Strategic Outsourcing in A Supply Chain

by

YANG, Xi

A Thesis Submitted to
The Hong Kong University of Science and Technology
in Partial Fulfillment of the Requirement for
the Degree of Doctor of Philosophy
in Industrial Engineering and Logistics Management

August 2010, Hong Kong
Authorization

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11 August 2010
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This is to certify that I have examined the above PhD thesis and have found that it is complete and satisfactory in all respects, and that any and all revisions required by the thesis examination committee have been made.

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Xi Yang
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Strategic Outsourcing in A Supply Chain

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Abstract

Nowadays outsourcing is a prevailing trend in industry, which allows brand name companies to wholly concentrate on their core competences, as well as introducing some risks when outsourcing its production and procurement activities. In this thesis, we study two issues on outsourcing management with information asymmetries.

We begin with the procurement outsourcing decisions. A brand name company may outsource the procurement activities along with production to a contract manufacturer in pursuit of a low cost, while such an approach may incur uncertainty on the quality of the materials and the final products. We consider a supply chain consisting of one brand name company, one contract manufacturer and a pool of material suppliers with distinct quality levels. The prices obtained from the suppliers depend on the bargaining power of the buyer, which is private information. We derive the optimal contracts under various scenarios to address whether the brand name company should outsource the procurement function and evaluate the value of procurement outsourcing strategy in offsetting supply chain risks. We also propose a quality management scheme as a means of fraud prevention in procurement outsourcing.

In the second problem, we study dynamic outsourcing mechanism design. We examine a multi-period game in which a brand name company outsources the production to a contract manufacturer. The unit production cost in each period consists of two parts: a random shock that is only observed by the contract manufacturer, and a deterministic cost representing the learning effect, which decreases in the accumulated production volume. Our analysis reveals that the order quantity in each period is not only determined...
by the cost in the current period, but also by the past orders. Thus we derive the optimal quantity-based contracts in each period, and compare the decisions in different scenarios and analyzed the impact of the learning rate in dynamic mechanism design.
Chapter 1

Introduction

1.1. Research Background

Nowadays, outsourcing is increasingly becoming a key business strategy. In a rapidly changing economy, every organization possesses some essential but non-core business functions and has difficulties in managing all the business aspects well. More and more companies begin to realize that it is time-consuming and expensive to run a large group of businesses. As brand name companies are forced to explore tactics to maintain their competitiveness, the topic of outsourcing, often referred to as the activity of subcontracting a service such as raw material procurement, product design or manufacturing to a third-party company, has gained much attention in the business world. Companies intend to identify and focus on core competencies, defined as “a unique combination of experience and expertise that would provide a source of competitive advantage in a given industry”, and let a third party take over other business functions. According to a report by Gartner Inc., in March 10, 2004, “The worldwide outsourcing market is expected to grow from $161.9 billion in 2002 to just over $235.6 billion in 2007 at a compound annual growth rate (CAGR) of 7.8 percent”. The article continues, “Because of the recent global economic downturn, cost reduction has been the primary driver for outsourcing over the past several years and continues as a strong driver even as economic growth returns”. Furthermore, according to a report by Canada-based research firm XMG Global, the global outsourcing market is projected to reach $373 billion in total revenue by the end of 2009, a 14.4 percent increase over 2008.
Brand name companies can outsource particular functions of their business for a number of reasons. Figure 1.1 demonstrates the main reasons why companies outsource their business processes which are rated from 1 to 12, with 1 as “the most important” and 12 as “the least important”. Average scores collected from a survey are shown in the figure. A main driver for the outsourcing trend is cost saving. Companies can achieve savings on time, effort, infrastructure and manpower, as outsourcing decisions gives them access to cost-effective services.

![Figure 1.1: Reasons for outsourcing.](image)

<table>
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Brand name companies are able to obtain low labor and material costs as well as reducing their overhead costs by outsourcing particular business functions to a third party...
who specializes in these activities. According to statistics, a given company benefits approximately 80% in cost savings by outsourcing application related work. An annual survey by accounting and consulting firm BDO Seidman LLP showed that the most popular destinations that companies are most likely to consider in 2009 are the United States (22%), China (16%) and India (13%).

Another common reason for outsourcing is the need to simplify business processes and to improve company focus. The need to concentrate on core business is a common motivation for brand name companies to outsource particular business processes. Companies are able to focus on the more important business activities associated with meeting customer needs by strategically outsourcing their non-core functions and defining their core competencies. They embrace outsourcing as a way to achieve higher overall efficiency, as core and non-core business functions are effectively performed in-house and by outsourcing partners, respectively.

Another reason that makes outsourcing strategy appealing is the need to get access to specialized services. By outsourcing its business functions, a brand name company is able to bring in some specialist expertise and skilled services, and thus can achieve higher efficiency and effectiveness. Some companies are also able to get access to new technology through such an approach. For example, small companies may outsource computer programming and other information technology functions as a means of maintaining high-level technology, as maintaining experts in-house may be too expensive.

