation damage are to be evaluated on the basis of the statistics of their net benefits. Different project evaluation rules are employed to determine the preferred design among the four levee systems.

To calculate the mean and variance of the reduced damage and the project net benefits, information on the frequency, discharge, stages, and damage at the site, that are extracted from a preliminary uncertainty analysis [38], is used to develop a frequency-damage function. The historical annual maximum discharge data of 65 years are used to determine the "without-project" frequency-discharge relationship (shown in Table 4.2) with the corresponding sample mean $m'_1 = 118.13$ m$^3$/s and variance $m_2 = 10106.21$ m$^6$/s$^2$. To prevent a negative-valued discharge for small non-exceedance probability, $m'_1$ is modified to 187.85 m$^3$/s, with unchanged $m_2$ and an estimated 100-yr flood. The corresponding Gumbel estimated parameters $\hat{\beta}$ and $\hat{\xi}$ can be computed by using $m'_1$ and $m_2$ in Eqs.(4-3a) and (4-3b). Since the discharge quantile is uncertain due to the use of a finite sample size, the associated standard errors for estimated flood quantiles can be determined from Table 4.2. The frequency-discharge relationship used in this example application is shown in Figure 4.2(a).
Table 4.2 “Without-project” discharge-frequency relationships in Chester Creek flood-damage-reduction case study (after USACE [38])

<table>
<thead>
<tr>
<th>Non-exceedance Probability</th>
<th>Return Period, $T$ (years)</th>
<th>Discharge ($m^3/s$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.998</td>
<td>500</td>
<td>898.8</td>
</tr>
<tr>
<td>0.995</td>
<td>200</td>
<td>676.1</td>
</tr>
<tr>
<td>0.99</td>
<td>100</td>
<td>538.5</td>
</tr>
<tr>
<td>0.98</td>
<td>50</td>
<td>423.0</td>
</tr>
<tr>
<td>0.95</td>
<td>20</td>
<td>298.8</td>
</tr>
<tr>
<td>0.9</td>
<td>10</td>
<td>222.5</td>
</tr>
<tr>
<td>0.8</td>
<td>5</td>
<td>158.4</td>
</tr>
<tr>
<td>0.5</td>
<td>2</td>
<td>87.0</td>
</tr>
<tr>
<td>0.2</td>
<td>1.25</td>
<td>50.9</td>
</tr>
<tr>
<td>0.1</td>
<td>1.11</td>
<td>39.4</td>
</tr>
<tr>
<td>0.05</td>
<td>1.05</td>
<td>32.3</td>
</tr>
<tr>
<td>0.01</td>
<td>1.01</td>
<td>22.9</td>
</tr>
</tbody>
</table>
To facilitate the mathematical manipulation, the data of the discharge-damage relationship extracted from USACE [38] is fitted by the following equation

$$\ln D = a + b(\ln Q)^2 + c / Q$$  \hspace{1cm} (4-13)

where $D$ represents the flood damage corresponding to a specific discharge $Q$; $a=15.38475$, $b=-0.09651$, and $c=-2101.77$ and the corresponding coefficient of determination ($R^2$) is 0.999998. The discharge-damage relationship is shown in Figure 4.2(b) from which the frequency-damage relationship can be established as shown in Figure 4.2(c).
The non-exceedance probabilities corresponding to the four different levee heights are interpolated from the fitted functions and are listed in Table 4.3 along with the deterministic annual installation costs.

**Table 4.3 Levee height, non-exceedance probability, and annual total installation cost (in 1978 U.S. dollars) for the four levee alternatives**

<table>
<thead>
<tr>
<th>Alternative (Levee Ht.)</th>
<th>Non-exceedance Probability ($F$)</th>
<th>Annual total installation cost ($10^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$ (6.68 m)</td>
<td>0.9931</td>
<td>19.8</td>
</tr>
<tr>
<td>$A_2$ (7.32 m)</td>
<td>0.9985</td>
<td>25.0</td>
</tr>
<tr>
<td>$A_3$ (7.77 m)</td>
<td>0.9996</td>
<td>30.6</td>
</tr>
<tr>
<td>$A_4$ (8.23 m)</td>
<td>0.9999</td>
<td>37.1</td>
</tr>
</tbody>
</table>

### 4.3.2 Flood Damage Statistics without Sampling Error

When the sampling error is neglected (assuming $n = \infty$), the mean and variance of the annual inundation-reduction benefits are calculated from Eqs.(4-6) and (4-7). The correlation coefficients of the annual inundation-reduction benefit between two project alternatives are calculated from Eq.(4-9). The calculated mean and standard deviation of the annual net benefit corresponding to the four levee height alternatives are listed in Table 4.4(a).

For this example, the correlation coefficient matrix for the random net benefits of the four levee alternatives is listed in Table 4.4(b). From Table 4.4(a) and (b), it can be observed that the degree of uncertainty in the net benefits associated with the four levee alternatives is quite considerable, with the coefficient of variation around 3.6~4.8, and all have significant positive inter-correlations. This implies that the conventional practice of considering only the
mean net benefit is not sufficient. Further consideration of the correlation and trade-off between the expected net benefits and their uncertainties is necessary.

| Table 4.4 Statistical features of annual net benefit for the five levee alternatives |
| (without considering sampling error $n = \infty$) |

(a) Means, standard deviations, and coefficients of variation

<table>
<thead>
<tr>
<th>Alternative $A_i$ (Levee Ht.)</th>
<th>Annual net benefit ($10^3$ in 1978 dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>$A_0$ (Status quo)</td>
<td>0</td>
</tr>
<tr>
<td>$A_1$ (6.68 m)</td>
<td>67.2</td>
</tr>
<tr>
<td>$A_2$ (7.32 m)</td>
<td>75.7</td>
</tr>
<tr>
<td>$A_3$ (7.77 m)</td>
<td>74.1</td>
</tr>
<tr>
<td>$A_4$ (8.23 m)</td>
<td>69.0</td>
</tr>
</tbody>
</table>

(b) Correlation coefficients

<table>
<thead>
<tr>
<th></th>
<th>$A_0$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_0$ (Status quo)</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$A_1$ (6.68 m)</td>
<td>0</td>
<td>1</td>
<td>0.783</td>
<td>0.726</td>
<td>0.705</td>
</tr>
<tr>
<td>$A_2$ (7.32 m)</td>
<td>0</td>
<td>0.783</td>
<td>1</td>
<td>0.928</td>
<td>0.901</td>
</tr>
<tr>
<td>$A_3$ (7.77 m)</td>
<td>0</td>
<td>0.726</td>
<td>0.928</td>
<td>1</td>
<td>0.972</td>
</tr>
<tr>
<td>$A_4$ (8.23 m)</td>
<td>0</td>
<td>0.705</td>
<td>0.901</td>
<td>0.972</td>
<td>1</td>
</tr>
</tbody>
</table>
4.3.3 Flood Damage Statistics Considering Sampling Error

Considering the sampling error of the estimated flood quantiles, the mean and standard deviation of the annual net benefit associated with the four levee alternatives and the covariance between net benefits of the alternatives can be computed by Eqs.(4-10), (4-11), and (4-12), respectively. In these equations, the sampling distribution $h_{Q_t}(q_t | n)$ is assumed to be a normal distribution [170], with the mean equal to the estimated $\tilde{q}_t$. The standard error of the flood quantile estimator is dependent on the parameter estimation method applied. The results of the mean and standard deviation of the annual net benefit associated with the four levee alternatives for a sample size of $n = 65$ by the MoM, MML, and PWM methods are listed in Table 4.5(a), 4.6(a), and 4.7(a), respectively. Using the three different parameter estimation methods, the corresponding correlation coefficient matrices of the annual net benefits among the levee alternatives are listed in Tables 4.5(b), 4.6(b), and 4.7(b). As can be seen in these tables, the three different parameter estimation methods produce very similar net benefit statistics, and the differences in correlation coefficients are even smaller. The values of the net benefit statistics calculated by the MoM are slightly larger than the PWM method, followed by the MML.

To quantify the effects of sampling error on the annual net benefit statistics, the relative changes in mean and standard deviation of the random annual net benefit, with and without considering the sampling error, are assessed, respectively, by

$$\frac{E[NB | n = 65] - E[NB | n = \infty]}{E[NB | n = \infty]} \times 100\%$$

$$\frac{\sigma[NB | n = 65] - \sigma[NB | n = \infty]}{\sigma[NB | n = \infty]} \times 100\%$$

where $E[NB | n = 65]$ and $\sigma[NB | n = 65]$ represent the mean and standard deviation of the net benefit for 65 years of data records, respectively; and $E[NB | n = \infty]$ and $\sigma[NB | n = \infty]$ represent the mean and standard deviation of the net benefit without sampling error, respectively. The results are listed in Tables 4.5(a), 4.6(a), and 4.7(a). As can be seen, the sampling errors contribute to an increase in the values of the mean and standard deviation of the net benefits.
The expected annual net benefits of the four levee alternatives increase by about 3.7%–5.4%. A more substantial increase in the standard deviations of the project net benefits of about 3.6%–8.2% can be observed, especially when the distribution parameters are estimated by the MoM (5.2%–8.2%). The coefficients of variation of the net benefits are around 3.65–4.83 for $n = 65$ which are higher than those without considering the sampling error when $n = \infty$. These results show that ignoring the sampling error would under-estimate the overall uncertainties.

<table>
<thead>
<tr>
<th>Alternative $A_i$ (Levee Ht.)</th>
<th>Annual net benefit ($10^3$)</th>
<th>Mean</th>
<th>Rel. change</th>
<th>Stdev</th>
<th>Rel. change</th>
<th>C.OV.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_0$ (Status quo)</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_1$ (6.68 m)</td>
<td></td>
<td>70.8</td>
<td>5.4%</td>
<td>261.7</td>
<td>8.2%</td>
<td>3.70</td>
</tr>
<tr>
<td>$A_2$ (7.32 m)</td>
<td></td>
<td>79.3</td>
<td>4.8%</td>
<td>321.2</td>
<td>6.2%</td>
<td>4.05</td>
</tr>
<tr>
<td>$A_3$ (7.77 m)</td>
<td></td>
<td>77.7</td>
<td>4.8%</td>
<td>342.4</td>
<td>5.5%</td>
<td>4.41</td>
</tr>
<tr>
<td>$A_4$ (8.23 m)</td>
<td></td>
<td>72.6</td>
<td>5.2%</td>
<td>350.9</td>
<td>5.2%</td>
<td>4.83</td>
</tr>
</tbody>
</table>

(b) Correlation coefficients

<table>
<thead>
<tr>
<th></th>
<th>$A_0$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_0$ (Status quo)</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$A_1$ (6.68 m)</td>
<td>0</td>
<td>1</td>
<td>0.800</td>
<td>0.746</td>
<td>0.727</td>
</tr>
<tr>
<td>$A_2$ (7.32 m)</td>
<td>0</td>
<td>0.800</td>
<td>1</td>
<td>0.934</td>
<td>0.910</td>
</tr>
<tr>
<td>$A_3$ (7.77 m)</td>
<td>0</td>
<td>0.746</td>
<td>0.934</td>
<td>1</td>
<td>0.975</td>
</tr>
<tr>
<td>$A_4$ (8.23 m)</td>
<td>0</td>
<td>0.727</td>
<td>0.910</td>
<td>0.975</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 4.6 Statistical features of annual net benefit for the five levee alternatives (considering sampling error $n=65$) using MML method

(a) Mean and standard deviations

<table>
<thead>
<tr>
<th>Alternative $A_i$ (Levee Ht.)</th>
<th>Annual net benefit ($10^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>$A_0$ (Status quo)</td>
<td>0</td>
</tr>
<tr>
<td>$A_1$ (6.68 m)</td>
<td>70.0</td>
</tr>
<tr>
<td>$A_2$ (7.32 m)</td>
<td>78.5</td>
</tr>
<tr>
<td>$A_3$ (7.77 m)</td>
<td>76.9</td>
</tr>
<tr>
<td>$A_4$ (8.23 m)</td>
<td>71.8</td>
</tr>
</tbody>
</table>

(b) Correlation coefficients

<table>
<thead>
<tr>
<th></th>
<th>$A_0$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_0$ (Status quo)</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$A_1$ (6.68 m)</td>
<td>0</td>
<td>1</td>
<td>0.796</td>
<td>0.741</td>
<td>0.721</td>
</tr>
<tr>
<td>$A_2$ (7.32 m)</td>
<td>0</td>
<td>0.796</td>
<td>1</td>
<td>0.932</td>
<td>0.908</td>
</tr>
<tr>
<td>$A_3$ (7.77 m)</td>
<td>0</td>
<td>0.741</td>
<td>0.932</td>
<td>1</td>
<td>0.974</td>
</tr>
<tr>
<td>$A_4$ (8.23 m)</td>
<td>0</td>
<td>0.721</td>
<td>0.908</td>
<td>0.974</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 4.7 Statistical features of annual net benefit for the five levee alternatives (considering sampling error \( n=65 \)) using PWM method

(a) Mean and standard deviations

<table>
<thead>
<tr>
<th>Alternative ( A_i ) (Levee Ht.)</th>
<th>Annual net benefit ($10^3$)</th>
<th>Mean</th>
<th>Rel. change</th>
<th>Stdev</th>
<th>Rel. change</th>
<th>C.O.V.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_0 ) (Status quo)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( A_1 ) (6.68 m)</td>
<td>70.4</td>
<td>4.8%</td>
<td>258.4</td>
<td>6.9%</td>
<td>3.7</td>
<td></td>
</tr>
<tr>
<td>( A_2 ) (7.32 m)</td>
<td>78.9</td>
<td>4.2%</td>
<td>317.9</td>
<td>5.1%</td>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td>( A_3 ) (7.77 m)</td>
<td>77.3</td>
<td>4.2%</td>
<td>339.3</td>
<td>4.6%</td>
<td>4.4</td>
<td></td>
</tr>
<tr>
<td>( A_4 ) (8.23 m)</td>
<td>72.2</td>
<td>4.5%</td>
<td>347.9</td>
<td>4.3%</td>
<td>4.8</td>
<td></td>
</tr>
</tbody>
</table>

(b) Correlation coefficients

<table>
<thead>
<tr>
<th></th>
<th>( A_0 )</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( A_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_0 ) (Status quo)</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( A_1 ) (6.68 m)</td>
<td>0</td>
<td>1</td>
<td>0.798</td>
<td>0.743</td>
<td>0.724</td>
</tr>
<tr>
<td>( A_2 ) (7.32 m)</td>
<td>0</td>
<td>0.798</td>
<td>1</td>
<td>0.933</td>
<td>0.909</td>
</tr>
<tr>
<td>( A_3 ) (7.77 m)</td>
<td>0</td>
<td>0.743</td>
<td>0.933</td>
<td>1</td>
<td>0.974</td>
</tr>
<tr>
<td>( A_4 ) (8.23 m)</td>
<td>0</td>
<td>0.724</td>
<td>0.909</td>
<td>0.974</td>
<td>1</td>
</tr>
</tbody>
</table>

4.3.4 Effect of Sampling Errors on Project Evaluation

The uncertainty due to sampling error in estimating \( Q_T \) is reducible and such reduction may lead to a decrease in the potential risk in terms of the EOL of a project. This section uses the rules mentioned previously to evaluate the economic merits of the four levee alternatives based on the calculated statistics of the net benefits. The rankings for the four levee alternatives are listed in Table 4.8 when the uncertainty due to sampling error is neglected, and in Table 4.9 when the sampling error is considered (\( n = 65 \)) and the distribution
parameters are estimated by the MoM. Without over-complicating the mathematical manipulations, a normal distribution is assumed for the random annual net benefit in the example. Tung [159] justified the appropriateness of the normality assumption for the net benefit based on the central limit theorem and the normal distribution for net benefits was also used by the USACE [9].