Other advantages of outsourcing also include to maintain supply chain flexibility and efficiency. Outsourcing can be a way to build in more flexibility to the operation in the sense that it helps companies to transfer fixed costs into variable costs, that it improves companies’ cash flow and that it increases the speed of business processes. Furthermore, outsourcing partners are able to respond more quickly, such as making faster deliveries to customers, which can enhance supply chain efficiency.
1.2. Information Asymmetry in Outsourcing Management

When brand name companies increasingly outsource their business functions, they may also experience unpleasant risks that mainly result from uncertainties about the contract manufacturer. One of the main disadvantages of outsourcing is the supply chain inefficiency due to hidden costs. Information asymmetry arises when a contract manufacturer conceals private information about his costs from his client and the latter may have difficulty in exploring the true information. If that is the case, the interest of the contract manufacturer may be different or even conflict with that of the brand name company. Therefore, information asymmetry plays an important role in determining supply chain performance and will affect the decision associated with outsourcing management. When CMs’ agendas conflict with their OEMs’, the OEMs should try to align them via the incentives they provide (Amaral Billington and Tsay 2006).

Another issue of outsourcing that managers should pay attention to is related to quality. Since the brand name company loses control over the business process, she may experience undesirable results. In the event that the quality of final products does not meet the standards, the company has to seek another service supplier or enforce the original outsourcing firm to repeat the manufacturing process, which is time-consuming and costly, and damages her reputation if defective products have been sold to customers. Furthermore, contract manufacturers may have less incentive to improve the quality of services as long as it meets the conditions of the contract. Without audits, the contract manufacturer may even violate the contract by using inferior materials to their own benefits. If that is the case, outsourcing may lead to unexpected product nonconformance that can damage the market share or even a brand name.

1.3. Incentive Contracts in Outsourcing Management

As discussed, information asymmetry can lead to inefficiency and supply chain risks when brand name companies are outsourcing their business functions. The contracts between the brand name company and the contract manufacturer can result in a total mismatch of policies, standards and interests of both firms. Therefore, it is critical to design
a mechanism that aligns the contract manufacturers’ agendas with their incentives and avoids interest conflicts.

Incentive contracts are widely used to deal with incomplete and asymmetric information. In particular, the principal-agent model is proposed to solve a problem when a client hires an agent to act on behalf of herself and to pursue the interests of the former. The agent holds private information and the principal offers a menu of contracts to maximize her expected profit.

Incentive contracts can be adopted into outsourcing management. The brand name company will offer a menu of incentive contracts, which states clearly the services the latter should provide, and the corresponding payments. Through the contracts, the client encourages desirable performance by attaching rewards to the contract manufacturer and maximizes her expected profit.

1.4. Research Significance

Motivated by industry practices, we look at related research issues on outsourcing problems in this thesis. It contains two theoretical models on outsourcing decisions with information asymmetry, including a procurement outsourcing decision problem and a dynamic outsourcing mechanism design problem.

1.4.1 Procurement Outsourcing Decisions

Procurement outsourcing is defined as transferring specified key activities relating to sourcing and supplier management to a third party, called the procurement contract manufacturer. When outsourcing its production to a contract manufacturer, a brand name company may also let the manufacturer take over the procurement of raw materials to reduce fixed cost, to maximize the return on existing procurement assets as well as to focus on its core competencies.

However, the brand name company may also lose control over the quality of raw materials and thus the final products. As the procurement process is managed by the contract manufacturer, the brand name company does not have sufficient information about the agent’s incentives and actions. In particular, the prices that the brand name company and
the contract manufacturer are able to obtain from the suppliers depend on their bargaining power which is private information. With hidden information, it is difficult for the brand name company to identify the cost as well as the incentive for the contract manufacturer. The brand name company will offer a menu of contracts in order to align the agenda of the agent with his interest and to induce him to adopt appropriate materials.

Furthermore, when quality is non-contractible, i.e., the contract manufacturer may use a different material from what is on the contract, referred to as contract violation, the brand name company may adopt some quality management procedures, e.g., inspecting the material purchased by the contract manufacturer before production begins. Quality management scheme is used as a means of fraud prevention and quality assurance in procurement outsourcing.

We derive the optimal contracts under various scenarios to address whether the brand name company should outsource the procurement function and evaluate the value of procurement outsourcing strategy in offsetting supply chain risks. We also propose a quality management scheme to cope with contract violation in procurement outsourcing, as well as extending the basic model to different scenarios to analyze the impact of manufacturing process uncertainty, demand uncertainty and control delegation.