According to the expected-value rule, levee alternative $A_2$ is the best because it has the highest expected annual net benefit value, followed by alternatives $A_3$ and $A_4$, with $A_1$ ranked last. When the utility function is defined as

$$u(x) = -\exp\left(-\frac{x}{5000}\right), \quad (4-15)$$

the expected utilities for the four levee alternatives are listed in Table 4.8 and 4.9, and the rankings are $A_2 > A_3 > A_1 > A_4$ according to the expected utility theory. Referring to Table 4.4(a) and 4.5(a), the M-V rule indicates that alternative $A_1$ (the worse one with reference to the expected-value rule) is not dominated by or is superior to any other alternative because it has the lowest mean annual net benefit and the lowest uncertainty. Hence, its ranking cannot be determined, whereas the remaining three alternatives can be ranked as $A_2 > A_3 > A_4$. By the stochastic dominance rules, the FSD rule does not yield a preference between any two alternatives whereas the SSD produces the same ranking result as the M-V rule. The probabilities of getting negative net benefits, $P_i(NB_i < 0)$, for each alternative are also listed in Table 4.8 and 4.9. The rankings, in accordance with the magnitudes of probability-of-loss, are $A_1 > A_2 > A_3 > A_4$. In practice, these four probability values are too close to guide any decisive and meaningful choices.
Table 4.8 Project ranking results without considering sampling error \((n = \infty)\)

<table>
<thead>
<tr>
<th>Alternative</th>
<th>E-V(^a) (E(NB)) ($10^3)</th>
<th>EU(^b) EU</th>
<th>Probability of loss</th>
<th>Probability of loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_1)</td>
<td>67.2</td>
<td>60.96</td>
<td>0.391</td>
<td>1</td>
</tr>
<tr>
<td>(A_2)</td>
<td>75.7</td>
<td>66.15</td>
<td>0.401</td>
<td>2</td>
</tr>
<tr>
<td>(A_3)</td>
<td>74.1</td>
<td>63.20</td>
<td>0.410</td>
<td>3</td>
</tr>
<tr>
<td>(A_4)</td>
<td>69.0</td>
<td>57.60</td>
<td>0.418</td>
<td>4</td>
</tr>
</tbody>
</table>

\(^a\)E-V= expected-value rule; \(^b\)EU=expected utility theory

Table 4.9 Project ranking considering sampling error \((n = 65)\) with distribution parameters estimated using MoM method

<table>
<thead>
<tr>
<th>Alternative</th>
<th>E-V (E(NB)) ($10^3)</th>
<th>EU</th>
<th>Probability of loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_1)</td>
<td>70.8</td>
<td>63.57</td>
<td>0.393</td>
</tr>
<tr>
<td>(A_2)</td>
<td>79.3</td>
<td>68.57</td>
<td>0.402</td>
</tr>
<tr>
<td>(A_3)</td>
<td>77.7</td>
<td>65.57</td>
<td>0.410</td>
</tr>
<tr>
<td>(A_4)</td>
<td>72.6</td>
<td>59.95</td>
<td>0.418</td>
</tr>
</tbody>
</table>

Using the proposed decision-making framework, an EOL matrix can be constructed using Eq.(3-4a) by comparing each pair of the five alternatives (including the status quo \(A_0\)). Table 4.10 and 4.11, respectively, show the EOL matrix corresponding to without and with consideration of the sampling error. For example, referring to Table 4.10, \(EOL(A_1^*, A_2)\) is $79,400 corresponding to the selection of alternative \(A_1\) instead of \(A_2\). The ranking of the candidate alternatives is determined based on the minimax EOL rule. The best feasible
alternative is the one having the lowest maximum $EOL$ and satisfying the decision maker’s AR. Table 4.10 and 4.11 show that the maximum value of $EOL$ in choosing alternative $A_2$ is the smallest when compared to any other alternatives (i.e. $86,500$ and $92,400$). Consequently, alternative $A_2$ is ranked the best because it has the lowest maximum $EOL$, followed by alternative $A_1$, although its expected net benefit is the lowest. Despite the least $E(NB)$ of alternative $A_1$, it becomes more competitive compared to the others because the uncertainty of its net benefit is significantly less than the other competitors. The coefficient of variation of alternative $A_1$ is $3.7$ compared to the others that are around $4.0 \sim 4.8$ (see also Table 4.4(a) and 4.5(a)). Alternative $A_0$ is the worst alternative due to the highest maximum $EOL$. According to the minimax $EOL$ rule, alternative $A_2$ is decisively more preferable to $A_3$, although the expected net benefits of the two are very close. This is because the uncertainty of the net benefit associated with alternative $A_3$ is larger than $A_2$.

Without considering the sampling error, one assumes that complete data information is available, i.e., $n = \infty$. Comparing the values of $EOL_{\text{max}}(A_i)$ in Tables 4.10 and 4.11, the values of pair-wise $EOL$ and $EOL_{\text{max}}(A_{\text{opt}})$ under $n = \infty$ are less than those with $n = 65$. When more information is obtained, or the record length is increased, the sample estimators are more reliable with lower sampling errors. This leads to reduced values of $EOL$ and $EOL_{\text{max}}(A_{\text{opt}})$. As $EOL_{\text{max}}(A_{\text{opt}})$ decreases, the minimum level of $AR$ for having at least one feasible project can also be decreased. For example, with $n = 65$, the minimum level of the decision maker’s $AR$ is $92,400$; but with $n = \infty$, the minimum level of the decision maker’s $AR$ reduces to $86,500$ - about $6\%$ lower. This decreased value of $EOL$ indicates that the potential loss in taking a course of action can be reduced as more information becomes available. This reduction in the risk value, due to increased information by having a longer record length, can be viewed as the value of information ($VoI$). In other words, the $VoI$ can be viewed as the amount that a decision maker is willing to pay for securing additional information to improve the quality of the decision. Although more information leads to an improved decision, the decision maker should consider the balance between the $VoI$ and the cost of obtaining more information. In the next section, the effect of reducing $EOL$ with a longer record length is investigated.
Chapter 4  Incorporating Sampling Error into Risk-based Decision Making

Comparing the results in Table 4.10 and 4.11, the ranking of the five levee alternatives does not change with or without considering the sampling error when \( n = 65 \). Section 4.3.5 shows that the ranking of design alternatives can be affected by the sample size.

### Table 4.10 EOL matrix ($10^3$) without considering sampling error (\( n = \infty \))

<table>
<thead>
<tr>
<th>Chosen alternatives</th>
<th>Competing Alternatives</th>
<th>( EOL_{max}(A_i) )</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_0 )</td>
<td>0.0 133.8 162.3 169.9 170.4 170.4</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>( A_1 )</td>
<td>66.6 0.0 79.4 92.6 95.4 95.4</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>( A_2 )</td>
<td>86.5 70.8 0.0 47.5 54.3 86.5</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>( A_3 )</td>
<td>95.8 85.7 49.1 0.0 28.9 95.8</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>( A_4 )</td>
<td>101.4 93.5 61.0 34.0 0.0 101.4</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

### Table 4.11 EOL matrix ($10^3$) considering the effect of sampling error (\( n = 65 \)) with distribution parameters estimated using MoM method

<table>
<thead>
<tr>
<th>Chosen alternatives</th>
<th>Competing Alternatives</th>
<th>( EOL_{max}(A_i) )</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_0 )</td>
<td>0.0 143.6 171.7 179.0 179.3 179.3</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>( A_1 )</td>
<td>72.8 0.0 81.2 94.4 97.1 97.1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>( A_2 )</td>
<td>92.4 72.7 0.0 47.9 54.6 92.4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>( A_3 )</td>
<td>101.3 87.5 49.5 0.0 28.9 101.3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>( A_4 )</td>
<td>106.7 95.3 61.4 34.0 0.0 106.7</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>
4.3.5 Value of Information for Decision Making

To investigate the effect of sample size on the project ranking, the first two statistical moments of the project annual net benefit (shown in Figures 4.3 and 4.4) are calculated for sample sizes ranging from 5 to 150 years by considering the uncertainty of the flood quantile estimator. These figures show that the mean and standard deviation of the annual net benefits for each levee alternative decrease as the sample size increases. Decreasing uncertainty in the annual net benefits renders a decreasing value of $EOL_{\text{max}}(A_{\text{opt}})$, as shown in Figure 4.5.

In terms of the values of $EOL_{\text{max}}(A_i)$, it is interesting to note that the ranking of levee alternatives can change with the sample size and parameter estimation method when the sampling error of the flood quantile estimator is considered. A decrease in $EOL_{\text{max}}(A_{\text{opt}})$ will influence the ranking of the alternatives and enhance their feasibility with reference to AR. In this case, the sample size that would change the ranking among the design alternatives is between 20~30 record years, depending on the parameter estimation method used. With different parameter estimation methods, alternative $A_1$ is ranked the best when the sample size is less than 30, 25, and 20 years, respectively, when sampling error is computed by the MoM, PWM, and MML methods. As the record length gets higher, alternative $A_2$ becomes the best.

The $VoI$ for choosing an optimum project can be expressed as the marginal rate of decrease in $EOL_{\text{max}}(A_{\text{opt}})$ with respect to sample size as

$$
\frac{dEOL_{\text{max}}(A_{\text{opt}})}{dn}
$$

(4-16)

Figure 4.6 shows the sensitivity of $EOL_{\text{max}}(A_{\text{opt}})$ with respect to sample size $n$ for the levee example problem. When more information is used, the potential risk is lowered and the required AR can be reduced. When $n$ is not large, the rate of marginal improvement in reducing the potential risk per unit increase in sample size is high. However, the marginal rate of improvement diminishes as the sample size $n$ gets larger. The required additional pieces of sample data to achieve the decision maker’s AR can be determined by interpolating the data points in Figure 4.5 or by using Eq.(4-16). For example, by the MoM, $EOL_{\text{max}}(A_{\text{opt}})$ is $92,400 according to the 65 years of data records. When $AR = \$90,000$, $EOL_{\text{max}}(A_{\text{opt}})$ is larger than the
decision maker’s AR and there is no feasible project. In order to reduce the uncertainty and therefore achieve the decision maker’s AR, at least 111 years of systematic data would be required according to Figure 4.5. So that the required additional record length to achieve the decision maker’s AR will be 46 years. The decision maker should not forget the trade-off between the cost of getting more information and the potential gain from making a better decision by using that information. If one is concerned with the sensitivity of the EOL$_{max}$ of a particular alternative $A_i$ with respect to sample size $n$, Eq. (4-15) can similarly be used in which EOL$_{max}(A_{opt})$ is replaced by EOL$_{max}(A_i)$.

Figure 4.3 Variation of mean annual net benefit
with respect to sample size

(a) Top left. Alternative $A_1$. (b) Top right. Alternative $A_2$.
(c) Bottom left. Alternative $A_3$. (d) Bottom right. Alternative $A_4$.

Note: inf = infinity
Figure 4.4 Variation of standard deviation of annual net benefit with respect to sample size

(a) Top left. Alternative $A_1$.
(b) Top right. Alternative $A_2$.
(c) Bottom left. Alternative $A_3$.
(d) Bottom right. Alternative $A_4$.

Note: inf = infinity
Figure 4.5 Variation of $EOL_{\text{max}}(A_{\text{opt}})$ with respect to sample size

![Graph showing variation of $EOL_{\text{max}}(A_{\text{opt}})$ with respect to sample size.](image)

Figure 4.6 Marginal rate of $EOL_{\text{max}}(A_{\text{opt}})$ with respect to sample size

![Graph showing marginal rate of $EOL_{\text{max}}(A_{\text{opt}})$ with respect to sample size.](image)
4.3.6 Effects of $AR$ on the Project Evaluation

The $AR$ can be used to check if the $EOL$, in selecting one alternative, would meet the decision maker’s risk tolerance in case the decision is wrong. A feasible alternative should have its maximum $EOL$ value lower than the $AR$. Referring to Table 4.10, when the $AR$ is under $86,500$, none of the alternatives are acceptable to the decision maker. In this case, the decision maker could try to reduce the uncertainty by conducting further research, obtaining more information, or increasing the contingency fund to prepare for any additional loss if the decision is incorrect. When the $AR$ is between $86,500$ and $95,400$, the best feasible option is alternative $A_2$, a plan that can be implemented. When the $AR$ is higher than $170,400$, all five alternatives (including the status quo $A_0$) have lower values are therefore feasible to the decision maker, meaning, in theory, any alternative can be chosen. However, in practice, it would be logical to select the alternative with the lowest maximum $EOL$.

4.3.7 Effects of Net Benefit Correlation on Alternative Ranking

As shown in Tables 4.4(b)–4.7(b), the random net benefits associated with the four levee alternatives are highly correlated. This correlation is mainly attributed to the dependence of the net benefits on the same common random factor, namely, the flood magnitude. To examine the effect of such correlation on decision making, an assumption is made that the net benefits among all four alternatives are independent and the values of $EOL$ are recalculated for subsequent alternative rankings.

Table 4.12 lists the maximum values of $EOL$ and the rankings of the four levee alternatives ($EOL_{max}(A_i)$, $i = 1, 2, 3, 4$) under the conditions that the net benefits are assumed uncorrelated ($\rho_{ij} = 0$) and $n = \infty$, when in reality they are actually correlated. The $EOL$ values are smaller when the net benefits between the alternatives are positively-correlated than when no correlation is assumed between the net benefits of the alternatives. Table 4.12 also shows that the rankings of the four levee alternatives are significantly affected by the presence of the correlation between the net benefits of the alternatives.
Table 4.12 Maximum EOL values and ranking for each alternative, with uncorrelated and correlated net benefits

<table>
<thead>
<tr>
<th>Chosen Alternative $A_i$</th>
<th>Uncorrelated net benefits</th>
<th>Correlated net benefits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$EOL_{\text{max}}(A_i)$ ($10^3$)</td>
<td>Ranking</td>
</tr>
<tr>
<td>$A_0$ (Status quo)</td>
<td>170.4</td>
<td>2</td>
</tr>
<tr>
<td>$A_1$ (6.68 m)</td>
<td>165.3</td>
<td>1</td>
</tr>
<tr>
<td>$A_2$ (7.32 m)</td>
<td>176.3</td>
<td>3</td>
</tr>
<tr>
<td>$A_3$ (7.77 m)</td>
<td>183.1</td>
<td>4</td>
</tr>
<tr>
<td>$A_4$ (8.23 m)</td>
<td>188.2</td>
<td>5</td>
</tr>
</tbody>
</table>

4.4 Summary and Conclusions

This chapter presents a framework for the risk-based evaluation of flood-damage-reduction projects, considering both aleatory and epistemic uncertainties. Both types of uncertainty contribute to the uncertainty of the project net benefits. Aleatory uncertainty arises from the randomness of the natural processes and is irreducible. Epistemic uncertainty is a result of knowledge deficiency on the system and is not commonly considered in risk-based design. In this study, the epistemic uncertainty associated with the sampling error of a quantile estimator due to the use of a finite sample size is considered to quantify the uncertainty features of project net benefits.

The methodological framework is applied to a levee design problem in which the flood frequency-damage relationships were extracted and modified from a particular case study. The standard error of the Gumbel flood quantiles were estimated by three parameter estimation methods. Statistical features of the annual net benefits associated with levee alternatives were assessed considering different types of the uncertainties. The statistical
features of annual net benefits were used in the evaluation of levee alternatives by different decision making rules. The findings of this study can be summarized as follows:

(1) Incorporating aleatory and epistemic uncertainties in risk-based project design provides more comprehensive information for project evaluation;

(2) Uncertainty in the project net benefits is found to be quite significant and should not be overlooked;

(3) Ignoring epistemic uncertainty due to sampling error could result in a significant under-estimation of the potential project risk, especially when the sample size is small.

(4) Having a longer record length improves the quality of information, enhances the reliability of the estimator, and reduces the sampling error. The expectation and the uncertainty of the project net benefits are also decreased.

(5) The potential risk in terms of $EOL$ decreases when the sampling error is decreased. Having a longer record length is a viable way to reduce the project potential risk. However, one still has to consider the balance between the cost of securing more information and the gain in making a better decision. In case no feasible project meets the decision maker’s acceptable risk, the relation between $EOL$ and record length can be used to determine the proper sample size to meet the acceptable risk.

(6) The ranking of project alternatives based on the minimax $EOL$ rule can be changed by adjusting the sample size.

(7) Different parameter estimation methods will influence the value of the statistics of the project net benefit and the maximum $EOL$ value of each project. Thus, parameter estimation methods will affect the ranking, especially when the sample size is small.

Bromley and Beattie [171] and Hoehn and Randall [172] show the limitations in the assessment of project benefits. Failure to consider other nonmonetary types of benefits may underestimate the project benefits. A more appropriate evaluation process is to involve other nonmonetary types of project benefits along with the monetary benefits in achieving the diverse objectives [173]. Therefore, extending the proposed risk-based project evaluation process to a risk-based multi-criteria decision-making framework is recommended.
CHAPTER 5
INCORPORATING UNCERTAINTY IN
MULTI-CRITERIA DECISION MAKING

A hydrosystem design project typically involves several objectives to be evaluated by different criteria. Issues related to the most commonly considered objectives are economic efficiency, economic development, technical feasibility, water quality and supply, and the fairness and equity of resources distribution. The performance of each design project in achieving different objectives are usually measured by different units. The non-commensurable and intangible factors can be just as important as the economic factors quantifiable by monetary terms in the decision making of public affairs. In this regard, multi-criteria decision making (MCDM) is a more appropriate methodology. MCDM helps to improve the decision quality by providing a systematic, comprehensive, transparent, and auditable framework to resolve conflict and obtain a compromise solution. It also complements conventional cost-benefit analysis by eliminating the bias when considering the economic efficiency as the only project evaluation criterion. As a result, it facilitates communication among analysts, decision makers, and stakeholders in different groups and disciplines and integrates diverse disciplines. Therefore, MCDM has received more and more attention for hydrosystems design and researches into the application of the existing MCDM techniques, and the development of innovative techniques was grown rapidly in recent decades.
5.1 Introduction

Most real-world decision-making problems are multi-criteria by nature. Decision alternatives (or course of actions) are evaluated according to a set of criteria. The criteria reflect the characteristics of the decision goals and are used to evaluate the achieving of the decision goals. These criteria, quantitative and qualitative (intangible), are often in conflict with each other and are non-commensurable. For sustainable hydrosystem planning and management, safety, economic, environmental, social, and technical criteria are often considered concurrently. In such cases, a solution that simultaneously optimizes all the criteria generally does not exist. A compromise solution would have to be made in an MCDM problem.

The procedure for MCDM analysis generally follows the following steps: (i) identifying the decision context, including the decision makers, stakeholders and decision constraints; (ii) defining the decision criteria; (iii) eliciting the relative importance of the criteria weights; (iv) generating a set of candidate decision alternatives; (v) evaluating the performance values of decision alternatives against the criteria; (vi) applying suitable techniques for MCDM; (vii) performing a sensitivity analysis; and (viii) making the final decision.

A wide variety of multi-criteria decision making (MCDM) methods have been developed to help decision makers in solving complex decision making situations of a multi-criteria nature. A detailed review of various existing MCDM techniques can be found in Section 2.4.2. By these methods, the alternatives are generally described, evaluated, ranked and selected on the basis of deterministic evaluation of the consequences of each alternative with respect to each criterion. However, various sources of uncertainties are present in the decision making process, including the external uncertainties (related to the imperfect information of the consequences of each alternative) and internal uncertainties (related to subjective human judgment). The objective of this chapter is to construct an MCDM framework that explicitly considers the external uncertainty on the consequences of the hydrosystem design and management alternatives. In particular, the concept of PROMETHEE (Preference Ranking Organization Method of Enrichment Evaluation) technique, one of the
most widely applied MCDM techniques, is selected in this study for two reasons: (1) the procedure of PROMETHEE is amenable to probabilistic treatment for dealing with a MCDM problem under uncertainty; and (2) the logic of the minimax \( EOL \) for single criterion decision making can easily be incorporated in the PROMETHEE because both are based on pair-wise comparison of the outcome difference (deviation) of the chosen alternative and a forgone alternative. The probabilistic features of the performance values associated with an alternative, instead of using the original deterministic performance values, is taken into account and used in the PROMETHEE MCDM technique. The integration of PROMETHEE and decision making under uncertainty is demonstrated through a case study considering sustainable water resources planning.

5.2 Information Requirement for a MCDM Analysis

Information concerning the performance of decision alternatives is the basis for comparing and evaluating the choices, and facilitates the selection of a satisfactory solution. Information on the performance values can be collected in a payoff matrix as shown in Table 5.1. Each row represents a different decision alternative in a set of decision alternatives \( \mathbf{A} = \{A_i | i = 1, 2, \ldots, N\} \) and each column represents a set of different evaluation criteria. A value \( x_{ik} \) in row \( i \) and column \( k \) of the payoff matrix represent the performance value for alternative \( A_i \) with respect to the \( k \)-th criterion \( C_k \).

The criteria weights are the required information in MCDM to represent a decision makers’ view on the relative importance of the criteria. They are rescaling factors that are also recognized as the swing weights [106]. The values of weight are non-negative and normally add up to one (normalized weights). These values can be elicited subjectively and objectively. Subjective weights can be determined by the decision maker directly or by means of the swing weight method [174], the rank-order centroid weighting method [175], the eigenvector method [125], the weighted least square method [176], the linear programming techniques for multi-dimensional analysis of preference [177], or the extreme weight approach [178]. Objective weights can be determined by the entropy method proposed by Shannon [179] and later applied and advocated by Capocelli and Luca [180].
### Table 5.1 General payoff matrix for MCDM

<table>
<thead>
<tr>
<th>Design alternatives</th>
<th>Decision criteria</th>
<th>Criteria weights</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_1$</td>
<td>$C_2$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>$x_{11}$</td>
<td>$x_{12}$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$x_{21}$</td>
<td>$x_{22}$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$A_i$</td>
<td>$x_{i1}$</td>
<td>$x_{i2}$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$A_N$</td>
<td>$x_{N1}$</td>
<td>$x_{N2}$</td>
</tr>
</tbody>
</table>

#### 5.3 Bran’s PROMETHEE Techniques

PROMETHEE was proposed by Brans in 1982 for constructing the dominance order in MCDM problems [119]. The PROMETHEE techniques (PROMETHEE-I, PROMETHEE-II, and PROMETHEE-GAIA visual interactive module) belong to the category of outranking methods that are based on the preference function between different alternatives. Some characteristic features of this multi-criteria technique are: (1) pair-wise comparison between every two decision alternatives are based on the deviations of the two performance values; (2) taking into account the scale effects; (3) either a partial or a complete ranking of the alternative can be constructed; and (4) the procedure is sufficiently simple and transparent.

Because of the transparent nature of the PROMETHEE techniques and that it can be easily understood by decision makers and analysts, the methods are widely used in many fields, such as banking, investments, medicine, and chemistry. Applications of PROMETHEE in water resources management problems can be found in ([17]; [113]; [114]; [117]; [134]; [140]; [153]).
The three phases for the PROMETHEE-I and II techniques are: (1) construction of preference functions; (2) determination of an outranking relation on the alternatives; and (3) evaluation of this relation in order to give a final decision. The PROMETHEE-I and II techniques can be applied when a payoff matrix, such as Table 5.1, is provided. The procedures for implementing PROMETHEE-I or -II are given in Figure 5.1. Supplementary information on the criteria weights, \( w_k \) for \( k = 1, \ldots, K \), which are considered as the relative importance among the criteria, also need to be specified, either subjectively by the decision-maker or objectively by the entropy method as mentioned earlier.

The first phase of Brans’ PROMETHEE technique is to construct the preference functions. The preference function is used to specify the degree of preference and the perception of scales within the criteria. In the manner of a pairwise comparison, it is a function of the deviation \( d_k \) between the performance values of two alternatives for each criterion \( C_k \) (i.e., \( d_k(A_i, A_j) = x_{ik} - x_{jk} \)). Please note that \( d_k(A_i, A_j) \) under one specific criterion has the same meaning of the outcome difference \( \Delta_{i*,j} \) for EOL calculation as defined in Section 3.2. Six types of preference functions recommended by Brans et al. [119], along with their required parameters, are shown in Table 5.2, in which type I to VI preference functions are the usual criterion, the U-shape criterion, the V-shape criterion, the level criterion, the V-shape criterion with indifference criterion, and the Guassian criterion, respectively. The indifference threshold \( (q) \), preference threshold \( (p) \) and intermediate value \( s \) between \( p \) and \( q \) may also have to be specified, depending on the selected preference function. The preference functions for each criterion are used to rescale the non-commensurable performance values \( d_k \) to a commensurable scale in terms of preference, called the generalized criterion \( P_k \). The preference value is a function of \( d_k \) and always lies between 0 and 1, i.e., \( 0 \leq P_k \leq 1 \).
Figure 5.1 The procedures for implementing PROMETHEE I and II.

1. Building the outranking relation
   - Determine the multicriteria preference index of $A_i$ over $A_j$:
     $$\pi(A_i, A_j) = \frac{1}{\sum_{i=1}^{K} w_k P_k(A_i, A_j)}$$
   - Determine the positive outranking index (leaving flow):
     $$\pi^+(A_i) = \frac{1}{N-1} \sum_{i \neq j} \pi(A_i, A_j)$$
   - Determine the negative outranking index (entering flow):
     $$\pi^-(A_i) = \frac{1}{N-1} \sum_{i \neq j} \pi(A_i, A_j)$$
   - Determine the net outranking index of $A_i$ for PROMETHEE II:
     $$\Pi(A_i) = \pi^+(A_i) - \pi^-(A_i)$$

2. Exploiting the outranking relation
   - PROMETHEE I:
     $A_i \succ A_j; A_j \succ A_i; A_i \sim A_j; \text{ or } A_i \parallel A_j$
   - PROMETHEE II:
     $A_i \succ^* A_j; A_j \succ^* A_i; \text{ or } A_i \sim^* A_j$

3. Evaluation of outranking relation to yield a final decision
## Table 5.2 Six types of preference functions (adapted from Brans et al. [119])

<table>
<thead>
<tr>
<th>Type of preference functions</th>
<th>Definition</th>
<th>Shape</th>
<th>Required parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type I:</strong> Usual function</td>
<td>$P_k(d_{i,j}) = \begin{cases} 0, &amp; d_{i,j} \leq 0; \ 1, &amp; d_{i,j} &gt; 0. \end{cases}$</td>
<td><img src="image" alt="Graph" /></td>
<td>--</td>
</tr>
<tr>
<td><strong>Type II:</strong> U-shape function</td>
<td>$P_k(d_{i,j}) = \begin{cases} 0, &amp; d_{i,j} \leq q; \ 1, &amp; d_{i,j} &gt; q. \end{cases}$</td>
<td><img src="image" alt="Graph" /></td>
<td>$q$</td>
</tr>
<tr>
<td><strong>Type III:</strong> V-shape function</td>
<td>$P_k(d_{i,j}) = \begin{cases} 0, &amp; d_{i,j} \leq 0; \ \frac{d_{i,j}}{p}, &amp; 0 &lt; d_{i,j} \leq p; \ 1, &amp; d_{i,j} &gt; p. \end{cases}$</td>
<td><img src="image" alt="Graph" /></td>
<td>$p$</td>
</tr>
<tr>
<td><strong>Type IV:</strong> Level function</td>
<td>$P_k(d_{i,j}) = \begin{cases} 0, &amp; d_{i,j} \leq q; \ \frac{1}{2}, &amp; q &lt; d_{i,j} \leq p; \ 1, &amp; d_{i,j} &gt; p. \end{cases}$</td>
<td><img src="image" alt="Graph" /></td>
<td>$q, p$</td>
</tr>
<tr>
<td><strong>Type V:</strong> V-shape function with indifference area</td>
<td>$P_k(d_{i,j}) = \begin{cases} 0, &amp; d_{i,j} \leq q; \ \frac{d_{i,j} - q}{p - q}, &amp; q &lt; d_{i,j} \leq p; \ 1, &amp; d_{i,j} &gt; p. \end{cases}$</td>
<td><img src="image" alt="Graph" /></td>
<td>$q, p$</td>
</tr>
<tr>
<td><strong>Type VI:</strong> Gaussian function</td>
<td>$P_k(d) = \begin{cases} 0, &amp; d \leq 0; \ 1 - e^{-d^2/s^2}, &amp; d &gt; 0. \end{cases}$</td>
<td><img src="image" alt="Graph" /></td>
<td>$s$</td>
</tr>
</tbody>
</table>
The second phase of the PROMETHEE technique is to build the outranking relation. The multi-criteria preference index for choosing decision alternative $A_i$ over $A_j$ is defined as

$$
\pi(A_i, A_j) = \frac{1}{\sum_{k=1}^{K} w_k} \sum_{k=1}^{K} w_k P_k(A_i, A_j)
$$

which is the weighted average of the preference functions, $P_k(A_i, A_j)$, aggregated over all the criteria, where $w_k$ is the weight assigned to criterion $C_k$.

The positive outranking index of alternative $A_i$, $\pi^+$, expresses the degree to which the alternative dominates all the other $(N-1)$ alternatives (also recognized as the leaving flow or the strength of $A_i$) and is defined as

$$
\pi^+(A_i) = \frac{1}{N-1} \sum_{j \neq i}^{N} \pi(A_i, A_j)
$$

On the other hand, the negative outranked index (outranked index) of alternative $A_i$, $\pi^-$, is defined as

$$
\pi^-(A_i) = \frac{1}{N-1} \sum_{j \neq i}^{N} \pi(A_j, A_i)
$$

which expresses how much an alternative is dominated by all the other $(N-1)$ alternatives. A schematic sketch of positive and negative outranked indices is shown in Figure 5.2.

### 5.3.1 PROMETHEE-I Partial Ranking

In PROMETHEE-I partial ranking methods (including possible incomparability relationship in the ranking result), the higher the outranking index and the lower the outranked index, the better the alternative. The leaving and entering flows induce the following pre-orders, respectively, on alternatives:
where \( > \), \( \sim \), and \( \parallel \) represent preference, indifference, and incomparability, respectively, between two alternatives based on the PROMETHEE-I method.

![Figure 5.2 An schematic sketch of positive and negative outranking indices](image_url)

5.3.2 PROMETHEE-II Complete Ranking:

To yield a complete ranking (no incomparability relationship remains in the ranking result), the net outranking index (net flow) of alternative \( A_i \) is defined as
\[ \Pi(A_i) = \pi^+(A_i) - \pi^-(A_i) \]  

(5-4)

which is the balance between the positive and negative outranking indices. The decision alternative having the largest net outranking index is considered as the best.

\[ A_i \triangleright^\Pi A_j \iff \Pi(A_i) > \Pi(A_j) \]

\[ A_i \sim^\Pi A_j \iff \Pi(A_i) = \Pi(A_j) \]

where \( \triangleright^\Pi \) and \( \sim^\Pi \) represent preference and indifference, respectively, between two alternatives based on the PROMETHEE-II method.

### 5.4 Proposed Approach for MCDM under Risk

In this section, the external uncertainties are considered in MCDM in which the performance values in the payoff matrix are uncertain. Instead of using the deterministic performance values in Brans’ PROMETHEE methods, they are denoted as random variables.

Figure 5.3 is the flowchart of the proposed procedure for an MCDM under uncertainty. After identifying the decision context, defining the decision criteria and the associated weights, and generating a set of decision alternatives, information on the probabilistic features (probability density functions and statistics) of the performance values for the alternatives are acquired.

To take into account the probabilistic features of consequences associated with an alternative, the expected net outranking index is derived and used for decision analysis. The derivation of the expected net outranking index for normally distributed performance values are shown in Sub-section 5.4.1 using the Type-VI preference function (Gaussian criterion). A sensitivity analysis is routinely recommended to be part of any MCDM application. After these processes, the best feasible alternative can be selected for implementation.
5.4.1 Expected Net Outranking Index

The expected net outranking index \( E[\Pi(A_i)] \) can be written as

\[
E[\Pi(A_i)] = E[\pi^+(A_i) - \pi^-(A_i)] = \frac{1}{N-1} \sum_{j=1}^{N} \sum_{k=1}^{K} w_k \left[ E[P_k(\Delta_{i,j}^*)] - E[P_k(\Delta_{j,i}^*)] \right]
\]  \hspace{1cm} (5-5)

where \( \Delta_{i,j}^* = X_{ik} - X_{jk} \) is the payoff difference of the selected alternative \( A_i \) and the unselected one \( A_j \) under a particular criterion \( C_k \). It is also a random variable because it is the difference of two random payoff values \( X_{ik} \) and \( X_{jk} \).

Referring to Eq.(3-20b), \( E[P_k(\Delta_{i,j}^*)] \) is equal to \( EOG(A_i^*, A_j) \) under criterion \( C_k \) when \( P_k(\cdot) = L(\cdot) \). Therefore, Eq. (5.5) can be also expressed as

\[
E[\Pi(A_i)] = \frac{1}{N-1} \sum_{j=1}^{N} \sum_{k=1}^{K} w_k \left[ EOG(A_i^*, A_j) - EOG(A_i, A_j) \right]
\]  \hspace{1cm} (5-6)

Assuming that \( \Delta_{i,j}^* \) is normally distributed with mean \( \mu_{\Delta} \) and standard deviation \( \sigma_{\Delta} \) and the Type-VI preference function (see Table 5.2) is used, the expected preference value conditioned on a particular criterion \( C_k \) in Eq.(5-5) can be derived as

\[
E[P_k(\Delta_{i,j}^*)] = \Phi \left( \frac{\mu_{\Delta}}{\sigma_{\Delta}} \right) - \int_{0}^{\infty} \exp \left( -\frac{\delta^2}{2\sigma_{\Delta}^2} \right) f_{\Delta_{i,j}^*}(\delta) d\delta
\]  \hspace{1cm} (5-7)

The expected net outranking index \( E[\Pi(A_i)] \) is then be derived as

\[
E[\Pi(A_i)] = \frac{1}{N-1} \sum_{j=1}^{N} \sum_{k=1}^{K} w_k \left[ 2\Phi(\beta) - \frac{\sigma'}{\sigma_{\Delta}} \exp(-\mu_{\Delta} \mu' \alpha_\lambda) (2\Phi(\beta') - 1) \right] \]  \hspace{1cm} (5-8)
where \( \alpha_k = \frac{1}{2s_k^2}, \quad \mu' = \frac{\mu_\Delta}{2\sigma_\Delta^2 \alpha_k + 1}, \quad \sigma'^2 = \frac{\sigma_\Delta^2}{2\sigma_\Delta^2 \alpha_k + 1}, \quad \beta = \frac{\mu_\Delta}{\sigma_\Delta} \) and \( \beta' = \frac{\mu'}{\sigma'} \). Detailed derivation of Eqs.(5-7) and (5-8) can be found in Appendix-D.

Figure 5.3 MCDM under uncertainty procedure

1. **Problem Identification**
   - Identify decision context:
     (constraints, decision makers and stakeholders)
   - Define criteria \((C_1, C_2, \ldots, C_K)\)
   - Generate possible alternatives \((A_1, A_2, \ldots, A_N)\)
   - Define criteria weights \((w_1, w_2, \ldots, w_K)\)
   - Assign probabilistic features of alternative performance values \((v_{ik})\)

2. **MCDM Analysis**
   - Define preference function for each criterion \(P_i(A_k, A_j)\)
   - Calculate expected net outranking index \(E[\Pi(A_i)]\)
   - Alternative ranking by PROMETHEE II

3. **Sensitivity Analysis**

4. **Making the final decision and implementing the chosen alternatives**
5.4.2 Ranking of Alternatives by Expected Net Outranking Index

Following the concept of Bran’s PROMETHEE technique and using the expected net outranking index of each alternative for ranking the decision alternatives, the preference order can be established as

\[ A_i \succ A_j \iff E[\Pi(A_i)] > E[\Pi(A_j)] \]

\[ A_i \sim A_j \iff E[\Pi(A_i)] = E[\Pi(A_j)] \]

where \( \succ \) and \( \sim \) represent preference and indifference, respectively, between two alternatives based on the PROMETHEE technique under uncertainty. The decision alternative having the largest expected net outranking index is considered to be the best.

5.5 Example Application of MCDM under uncertainty

In this section, an example case study extracted from Raju et al. [134], is used to illustrate the application of the proposed framework of an MCDM under uncertainty and to investigate how the uncertainty degrees of the consequences influence the ranking result. Discussion is also made about the influence of the outcome correlations and criteria weights on the final decision.

The example study area is located in the Flumen Monegros irrigation region in the Huesca Province in northeastern Spain. The problems confronting the decision-makers are the growing requirements of the irrigation systems all around the Mediterranean, and concern about water sustainability due to the intensive use of water resources. The decision makers need to find a solution for an irrigation system, while considering the sustainability of water resources. Three categories of criteria for evaluating the alternatives include (i) economic factors such as the initial cost of the irrigation system, maintenance cost, profitability of the crops, and the extent of European Union subsidies; (ii) environmental factors such as irrigation water volume, water quality after irrigation, efficiency of water use, and resistance to floods or droughts; and (iii) social factors including local employment, and land areas
which are not cultivated [134]. Three groups of decision makers (actor 1, actor 2, and actor 3) prioritize the factor categories of economic effects, environmental (sustainability) effects, and social effects, respectively. Their inclined attitudes are shown in three sets of criterion weights. The description of the criteria and the three sets of criteria weights are shown in Table 5.3.

Seven management alternatives are considered by the analysts to regulate the increase in water use. The descriptions of these alternatives with respect to different irrigation plans are shown in Table 5.4.

---

**Table 5.3 Criteria descriptions and weights for three groups of decision makers for the case study in Flumen Monegros (Adapted from Raju et al. [134]).**

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Descriptions of criteria</th>
<th>Criterion weights</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Actor 1</td>
</tr>
<tr>
<td>Economic factors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_1$ Initial cost often paid by the State</td>
<td>0.1</td>
<td>0.06</td>
</tr>
<tr>
<td>$C_2$ Maintenance cost</td>
<td>0.1</td>
<td>0.06</td>
</tr>
<tr>
<td>$C_3$ Profitability of crops</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>$C_4$ Extent of European Union subsidies</td>
<td>0.1</td>
<td>0.03</td>
</tr>
<tr>
<td>Environmental (sustainability based) factors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_5$ Irrigation water volume</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$C_6$ Water quality after irrigation</td>
<td>0.06</td>
<td>0.1</td>
</tr>
<tr>
<td>$C_7$ Efficiency of water use</td>
<td>0.06</td>
<td>0.2</td>
</tr>
<tr>
<td>$C_8$ Resistance to floods or droughts</td>
<td>0.03</td>
<td>0.1</td>
</tr>
<tr>
<td>Social factors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_9$ Employment of the population</td>
<td>0.125</td>
<td>0.125</td>
</tr>
<tr>
<td>$C_{10}$ Land area which is not cultivated</td>
<td>0.125</td>
<td>0.125</td>
</tr>
</tbody>
</table>
Table 5.4 Irrigation planning alternatives for case study in Flumen Monegros  
(Adapted from Raju et al. [134]).

<table>
<thead>
<tr>
<th>Alts</th>
<th>Descriptions of management alternatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>Combination of sprinkler irrigation system without changing the existing water pricing and allocation policy</td>
</tr>
<tr>
<td>$A_2$</td>
<td>Modification of $A_1$ without changing the existing cropping pattern</td>
</tr>
<tr>
<td>$A_3$</td>
<td>Modification of $A_1$ and increasing water pricing to 10 ptas/m³</td>
</tr>
<tr>
<td>$A_4$</td>
<td>Modification of $A_1$ and increasing water pricing to 20 ptas/m³</td>
</tr>
<tr>
<td>$A_5$</td>
<td>Combination of drip irrigation system and changing the existing water pricing to 20 ptas/m³ with existing allocation policy</td>
</tr>
<tr>
<td>$A_6$</td>
<td>Combination of drip irrigation system and changing the existing water pricing to 10 ptas/m³ with introduction of water quotas</td>
</tr>
<tr>
<td>$A_7$</td>
<td>Combination of sprinkler irrigation system and changing the existing water pricing to 20 ptas/m³ with introduction of market quotas</td>
</tr>
</tbody>
</table>

Note: 1 USD $\approx$ 125.603 ptas

Table 5.5 shows the payoff matrix representing the performance values of each alternative for the ten criteria. These values are evaluated by experts who are familiar with the planning project. Non-numerical indicators are converted into numerical values. In this study, these values are assumed to be the mean performance values. The probability distributions of performance values of all cases are assumed to be normally distributed.
5.5.1 Application of the Proposed Approach for MCDM under Uncertainty

The following assumptions are made in the first part of the case study for demonstration purposes. The coefficient of variation ($\Omega$) of the performance values is assumed to be 0.3, and represents a general degree of uncertainty. In reality, the performance values under each particular criterion can be inter-correlated. The correlation of performance values for each pair of alternatives ($A_i$ and $A_j$), $\rho_{ij}$, are assumed to be 0.7. The correlations of performance values across two different criteria are neglected. The first group of decision maker’s criteria weights are used in conjunction with the Type-VI preference function for each criterion. The preference function parameter $s$ is defined as 5.0, following the setting used in [134]. The uncertainty of the criteria weights is not considered as it belongs to the internal uncertainty and is out of the scope of this study.

Table 5.6 shows the multi-criteria preference index obtained from Eq.(5-1). The expected positive outranking index is the average of each row and the expected negative outranking index is the average of each column of the matrix of $E[\pi(A_i, A_j)]$. The expected net outranking index, shown in Table 5.7, is the difference between the expected positive and
negative outranking indices as Eq.(5-4). For example, $E[\Pi(A_i)] = 0.61 - 0.28 = 0.33$. The result shows that the alternative $A_1$, having the highest expected net outranking index, would be considered the best. The ranking patterns are also shown in Table 5.7.

5.5.2 Sensitivity Analysis

The sensitivity of (1) uncertainty degrees of the performance values, (2) correlation of the performance values within each particular criterion, and (3) the adopted criteria weights by different decision maker’s opinions on the ranking results are discussed. Sensitivity analysis of the three parts are made by designing different parameters from the baseline of the previous settings (i.e., the performance values are uncertain $(\Omega = 0.3)$ and inter-correlated $(\rho_{i,j} = 0.7)$ within a criterion, but not correlated across different criteria. The first group of decision maker’s criteria weights and Type-VI preference function $(s = 5)$ for each criterion are used. First, values of $\Omega$ (0.0, 0.3, and 0.8) are used to investigate the effect of the uncertainty degree on the ranking of alternatives. Second, the influence of the correlation of the performance values within each particular criterion is investigated by using two values of $\rho_{i,j}$ (0.0 and 0.7). Third, the influence of the criteria weights assigned by decision makers with different orientations towards the importance of the economic, environmental, or social factors on the results are discussed.

Table 5.8 shows the expected net outranking indices and the ranking of the seven alternatives by assuming different combinations of $\Omega$ and $\rho_{i,j}$. In Table 5.8, the expected net outranking indices for $\Omega = 0.0$ and $\rho_{i,j} = 0.0$ are similar to those values in Raju et al. [134]. The difference is come from different preference functions that are used.
Table 5.6 Expected multi-criteria preference index values and expected positive and negative outranking index values for the first set of criteria weights ($\Omega = 0.3$ and $\rho_{ij} = 0.7$).

<table>
<thead>
<tr>
<th></th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
<th>$A_5$</th>
<th>$A_6$</th>
<th>$A_7$</th>
<th>$E[\pi^+(A_i)]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0.000</td>
<td>0.462</td>
<td>0.518</td>
<td>0.553</td>
<td>0.667</td>
<td>0.690</td>
<td>0.743</td>
<td>0.606</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.386</td>
<td>0.000</td>
<td>0.497</td>
<td>0.543</td>
<td>0.668</td>
<td>0.681</td>
<td>0.761</td>
<td>0.589</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.321</td>
<td>0.359</td>
<td>0.000</td>
<td>0.448</td>
<td>0.615</td>
<td>0.640</td>
<td>0.692</td>
<td>0.513</td>
</tr>
<tr>
<td>$A_4$</td>
<td>0.299</td>
<td>0.328</td>
<td>0.373</td>
<td>0.000</td>
<td>0.587</td>
<td>0.604</td>
<td>0.673</td>
<td>0.477</td>
</tr>
<tr>
<td>$A_5$</td>
<td>0.236</td>
<td>0.242</td>
<td>0.264</td>
<td>0.278</td>
<td>0.000</td>
<td>0.427</td>
<td>0.558</td>
<td>0.334</td>
</tr>
<tr>
<td>$A_6$</td>
<td>0.228</td>
<td>0.252</td>
<td>0.254</td>
<td>0.272</td>
<td>0.404</td>
<td>0.000</td>
<td>0.560</td>
<td>0.328</td>
</tr>
<tr>
<td>$A_7$</td>
<td>0.198</td>
<td>0.176</td>
<td>0.221</td>
<td>0.225</td>
<td>0.313</td>
<td>0.318</td>
<td>0.000</td>
<td>0.242</td>
</tr>
<tr>
<td>$E[\pi^-(A_i)]$</td>
<td>0.278</td>
<td>0.303</td>
<td>0.355</td>
<td>0.387</td>
<td>0.542</td>
<td>0.560</td>
<td>0.665</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.7 Expected net outranking index values and ranking patterns for the first set of criteria weights ($\Omega = 0.3$ and $\rho_{ij} = 0.7$).

<table>
<thead>
<tr>
<th></th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
<th>$A_5$</th>
<th>$A_6$</th>
<th>$A_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[\Pi(A_i)]$</td>
<td>0.33</td>
<td>0.29</td>
<td>0.16</td>
<td>0.09</td>
<td>-0.21</td>
<td>-0.23</td>
<td>-0.42</td>
</tr>
<tr>
<td>Rank</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>
Table 5.8 Expected net outranking index values and ranking patterns for the first set of criteria weights and different combinations of $\Omega$ and $\rho_{ij}$.

<table>
<thead>
<tr>
<th>$\Omega$</th>
<th>$\rho$</th>
<th>$E[\Pi(A_i)]$</th>
<th>Rankings($A_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.44 0.41 0.16 0.07 -0.30 -0.29 -0.48</td>
<td>1 2 3 4 6 5 7</td>
</tr>
<tr>
<td>0.3</td>
<td>0</td>
<td>0.24 0.21 0.12 0.08 -0.14 -0.16 -0.35</td>
<td>1 2 3 4 5 6 7</td>
</tr>
<tr>
<td>0.8</td>
<td>0</td>
<td>0.11 0.09 0.06 0.04 -0.05 -0.06 -0.18</td>
<td>1 2 3 4 5 6 7</td>
</tr>
<tr>
<td>0</td>
<td>0.7</td>
<td>0.44 0.41 0.16 0.07 -0.30 -0.29 -0.48</td>
<td>1 2 3 4 6 5 7</td>
</tr>
<tr>
<td>0.3</td>
<td>0.7</td>
<td>0.33 0.29 0.16 0.09 -0.21 -0.23 -0.42</td>
<td>1 2 3 4 5 6 7</td>
</tr>
<tr>
<td>0.8</td>
<td>0.7</td>
<td>0.17 0.14 0.08 0.05 -0.09 -0.11 -0.25</td>
<td>1 2 3 4 5 6 7</td>
</tr>
</tbody>
</table>

Figure 5.4 shows the influence of uncertainty on the ranking results. The relative performance of different alternatives becomes less distinct when the uncertainty degree of the preference values increases. The ranking orders for different values of $\Omega$ remain very much the same; only the ranking orders between $A_5$ and $A_6$ are changed by the level of uncertainty.

The correlation between the performance values of each alternative can also influence the values of the expected net outranking index (except for $\Omega = 0.0$). A higher positive correlation coefficient will result in higher values of the positive-valued expected net outranking index or smaller values of the negative-valued expected net outranking index. In other words, a better alternative will become even better as the $\rho_{ij}$ is changed from negative to positive in terms of expected net outranking index as shown in Figure 5.5. The ranking orders are not much affected by changing the correlation in this example.

On the sensitivity of criteria weights on the final decision, Table 5.9 and Figure 5.6 show that the values of the expected net outranking index are changed according to different sets of criteria weights associated with different decision makers. The ranking orders are not changed greatly except for the reversed ranking orders between $(A_2, A_3)$, and $(A_5, A_6)$. $A_1$ is always considered as the best solution by all of the three groups of decision makers. This
means that the final ranking of the alternatives is insensitive to the original settings of the criteria weights. The decision makers would feel confident in choosing the first-ranked alternative.

In addition to the three sets of criteria weights shown in Table 5.3, three extreme sets of criteria weights are considered. Equal weights are assigned to the focused factors (economic, environmental, or social, respectively) and zero weights are assigned to the factors that are ignored. The extreme weights for the ten criteria for an economic-focused decision maker, an environmental-focused decision maker, and a social-focused decision maker are listed in Table 5.10. Surprisingly, when the extreme weightings are adopted, the rankings are found to change greatly. Alternative $A_2$ is the best, considering either the economic or social factors only, but it becomes the worst choice when considering only the environmental factors. This means that changing the existing crop pattern (rice) requires a lot of initial costs and huge work force, and increasing the area of the non-cultivated land, but it has a huge positive benefit for water sustainability. This result also shows the remarkable conflict between economic and environmental criteria or social and environmental criteria.

<table>
<thead>
<tr>
<th>Criteria weights</th>
<th>$E[\Pi(A_i)]$</th>
<th>Rankings($A_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actor 1</td>
<td>0.33 0.29 0.16 0.09 -0.21 -0.23 -0.42</td>
<td>1 2 3 4 5 6 7</td>
</tr>
<tr>
<td>Actor 2</td>
<td>0.24 0.11 0.13 0.10 -0.13 0.01 -0.45</td>
<td>1 3 2 4 6 5 7</td>
</tr>
<tr>
<td>Actor 3</td>
<td>0.34 0.32 0.19 0.12 -0.25 -0.24 -0.49</td>
<td>1 2 3 4 6 5 7</td>
</tr>
<tr>
<td>Economic-focused</td>
<td>0.42 0.48 0.16 0.08 -0.20 -0.53 -0.42</td>
<td>2 1 3 4 5 7 6</td>
</tr>
<tr>
<td>Environmental-focused</td>
<td>0.00 -0.33 0.00 0.12 -0.13 0.48 -0.14</td>
<td>3 7 4 2 5 1 6</td>
</tr>
<tr>
<td>Social-focused</td>
<td>0.52 0.59 0.36 0.19 -0.37 -0.47 -0.82</td>
<td>2 1 3 4 5 6 7</td>
</tr>
</tbody>
</table>
Table 5.10 Hypothetical sets of criteria weights for three groups of decision makers.

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_6$</th>
<th>$C_7$</th>
<th>$C_8$</th>
<th>$C_9$</th>
<th>$C_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economic-focused</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.00</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>decision maker</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Environmental-focused decision maker</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Social-focused</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>decision maker</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.4 Effect of performance value uncertainty on $E[\Pi(A_i)]$
Figure 5.5 Effect of performance value correlation on $E[\Pi(A_i)]$

![Graph showing the effect of performance value correlation on $E[\Pi(A_i)]$](image)

Figure 5.6 Expected net outranking index for different decision maker’s criteria weights

![Graph showing the expected net outranking index for different decision maker’s criteria weights](image)
5.6 Summary

This study provides a new method for incorporating the external uncertainties of the performance value of each alternative into the framework of MCDM. By assigning probability features to the performance values, the expected net outranking index can be calculated and used for alternatives ranking. The results of the case study illustrate that the uncertainty degree of the performance value of each alternative and the correlation between each of them can influence the values of the expected net outranking index. A sensitivity analysis of the MCDM approach is recommended to examine the robustness of the final decision with respect to changes in the uncertainty degree of the performance values, correlations between the performance values, and the criteria weights. The results of these sensitivity analyses show the robustness of the three sets of criteria weights.
CHAPTER 6
CONCLUSIONS AND RECOMMENDATIONS

6.1 Summary and Conclusions

The frequency-based approach has been the mainstream method used in hydrosystems design, which pre-selects the design frequency as the protection level for a considered area. Due to the presence of uncertainties, damage arising from a project failure and the cost and benefit of different projects are random variables with their own statistical moments and distributions. As of today, conceptual advancement on the risk-based design approach involves tradeoffs between the project investment cost and the expected damage cost due to system failure. However, the majority of risk-based designs in practice are limited only to consideration of the expected value of failure damage or other economic performance indicators. Information about the variability of the project outcome and potential adverse consequences are implicitly condensed into one single number as an indicator and the decision maker is not informed of this information.

When a decision maker is faced with the choice of various alternatives with uncertain outcomes, it is difficult to ensure that the chosen alternative is really the best one. There is a likelihood that any chosen alternative could turn out to be wrong because of the uncertain outcomes. The selection of different project alternative of varying probability distributions for the total annual cost or benefit can be regarded as the problem of decision making under risk. In addition, quantitative indicators or criteria that account for the full uncertainty features of
alternative outcomes is needed to assist the decision maker in screening and ranking alternatives. A risk-based decision model, i.e., minimax \( EOL \) decision rule, for alternative ranking and selection under risk is proposed in this research study (see Chapter 3).

It is also recognized that when the outcome of a decision (or design) is uncertain, the selection of any alternative, regardless of its highly expected desirable outcome, can still potentially be outperformed by other competing alternatives. Hence, the concept of opportunity cost is incorporated in the proposed risk-based decision model for enhancing the precautionary design of hydrosystem infrastructures. By considering the opportunity cost, the notion is accepted that having an alternative with a lower \( EOL \) is preferable to the one with a higher \( EOL \). In conjunction with the evaluation of \( EOL \), Savage’s minimax principle was employed for alternative ranking. Different from Savage’s minimax regret rule, by using the \( EOL \), the proposed minimax \( EOL \) decision rule is more generalized in which the consequences for varying state of nature is described by continuous random variables. If the adverse consequences of making a wrong decision can be evaluated, proper preparations (e.g., contingency or insurance) can be made to deal with the unwanted adverse outcomes.

The purpose and strength of the proposed decision-making framework is to help improve the quality of decisions by:

- Providing a logical and defensible decision framework for hydrosystems design and management when the decision outcomes are subject to uncertainty arising from hydrological, hydraulic, economic, and other aspects;

- Allowing water resources engineers to efficiently consider the tradeoffs between the project cost, failure damage, and their probabilistic features in the decision process;

- Providing a framework which explicitly quantifies various uncertainty features, in addition to the mean values, associated with project performance indicators, such as failure damage, net benefit, and cost, rendering a risk-informed decision making;

- Formulating an \( EOL \)-based risk measure which rationally reflects the potential loss in case the selected alternative turns out to be inferior to its competitors. The decision
rule is capable of dealing with the effect of correlation between two competing alternative outcomes;

- Constructing a decision rule that permits joint consideration of a decision maker’s risk attitude, as most engineers are reluctant to take risks on infrastructural system failure. Savage’s minimax regret principle is employed in conjunction with the evaluation of $EOL$ for alternative ranking. Based on the $EOL$, the decision maker could select an alternative with minimal expected loss when the decision turns out to be wrong. The decision maker’s acceptable risk is incorporated in the decision process.

From the numerical examples illustrated in this thesis, the following important observations are noted:

- The value of $EOL$ of any two competing alternatives under consideration is dependent on the relative magnitudes of their means, standard deviations, and correlation. Choice of a design alternative having a smaller mean outcome ratio, higher uncertainty, and/or large negative correlation with respect to its competing alternative could result in a larger value of $EOL$.

- The $EOL$ reflects more accurately the relative merit of two competing alternatives without suffering the pessimism of the conditional risk measure and the counter-intuition of the two previous risk measures: XRM and CRM [49, 50].

- The results of the example applications have indicated that the uncertainty of the project net benefits is quite significant and its effect on economic merit should not be ignored in decision making under uncertainty. Consideration of alternative outcomes uncertainty is important in the selection of alternatives.

- Alternative outcomes in the design of hydro-infrastructural systems are rarely independent in real-life and their correlation levels could be significant enough to affect the relative merit of the alternatives under consideration.

- The $EOL$ can be generalized by adopting various forms of loss function, which is a
function of the outcome difference. When the loss function is the $m^{th}$ order power function (with $m$ being an integer), the analytical expressions of $EOL$ for normal random outcomes are derived. For non-integer $m$ values, empirical relations of normalized $EOL$ with $m$ and $\beta = -\mu_\Delta / \sigma_\Delta$ are provided. The form of loss functions can influence the value of $EOL$ and the ranking result. Higher order of the loss function means the decision maker is more conservative and risk-averse.

- When sample size is small, ignoring the sampling error will underestimate the potential risk of each project. It is clearly shown that adding data to existing sample observations leads to improved quality of information, enhanced reliability of the estimators, reduced sampling error and less uncertainty of the project net benefits. Through the proposed framework, the proper length of the extended record period for risk reduction can be determined to achieve the required level of acceptable risk.

Once the ranking of alternatives is identified on the basis of the lowest maximum $EOL$, the acceptable risk of the decision maker can be used to examine the feasibility of the alternatives. When the value of the maximum $EOL$ of a chosen alternative is lower than the acceptable risk, it may be considered implementable by the decision maker. Otherwise the following course of actions might be considered: (1) formulating a new set of alternatives that are not being currently considered; (2) conducting more research to reduce outcome uncertainties associated with currently considered alternatives; or (3) increasing the budgetary reserves for a contingency or acceptable risk by the decision maker [53].

Moreover, the risk-based framework can be expanded into risk-based multi-criteria decision making. In this case, the alternatives of hydrosystems design are evaluated based on the uncertain performance values associated with more than one decision criterion.

A hydrosystem design project typically involves several alternatives that are evaluated by different criteria. The most commonly considered objectives are related to the issues about the economic efficiency, the economic development, the technical feasibility, the water quality and supply, and the fairness and equity of the resources distribution issues. The performance of each project in achieving different objectives is usually measured by different
units. The non-commensurable and intangible factors can be as important as the economic factors quantifiable in monetary terms for public decision making areas. In this regard, MCDM is a more appropriate methodology and it has received growing attention in these few decades. In general, multi-criteria decision analysis helps to improve the decision quality in the following aspects:

- Enhances the decision maker’s understanding of the decision context;
- Provides a systematic, comprehensive, transparent, and auditable framework to resolve conflict and obtain an equitable compromise solution;
- Complements conventional cost-benefit analysis by eliminating the bias of considering economic efficiency as the only project evaluation criterion; and
- Facilitates communication among analysts, decision makers, and stakeholders in different groups and disciplines and integrates diverse disciplines.

Two types of uncertainties are present in the MCDM process, namely the external and internal uncertainties. The external uncertainty is related to the imperfect information on the consequences of each alternative (e.g., performance values) and internal related to subjective human judgment (e.g., criteria weights). Most readily available MCDM techniques generally use the deterministic performance values of each alternative against a given criterion to describe the design alternatives and to evaluate, rank and select the best solution. The uncertainties in the consequences and weights of the criteria are merely considered by sensitivity analysis. Explicit consideration of the external uncertainties is rarely found.

The other objective of this study is to construct a MCDM framework that explicitly considers the external uncertainty on the consequences of hydrosystem design or management alternatives. The probabilistic features of the consequences associated with an alternative is taken into account and used in the PROMETHEE (Preference Ranking Organization Method of Enrichment Evaluation) MCDM technique, instead of using the original deterministic performance values.
Finally, uncertainty can cause hesitation in making a decision. Without a normative decision making rule, information on uncertainty can only help to improve understanding on decision problems. This study provides a new aspect on how to utilize this descriptive information on uncertainty to prescriptively assist decision making in a highly variable environment.

6.2 Recommendations for Future Work

It is recommended that the following topics can be further studied and investigated.

- The proposed risk-based decision making methodology appears to have great potential for a wide range of applications in a variety of fields other than water resources management and flood-damage-reduction project evaluation. It can be also applied to site selection for renewable energy power stations, emergency evacuation in response to incoming flood, among others.

- Although this study focuses on the EOL which is suitable for risk-averse decision makers, its counterpart, the EOG, can be equally applied to describe risk-seeking behavior in which the principle of maximin can be utilized. Other variations of practical decision criteria can be derived on the basis of EOL and EOG.

- In this research, the hydrologic randomness of the flood magnitudes and the parameter uncertainty are incorporated in the process of flood-damage-reduction project evaluation. Other aspects of uncertainties, such as those in the stage-discharge relationship, the damage-stage function, and economic parameters, could also contribute to the uncertainty level of the alternative net benefits. These uncertainties can be further incorporated in the estimation of the uncertainty associated with the project net benefit for project evaluation.

- In reality, decision maker’s acceptable risk could also be uncertain. The influence of an uncertain acceptable risk on the implementation of the EOL-based decision making should be further studied. In this circumstance, higher order moments of opportunity loss should be derived and utilized to evaluate the likelihood of each
candidate project alternative being feasible.

- To avoid unnecessary complexity, the case studies in this thesis assumed that the net benefits are normal random variables. The effect of the probability distribution of a project performance indicator on the decision should be investigated. This can be done by assessing the higher-order moments of the performance indicators. The consideration of non-normality would introduce complications in defining the joint probability distribution and in the associated mathematical complexity in evaluating the EOL.

- For the proposed risk-based MCDM methodology, in addition to the uncertainty of the performance values, the weighting factors reflecting the relative importance of different criteria are also uncertain. A number of methods for eliciting the criteria weights are available but there is no consensus on which method has the most advantage for a particular decision problem. Therefore, there are inherent uncertainty results from the subjective judgments of the decision maker. Moreover, there are also correlations between the weights. A joint consideration of the uncertainties of the performance values and the criteria weights, and correlations of weights between criteria, would lead to a more comprehensive evaluation.
REFERENCES


Army Corps of Engineers. IWR 92-R-1 and IWR 92-R-2, 1992.


Bibliography and References


From Eqs.(3-4a) and (3-6), the proposed risk measure of the EOL and EOG in choosing alternative \( A_i \) over \( A_j \) can be defined as

\[
EOL(A_i^*, A_j) = -\int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} (x_i - x_j) f_{i,j}(x_i, x_j) \, dx_j \right] \, dx_i
= -\int_{-\infty}^{\infty} \delta f_{A_{i,j}}(\delta) \, d\delta
\]

(A-1a)

\[
EOG(A_i^*, A_j) = \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} (x_i - x_j) f_{i,j}(x_i, x_j) \, dx_i \right] \, dx_j
= \int_{-\infty}^{\infty} \delta f_{A_{i,j}}(\delta) \, d\delta, \text{ for } \Delta_{i,j} > 0
\]

(A-1b)

where \( EOL(A_i^*, A_j) \) and \( EOG(A_i^*, A_j) \) are the EOL and EOG for the chosen alternative \( A_i \) with reference to \( A_j \), respectively; \( X_i \) and \( X_j \) are the random outcomes of alternatives \( A_i \) and \( A_j \), respectively.

The relationship between \( EOL \) and \( EOG \) can be expressed as

\[
EOL[A_i^*, A_j] = EOG[A_i, A_j^*] \geq 0 \quad (A-2a)
\]

\[
EOL[A_i, A_j^*] = EOG[A_i^*, A_j] \geq 0 \quad (A-2b)
\]

Let the long term expectation of two random outcomes \( X_i \) and \( X_j \) be

\[
E[X_i - X_j] = \mu_i - \mu_j \quad (A-3a)
\]

\[
E[X_j - X_i] = \mu_j - \mu_i \quad (A-3a)
\]

The \( EOG \) is a complement of \( EOL \). The long term expectation of outcome difference between \( X_i \) and \( X_j \) can be further expressed by \( EOL \) and \( EOG \) as
\[
= EOG[A_i^*, A_j] - EOG[A_i, A_j^*] \\
= EOL[A_i, A_j^*] - EOL[A_i^*, A_j] 
\] (A-4a)

and

\[
E[X_j - X_i] = EOG[A_i, A_j^*] - EOL[A_i, A_j^*] \\
= EOG[A_i, A_j^*] - EOG[A_i^*, A_j] \\
= EOL[A_i^*, A_j] - EOL[A_i, A_j^*] 
\] (A-4b)

If the expectation of \(X_i\) and \(X_j\) are identical, i.e., \(\mu_i = \mu_j\), then

\[
\] (A-5)

Otherwise, if \(\mu_i > \mu_j\), then \(E[X_i - X_j] > 0\), and

\[
EOL[A_i^*, A_j] < EOL[A_i, A_j^*] 
\] (A-6a)

\[
EOG[A_i^*, A_j] > EOG[A_i, A_j^*] 
\] (A-6b)

On the other hand, when \(\mu_i < \mu_j\), then \(E[X_i - X_j] < 0\), and

\[
EOL[A_i^*, A_j] > EOL[A_i, A_j^*] 
\] (A-7a)

\[
EOG[A_i^*, A_j] < EOG[A_i, A_j^*] 
\] (A-7b)

Considering three alternatives \(A_i, A_j,\) and \(A_k,\) and \(X_i, X_j,\) and \(X_k\) are their corresponding random outcomes, respectively. If \(0 < \mu_k \leq \mu_j \leq \mu_i\), and \(\sigma_j > \sigma_i\), then as shown in Figure 3.7,

\[
EOL[A_j^*, A_k] > EOL[A_i^*, A_k] 
\] (A-8a)

Or if \(0 < \mu_k \leq \mu_j \leq \mu_i\), and \(\sigma_k > \sigma_j\), then

\[
EOL[A_j^*, A_k] > EOL[A_i^*, A_k] 
\] (A-8b)
APPENDIX – B

DETERMINATION OF THE INTERSECTION POINTS OF XRM AND EOL CURVES IN 1-D DECISION PROBLEM

Referring to Figure 3.3, the mathematical expressions of XRM and EOL, for \( x_i/\mu_j \geq 1 \), can be written, respectively, as

\[
XRM \left( A_i', A_j \right) = (x_i - \mu_j) \times P_r \left( x_i \leq X_j \right) \\
= (x_i - \mu_j) \left[ 1 - F_j \left( x_i \right) \right] 
\]

\[
EOL \left( A_i', A_j \right) = -\int_{x_i}^{\infty} \left( x_i - x_j \right) f_j \left( x_j \right) dx_j \\
= \int_{x_i}^{\infty} x_j f_j \left( x_j \right) dx_j - x_i \int_{x_i}^{\infty} f_j \left( x_j \right) dx_j \\
= \left( \mu_j - \mu_{d(x_i,x_i)} \right) - x_i \left[ 1 - F_j \left( x_i \right) \right] 
\]

where \( f_j(*) \) and \( F_j(*) \) are, respectively, the PDF and CDF of the random outcome \( X_j \) with the mean \( \mu_j \) and standard deviation \( \sigma_j \); and \( \mu_{d(x_i,x_i)} \) is lower partial mean of \( X_j \) over partial range of \(( -\infty, x_i^* ) \) which can be expressed as

\[
\mu_{d(x_i,x_i)} = \int_{-\infty}^{x_i^*} x_j f_j \left( x_j \right) dx_j 
\]

Let the value of \( XRM \) equal to \( EOL \) in the region of \( x_i/\mu_j \geq 1 \), then the following relationship can be established

\[
\left( 2x_i - \mu_j \right) \left[ 1 - F_j \left( x_i \right) \right] = \mu_j - \mu_{d(x_i,x_i)} 
\]

Similarly, for \( x_i/\mu_j \leq 1 \), \( A_j \) is chosen and the corresponding \( XRM \) and \( EOL \) can be expressed as

\[
XRM \left( A_i', A_j \right) = (\mu_j - x_i) F_j \left( x_i \right) 
\]
\[ EOL(A^*_i, A_j) = -\int_{-\infty}^{x_i^*} (x_j - x_i^*) f_j(x_j) \, dx_j \]
\[ = x_i^* \int_{-\infty}^{x_i^*} f_j(x_j) \, dx_j - \int_{-\infty}^{x_i^*} x_j f_j(x_j) \, dx_j \]
\[ = x_i^* F_j(x_i^*) - \mu_{d(x_j, x_i^*)} \quad (B-4b) \]

Let \( XRM \) equal to \( EOL \) in \( x_i^*/\mu_j \leq 1 \), one has
\[ (2x_i^* - \mu_j) \cdot F_j(x_i^*) = \mu_{d(x_j, x_i^*)} \quad (B-5) \]

When random outcome \( X_j \) follows a normal distribution, the lower partial mean can be expressed as [160]
\[ \mu_{d(x_j, x_i^*)} = \mu_j \cdot \Phi \left( \frac{x_i^* - \mu_j}{\sigma_j} \right) - \sigma_j \cdot \phi \left( \frac{x_i^* - \mu_j}{\sigma_j} \right) \quad (B-6) \]

where \( \phi(\cdot) \) and \( \Phi(\cdot) \) are the standard normal PDF and CDF, respectively.

Substituting Eq.(B-6) into Eq.(B-3) and (B-5), one has

(i) For \( x_i^*/x_j \leq 1 \) and choose \( A_i \),
\[ 2(z_2) = \frac{\phi(z_2)}{1 - \Phi(z_2)} \quad (B-7a) \]

(ii) For \( x_i^*/x_j \geq 1 \) and choose \( A_j \),
\[ -2(z_1) = \frac{\phi(z_1)}{\Phi(z_1)} \quad (B-7b) \]

where \( z_1 = r_1^{-1}/\Omega_j \); \( z_2 = r_2^{-1}/\Omega_j \) with \( \Omega_j = \sigma_j/\mu_j \). Solving \( r_1 \) and \( r_2 \) would yield the ratios of \( x_i^*/\mu_j \) where the two risk measures intersect.
APPENDIX – C

ANALYTICAL DERIVATION OF \( m \)th-ORDER EOL

When the loss function of \( \Delta_{i,j} \) is restricted to be a power function, \( EOL_m(\Delta_{i,j}^*, A_i) \), as Eq.(3-4a), is the \( m \)th-order lower partial moment of \( \Delta_{i,j} \) for \( -\infty \leq \Delta_{i,j} \leq 0 \). The analytical expression of a partial raw moment with integer-valued orders can be determined by

\[
EOL_m \left( \Delta_{i,j}^*, A_i \right) = \left[ D^m M(t) \right]_{t=0} \tag{C-1}
\]

where \( D^m = d^m/dt^m \) is the \( m \)th-order derivatives with respect to \( t \), \( M(t) = E[\exp(t\delta_{i,j}) | \delta_{i,j} \in A] \) is the partial moment generating function (MGF) of \( \Delta_{i,j} \) in a domain of \( A \). When \( \Delta_{i,j} \) is in the domain of \( a_1 \leq \Delta_{i,j} \leq a_2 \), \( M(t) \) is defined as

\[
M(t) = \int_{a_1}^{a_2} \exp(t\delta) f_{\Delta_{i,j}}(\delta) d\delta \tag{C-2}
\]

Under the assumption of normally distributed \( \Delta_{i,j} \), Eq.(C-2) can further be expressed as

\[
M(t) = \int_{a_1}^{a_2} e^{it\delta} \cdot \frac{1}{\sigma_\Delta \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\delta - \mu_\Delta}{\sigma_\Delta} \right)^2} d\delta
\]

\[
= \frac{1}{\sigma_\Delta \sqrt{2\pi}} \int_{a_1}^{a_2} e^{-\frac{1}{2} \left( \frac{\delta - (\mu_\Delta + \sigma_\Delta^2)^2}{\sigma_\Delta} \right)} d\delta
\]

\[
= e^{\frac{1}{2} \left( \frac{\mu_\Delta + \sigma_\Delta^2}{\sigma_\Delta^2} \right)^2} \Phi \left( \frac{a_2 - \mu_\Delta}{\sigma_\Delta} - \sigma_\Delta t \right) - \Phi \left( \frac{a_1 - \mu_\Delta}{\sigma_\Delta} - \sigma_\Delta t \right)
\]

\[
= e^{\frac{1}{2} \left( \frac{\mu_\Delta + \sigma_\Delta^2}{\sigma_\Delta^2} \right)^2} \left[ \Phi \left( \frac{a_2 - \mu_\Delta}{\sigma_\Delta} - \sigma_\Delta t \right) - \Phi \left( \frac{a_1 - \mu_\Delta}{\sigma_\Delta} - \sigma_\Delta t \right) \right]
\]

where \( \mu_\Delta \) and \( \sigma_\Delta \) are the mean and standard deviation of random variable \( \Delta_{i,j} \), respectively, readily obtainable from Eq.(3-2a) and (3-2b) as repeated below.

\[
\mu_\Delta = E[\Delta_{i,j}] = \mu_i - \mu_j \quad \text{and} \quad \sigma_\Delta^2 = Var[\Delta_{i,j}] = \sigma_i^2 + \sigma_j^2 - 2\rho_{i,j} \sigma_i \sigma_j
\]
By letting

$$\Xi(t) = e^{\mu t + \sigma^2 t^2 / 2}$$

$$z_1(t) = \frac{a_1 - \mu}{\sigma} - \sigma t$$ and $$z_2(t) = \frac{a_2 - \mu}{\sigma} - \sigma t,$$

Eq.(C-3) can be simplified as

$$M(t) = \Xi(t) \times \left[ \Phi(z_2) - \Phi(z_1) \right] \quad \text{(C-4)}$$

The analytical expression of $EOL_m(A_i^*, A_j)$ can be derived from Eq.(C-1) by differentiating the partial MGF $m$ times with respect to $t$ and setting $t = 0$. Using binomial expansion, the $m^{th}$-order derivatives of $M(t)$ with respect to $t$ can be expressed as

$$D^m M(t) = \sum_{k=0}^{m} \binom{m}{k} \left( D^{m-k} \Xi(t) \right) \left( D^k \Phi(z_2) - D^k \Phi(z_1) \right) \quad \text{(C-5)}$$

where $D^m[\Xi(t)]$ is the $m^{th}$-order derivatives of $\Xi(t)$ with respect to $t$ and $D^k \Phi(t)$ is the $k^{th}$-order derivative of $\Phi$ with respect to $t$. The general form of $m^{th}$-order derivatives of $\Xi$ and $\Phi$, with $m$ being an integer, can be derived and given below.

The first five derivatives of $\Xi(t)$ are

$$\frac{d}{dt} \exp(\mu t + 0.5\sigma^2 t^2) = \left( \mu + \sigma^2 t \right) \exp(\mu t + 0.5\sigma^2 t^2)$$

$$\frac{d^2}{dt^2} \exp(\mu t + 0.5\sigma^2 t^2) = \left[ \sigma^2 + \left( \mu + \sigma^2 t \right)^2 \right] \exp(\mu t + 0.5\sigma^2 t^2)$$

$$\frac{d^3}{dt^3} \exp(\mu t + 0.5\sigma^2 t^2) = \left[ 3\sigma^2 \left( \mu + \sigma^2 t \right) + \left( \mu + \sigma^2 t \right)^3 \right] \exp(\mu t + 0.5\sigma^2 t^2)$$

$$\frac{d^4}{dt^4} \exp(\mu t + 0.5\sigma^2 t^2) = \left[ 3\sigma^4 + 6\sigma^2 \left( \mu + \sigma^2 t \right)^2 + \left( \mu + \sigma^2 t \right)^4 \right] \times \exp(\mu t + 0.5\sigma^2 t^2) \quad \text{(C-6)}$$

$$\frac{d^5}{dt^5} \exp(\mu t + 0.5\sigma^2 t^2) = \left[ 15\sigma^4 \left( \mu + \sigma^2 t \right) + 10\sigma^2 \left( \mu + \sigma^2 t \right)^3 + \left( \mu + \sigma^2 t \right)^5 \right] \times \exp(\mu t + 0.5\sigma^2 t^2)$$
The form of the $m^{\text{th}}$-order derivatives of $\Xi(t)$, for $m > 2k$, can be derived as:

(1) $m = 2n-1$ ($m$ is an odd number), $n=1, 2, \ldots$

$$D^m \Xi(t) = \sum_{k=0}^{(m-1)/2} \frac{m!}{2^k k! (m-2k)!} \sigma_{\Delta}^{2k} (\mu_{\Delta} + \sigma_{\Delta}^2 t)^{m-2k} \Xi(t) \quad (C-7a)$$

(2) $m = 2n$ ($m$ is an even number) $n = 1, 2, \ldots$

$$D^m \Xi(t) = \sum_{k=0}^{m/2} \frac{m!}{2^k k! (m-2k)!} \sigma_{\Delta}^{2k} (\mu_{\Delta} + \sigma_{\Delta}^2 t)^{m-2k} \Xi(t) \quad (C-7b)$$

All of these are polynomials in $\sigma_{\Delta}$ and $\mu_{\Delta} + \sigma_{\Delta}^2 t$ multiplied by $\Xi(t)$.

When $z(t)$ is in the form of \left[(a - \mu_{\Delta})/\sigma_{\Delta}\right] - \sigma_{\Delta} t$ with $a$ being a constant, the first four derivatives of normal CDF $\Phi(z)$ with respect to $t$ are polynomials in $z$ multiplied by $\sigma_{\Delta}^m$ and the normal PDF, $\phi(z)$ [164]. The polynomials are called the Chebyshev-Hermite polynomials [164], defined as

$$(-D)^m \phi(z) = H_m(z) \phi(z) \quad (C-8)$$

The first five derivatives of normal CDF, $\Phi(z)$, are

$$\frac{d\Phi}{dt} = -\sigma_{\Delta} \phi(z) = -\sigma_{\Delta} H_0(z) \phi(z)$$
$$\frac{d^2\Phi}{dt^2} = -\sigma_{\Delta}^2 z \phi(z) = -\sigma_{\Delta}^2 H_1(z) \phi(z)$$
$$\frac{d^3\Phi}{dt^3} = -\sigma_{\Delta}^3 (z^2 - 1) \phi(z) = -\sigma_{\Delta}^3 H_2(z) \phi(z) \quad (C-9)$$
$$\frac{d^4\Phi}{dt^4} = -\sigma_{\Delta}^4 (z^3 - 3z) \phi(z) = -\sigma_{\Delta}^4 H_3(z) \phi(z)$$
$$\frac{d^5\Phi}{dt^5} = -\sigma_{\Delta}^5 (z^4 - 6z^2 + 3) \phi(z) = -\sigma_{\Delta}^5 H_4(z) \phi(z)$$

The general form of the $m^{\text{th}}$-order derivatives of normal CDF, $\Phi(z)$, is [164]
\[ D^n \Phi(z) = -\sigma^n_\Delta H_{m-1}(z) \phi(z) \]  
(C-10)

where

(1) for \( m > 2k, m = 2n-1 \) (\( m \) is an odd number), \( n = 1, 2, \ldots \)

\[ H_m(z) = \sum_{k=0}^{(m-1)/2} \frac{(-1)^k m!}{2^k k!(m-2k)!} z^{m-2k} \]  
(C-11a)

(2) for \( m > 2k \) (\( m \) is an even number) \( m = 2n, n = 1, 2, \ldots \)

\[ H_m(z) = \sum_{k=0}^{m/2} \frac{(-1)^k m!}{2^k k!(m-2k)!} z^{m-2k} \]  
(C-11b)

Combining Eqs.(C-7a),(C-7b), (C-10), (C-11a) and (C-11b) into Eq.(C-5), the first five derivatives of \( M(t) \) are

\[ M^{(1)}(t) = (\mu_\Delta + \sigma^2_\Delta t) e^{(\mu_\Delta + \sigma^2_\Delta t)^2} \left[ \Phi(z_2) - \Phi(z_1) \right] - \sigma_\Delta e^{(\mu_\Delta + \sigma^2_\Delta t)^2} \left[ \phi(z_2) - \phi(z_1) \right] \]

\[ M^{(2)}(t) = \left[ \sigma^2_\Delta + (\mu_\Delta + \sigma^2_\Delta t)^2 \right] e^{(\mu_\Delta + \sigma^2_\Delta t)^2} \left[ \Phi(z_2) - \Phi(z_1) \right] \]

\[ - 2\sigma_\Delta (\mu_\Delta + \sigma^2_\Delta t) e^{(\mu_\Delta + \sigma^2_\Delta t)^2} \left[ \phi(z_2) - \phi(z_1) \right] \]

\[ - \sigma^2_\Delta e^{(\mu_\Delta + \sigma^2_\Delta t)^2} \left[ z_2 \phi(z_2) - z_1 \phi(z_1) \right] \]

\[ M^{(3)}(t) = (\mu_\Delta + \sigma^2_\Delta t) \left[ (\mu_\Delta + \sigma^2_\Delta t)^2 + 3\sigma^2_\Delta \right] e^{(\mu_\Delta + \sigma^2_\Delta t)^2} \left[ \Phi(z_2) - \Phi(z_1) \right] \]

\[ - 3\sigma^2_\Delta \left[ (\mu_\Delta + \sigma^2_\Delta t)^2 + \sigma^2_\Delta \right] e^{(\mu_\Delta + \sigma^2_\Delta t)^2} \left[ \phi(z_2) - \phi(z_1) \right] \]

\[ - 3\sigma^2_\Delta (\mu_\Delta + \sigma^2_\Delta t) e^{(\mu_\Delta + \sigma^2_\Delta t)^2} \left[ z_2 \phi(z_2) - z_1 \phi(z_1) \right] \]

\[ - \sigma^3_\Delta e^{(\mu_\Delta + \sigma^2_\Delta t)^2} \left[ (z_2^2 - 1) \phi(z_2) - (z_1^2 - 1) \phi(z_1) \right] \]
\[ M^{(4)}(t) = \left[ (\mu_\Delta + \sigma_\Delta^2) + 6\sigma_\Delta^2 (\mu_\Delta + \sigma_\Delta^2) + 4\sigma_\Delta^4 \right] e^{(\mu_\Delta + \sigma_\Delta^2)/2} \left[ \Phi(z_2) - \Phi(z_1) \right] \]
\[- 4\sigma_\Delta \left[ (\mu_\Delta + \sigma_\Delta^2) + 3\sigma_\Delta^2 (\mu_\Delta + \sigma_\Delta^2) \right] e^{(\mu_\Delta + \sigma_\Delta^2)/2} \left[ \phi(z_2) - \phi(z_1) \right] \]
\[- 6\sigma_\Delta^2 \left[ (\mu_\Delta + \sigma_\Delta^2) + 2\sigma_\Delta^2 \right] e^{(\mu_\Delta + \sigma_\Delta^2)/2} \left[ z_2 \phi(z_2) - z_1 \phi(z_1) \right] \]
\[- 4\sigma_\Delta^4 e^{(\mu_\Delta + \sigma_\Delta^2)/2} \left[ (z_2^2 - 1) \phi(z_2) - (z_1^2 - 1) \phi(z_1) \right] \]
\[- \sigma_\Delta^4 e^{(\mu_\Delta + \sigma_\Delta^2)/2} \left[ (z_2^3 - 3z_2) \phi(z_2) - (z_1^3 - 3z_1) \phi(z_1) \right] \]
\[ M^{(5)}(t) = \left[ (\mu_\Delta + \sigma_\Delta^2) + 10\sigma_\Delta^2 (\mu_\Delta + \sigma_\Delta^2) + 15\sigma_\Delta^4 (\mu_\Delta + \sigma_\Delta^2) \right] e^{(\mu_\Delta + \sigma_\Delta^2)/2} \left[ \Phi(z_2) - \Phi(z_1) \right] \]
\[- 5\sigma_\Delta \left[ (\mu_\Delta + \sigma_\Delta^2) + 6\sigma_\Delta^2 (\mu_\Delta + \sigma_\Delta^2) + 3\sigma_\Delta^4 \right] e^{(\mu_\Delta + \sigma_\Delta^2)/2} \left[ \phi(z_2) - \phi(z_1) \right] \]
\[- 10\sigma_\Delta^2 \left[ (\mu_\Delta + \sigma_\Delta^2) + 3\sigma_\Delta^2 (\mu_\Delta + \sigma_\Delta^2) \right] e^{(\mu_\Delta + \sigma_\Delta^2)/2} \left[ z_2 \phi(z_2) - z_1 \phi(z_1) \right] \]
\[- 10\sigma_\Delta^3 \left[ (\mu_\Delta + \sigma_\Delta^2) + 2\sigma_\Delta^2 \right] e^{(\mu_\Delta + \sigma_\Delta^2)/2} \left[ (z_2^2 - 1) \phi(z_2) - (z_1^2 - 1) \phi(z_1) \right] \]
\[- 5\sigma_\Delta^4 e^{(\mu_\Delta + \sigma_\Delta^2)/2} \left[ (z_2^3 - 3z_2) \phi(z_2) - (z_1^3 - 3z_1) \phi(z_1) \right] \]
\[- \sigma_\Delta^5 e^{(\mu_\Delta + \sigma_\Delta^2)/2} \left[ (z_2^4 - 6z_2^3 + 3z_2) \phi(z_2) - (z_1^4 - 6z_1^3 + 3z_1) \phi(z_1) \right] \]

The \( m \)-th order EOL of \( A_{*,ij} \), for \(-\infty \leq A_{*,ij} \leq 0\), can be derived analytically by letting \( t = 0,\ a_1 = -\infty \) and \( a_2 = 0 \). The corresponding \( z_1(t) = -\infty \) and \( z_2 = -\mu_\Delta/\sigma_\Delta \). The functions of \( EOL_m \) for \( 0 \leq m \leq 4 \) can be derived as

\[ EOL_1 = \mu_\Delta \Phi(\beta) - \sigma_\Delta \phi(\beta) \]
\[ EOL_2 = (\mu_\Delta^2 + \sigma_\Delta^2) \Phi(\beta) - 2\mu_\Delta \sigma_\Delta \Phi(\beta) - \beta \sigma_\Delta^2 \phi(\beta) \]
\[ EOL_3 = (\mu_\Delta^3 + 3\mu_\Delta \sigma_\Delta^2) \Phi(\beta) - 3\sigma_\Delta (\mu_\Delta^2 + \sigma_\Delta^2) \phi(\beta) - 3\mu_\Delta \sigma_\Delta^2 \phi(\beta) - (\beta^2 - 1) \sigma_\Delta^2 \phi(\beta) \]
\[ EOL_4 = (\mu_\Delta^4 + 6\mu_\Delta^2 \sigma_\Delta^2 + 3\sigma_\Delta^4) \Phi(\beta) - 4\sigma_\Delta (\mu_\Delta^3 + 3\mu_\Delta^2 \sigma_\Delta) \phi(\beta) - 6\sigma_\Delta^2 (\mu_\Delta^2 + \sigma_\Delta^2) \phi(\beta) - 4(\beta^2 - 1) \mu_\Delta \sigma_\Delta^2 \phi(\beta) - (\beta^3 - 3\beta) \sigma_\Delta^4 \phi(\beta) \]
\[ EOL_\beta = (\mu_\beta^2 + 10\mu_\beta^3 \sigma_\beta^2 + 15\mu_\beta \sigma_\beta^4) \Phi(\beta) - 5\sigma(\mu_\beta^4 + 6\mu_\beta^2 \sigma_\beta^2 + 3\sigma_\beta^4) \phi(\beta) \]
\[ -10 \beta \sigma_\beta^2 (\mu_\beta^3 + 3\mu_\beta \sigma_\beta^2) \phi(\beta) - 10(\beta^2 - 1) \sigma_\beta^4 (\mu_\beta^2 + \sigma_\beta^2) \phi(\beta) \]
\[ -5(\beta^3 - 3\beta) \mu_\beta \sigma_\beta^4 \phi(\beta) - (\beta^4 - 6\beta^2 + 3) \sigma_\beta^6 \phi(\beta) \]

and listed in Table 3.2 where \( \beta = -\mu_\beta / \sigma_\beta \).
APPENDIX – D

ANALYTICAL DERIVATION OF THE EXPECTED NET OUTRANKING INDEX

The expected net outranking index $E\left[\Pi(A_i)\right]$ can be written as

$$
E\left[\Pi(A_i)\right] = E\left[\pi^+ (A_i) - \pi^- (A_i)\right] = \frac{1}{n-1} \sum_{j=1}^{N} \sum_{k=1}^{K} w_k \left\{ E\left[P_k \left(\Delta_{i,j}\right)\right] - E\left[P_k \left(\Delta_{j,i}\right)\right]\right\}
$$

(D-1)

where $\Delta_{i,j} = x_{i,k} - x_{j,k}$ is the payoff difference of the selected alternative $A_i$ and unselected one $A_j$ under a particular criterion $C_k$. It is also a random variable because it is the difference of two random variables $X_{i,k}$ and $X_{j,k}$.

Assuming that $\Delta_{i,j}$ is normally distributed and the Type-VI preference function is used, the expected preference value conditioned on a particular criterion $C_k$ in Eq.(D-1) can be derived as

$$
E\left[P_k \left(\Delta_{i,j}\right)\right] = \int_{\delta}^{\infty} P_k (\delta) f_{\Delta_{i,j}} (\delta) d\delta
$$

$$
= \int_{\delta}^{\infty} \left[ 1 - \exp\left(-\frac{\delta^2}{2\sigma_k^2}\right) \right] f_{\Delta_{i,j}} (\delta) d\delta
$$

$$
= \int_{\delta}^{\infty} f_{\Delta_{i,j}} (\delta) d\delta - \int_{0}^{\infty} \exp\left(-\frac{\delta^2}{2\sigma_k^2}\right) f_{\Delta_{i,j}} (\delta) d\delta
$$

$$
= \Phi \left(\frac{\mu}{\sigma}\right) - \int_{0}^{\infty} \exp\left(-\frac{\delta^2}{2\sigma_k^2}\right) f_{\Delta_{i,j}} (\delta) d\delta
$$

(D-2)
where $\Phi(\bullet)$ is the standard normal cumulative distribution function; $\mu_\Delta$ and $\sigma_\Delta$ are the mean and standard deviation of $\Delta_{i,x}$, respectively. Let $\alpha_k = \frac{1}{2s_k^2}$, the second term in Eq.(D-2) can be rewritten as

$$
\int_{-\infty}^\infty \exp \left( -\alpha_k \delta^i \right) f_{\Delta} \left( \delta \right) d\delta = \frac{1}{\sigma_\Delta \sqrt{2\pi}} \int_{-\infty}^\infty \exp \left( -\alpha_k \delta^i \right) \exp \left( -\frac{1}{2} \left( \frac{\delta - \mu_\Delta}{\sigma_\Delta} \right)^2 \right) d\delta
$$

$$
= \frac{1}{\sigma_\Delta \sqrt{2\pi}} \int_{-\infty}^\infty \exp \left( -\alpha_k \delta^i \right) \exp \left( -\frac{1}{2} \left( \frac{\delta - \mu_\Delta}{\sigma_\Delta} \right)^2 \right) d\delta
$$

$$
= \frac{1}{\sigma_\Delta \sqrt{2\pi}} \int_{-\infty}^\infty \exp \left[ -2\alpha_k \delta^i + \frac{1}{2} \left( \frac{\delta - \mu_\Delta}{\sigma_\Delta} \right)^2 \left( \frac{\delta - \mu_\Delta}{\sigma_\Delta} \right) \right] \left( \frac{\mu_\Delta}{\sigma_\Delta} \right) \left( \frac{\sigma_\Delta}{\mu_\Delta} \right) d\delta
$$

$$
= \frac{1}{\sigma_\Delta \sqrt{2\pi}} \int_{-\infty}^\infty \exp \left[ -2\alpha_k \delta^i + \frac{1}{2} \left( \frac{\delta - \mu_\Delta}{\sigma_\Delta} \right)^2 \right] \left( \frac{\mu_\Delta}{\sigma_\Delta} \right) \left( \frac{\sigma_\Delta}{\mu_\Delta} \right) d\delta
$$

$$
= \frac{1}{\sigma_\Delta \sqrt{2\pi}} \int_{-\infty}^\infty \exp \left[ -\alpha_k \delta^i \right] \exp \left( -\frac{1}{2} \left( \frac{\delta - \mu_\Delta}{\sigma_\Delta} \right)^2 \right) d\delta
$$

By letting $\mu' = \frac{\mu_\Delta}{2\alpha_k \sigma_\Delta + 1}$, and $\sigma'^2 = \frac{\sigma_\Delta^2}{2\alpha_k \sigma_\Delta + 1}$, the above derivation becomes

$$
\int_{0}^{\infty} \exp \left( -\alpha_k \delta^i \right) f_{\Delta} \left( \delta \right) d\delta = \frac{\sigma'}{\sigma_\Delta} \exp \left( -\mu_\Delta \mu' \alpha_k \right) \int_{0}^{\infty} \frac{1}{\sigma' \sqrt{2\pi}} \exp \left\{ -\frac{1}{2} \left( \frac{\delta - \mu'}{\sigma'} \right)^2 \right\} d\delta
$$

$$
= \frac{\sigma'}{\sigma_\Delta} \exp \left( -\mu_\Delta \mu' \alpha_k \right) \Phi \left( \frac{\mu'}{\sigma'} \right)
$$

, and Eq.(D-1) can be written as

$$
E \left[ P_i \left( \Delta_{i,x} \right) \right] = \Phi \left( \frac{\mu_\Delta}{\sigma_\Delta} \right) - \frac{\sigma'}{\sigma_\Delta} \exp \left( -\mu_\Delta \mu' \alpha_k \right) \Phi \left( \frac{\mu'}{\sigma'} \right)
$$

(D-3)

The expected net outranking index $E \left[ \Pi(A_i) \right]$ can then be derived as
\[ E[\Pi(A)] = \frac{1}{n-1} \sum_{j_0} \left\{ \sum_{j_{-1}} \left[ E[P_{\beta_{j_0}}] - E[P_{\beta_{j_{-1}}}] \right] \right\} \]

\[ = \frac{1}{n-1} \sum_{j_0} \left\{ \sum_{j_{-1}} \left[ \Phi(\beta) - \frac{\sigma'}{\sigma} \exp(-\mu' \alpha') \Phi(\beta') - \left( \Phi(-\beta) - \frac{\sigma'}{\sigma} \exp(-\mu' \alpha') \Phi(-\beta') \right) \right] \right\} \]

\[ = \frac{1}{N-1} \sum_{j_0} \left\{ \sum_{j_{-1}} \left[ 2\Phi(\beta) - 1 + \frac{\sigma'}{\sigma} \exp(-\mu' \alpha') + 2 \frac{\sigma'}{\sigma} \exp(-\mu' \alpha') \Phi(\beta') \right] \right\} \]

\[ = \frac{1}{N-1} \sum_{j_0} \left\{ \sum_{j_{-1}} \left[ 2\Phi(\beta) - 1 - \frac{\sigma'}{\sigma} \exp(-\mu' \alpha') (2\Phi(\beta') - 1) \right] \right\} \]

(E-4)

where \( \beta = \frac{\mu_{\Delta}}{\sigma_{\Delta}} \) and \( \beta' = \frac{\mu'}{\sigma'} \).