Whether to outsource the procurement function along with production and how to design incentive contracts to motivate the contract manufacturers are thorny problems in practice for brand name companies. The outcomes of our study will have significant impacts in both academia and industry. Procurement outsourcing can be extremely beneficial for an organization if planned and executed correctly and quality-related cost is a key factor for excelling in competition. However, quality, information asymmetry and incentive contract design in procurement outsourcing have not been well studied in the literature, and its significance in supply chain management has not been well understood. This study can complement the existing literature by adding this new dimension. In practice, the outcomes of this research can help managers to gain managerial insights and thus to improve decision-making processes on procurement outsourcing in supply chain management.
1.4.2 Dynamic Outsourcing Mechanism Design

In a lot of practices, outsourcing decision has been an effective strategy necessary for long-term success. More and more brand name companies are driven by cost reduction (capital investment, labor and material cost, operating cost and etc) and the need to focus on the core businesses in order to maintain advantages over their competitors. When outsourcing its business processes to a third party who specializes in these activities, band name companies intend to build in solid relationships with their service providers and to sign contracts serving as an agreement that outsourcing is a long-term relationship and an ongoing process. That way, both parties can have better knowledge of each other’s business mode, company culture, weaknesses and strengths, and maintain mutual trust between the two parties.

Although brand name companies may prefer to maintain long-term relationships with their contract manufacturers, it may not be an optimal decision for them to sign a single long-term contract that is initiated at the beginning of cooperation. A single contract might hinder the flexibility since that the brand name company lacks the capability of trimming its strategy according to the market change and that the contract manufacturer may not have enough incentives to improve the service to cater to the changes of customer needs. Furthermore, with a single contract, it is difficult for a brand name company to share the on-going cost savings if the agent holds private information. Therefore, many companies will seek a dynamic mechanism to better maintain long-term relationships with their outsourcing companies, to achieve improvement along with time, and to revise their plans or to renew their assessments.

We consider the design of incentive contracts in a dynamic environment in which a contract manufacturer observes private information about production cost over time. Unit production cost decreases in the accumulated production volume as experience is gained from the learning-by-doing process. We derive the optimal quantity-based contracts in each period to study how the brand name company offers incentives to align the interests of both parties, and how a dynamic outsourcing mechanism is designed to maximize the brand name company’s profit. We have also compared decisions in different scenarios to analyze the impact of the learning rate on dynamic mechanism design.

As dynamic mechanism design is a relatively new topic and only a few papers (most
of which are research papers in Economics) have studied the issue, our study contributes to the literature analyzing mechanism design in supply chain management.

1.5. Thesis Organization

The remainder of this thesis is organized as follows. In Chapters 2 and 3, we present a procurement outsourcing problem and a dynamic mechanism design problem, respectively. In each chapter, we begin with industry background and business cases, review relevant research papers, and then introduce the formulation and solution methodology. Finally, we summarize in Chapter 4 the findings and contributions of the thesis, and propose areas for future research.
Chapter 2

Procurement Outsourcing Management

2.1. Introduction

Nowadays, more and more brand name companies outsource the production of their products to contract manufacturers, often located in developing countries, in pursuit of low labor and material costs. While outsourcing its production, a brand name company may choose to keep raw material purchasing decisions in-house. It identifies qualified suppliers, purchases raw materials from the suppliers, and arranges deliveries of the materials to its contract manufacturer. That way, the company can ensure quality inputs into the contract manufacturer’s production process, which has a direct impact on the final products delivered by the manufacturer. While the brand name company has more control over the quality of its products by keeping the purchasing function in-house, such an approach incurs high overhead costs as the company has to deal with the bargaining, administration, and logistics of raw material purchasing by itself, which are most likely not its core competencies. In addition, suppliers of the raw material may often be located far away from the company, possibly in a different country or even on a different continent, which further complicates the issues.

Many companies are now switching to a new approach to raw material procurement. They may outsource their raw material purchasing decisions along with the production to their contract manufacturers. According to a recent survey by Aberdeen Group (2006), nearly half of the companies surveyed indicated that they were outsourcing or planning to outsource some portion of their procurement activities. Furthermore, companies that had
already outsourced some procurement activities were planning to increase their outsourcing activities by more than 16% within one year. The benefits of outsourcing procurement are obvious and well recognized, including fixed-cost and cycle-time reduction, more dynamic and responsive systems, and fewer procurement employees.

However, despite of all the benefits from procurement outsourcing, a brand name company may lose control over the quality of procured materials and thus the quality of the final products. The reason is that there may be information asymmetries and incentive conflicts between a brand name company and its contract manufacturer. For example, the brand name company may not have sufficient information about the contract manufacturer’s procurement capabilities. These include the contract manufacturer’s domain knowledge of or past experience on procurement activities, its quality inspection mechanism for purchased materials, and/or its bargaining power over the suppliers of raw materials. Furthermore, the contract manufacturer has its own profit incentive that may not be aligned with that of the brand name company and thus may seek less expensive/low-quality materials if assigned procurement decisions. Thus, information asymmetries and incentive conflicts in procurement outsourcing may cause product nonconformance which may in turn damage the market share of the brand name products and lead to devastating consequences. This is evident in the case where Mattel Inc., a leading toy company in U.S., had to recall over one million units of its toys because one of its contract manufacturers in China sourced and used paint containing a hazardous level of lead. Therefore, when outsourcing procurement functions, a brand name company needs to put in place effective mechanisms to motivate its contract manufacturers to strike the right balance between price and quality in raw material purchasing.

In this chapter, we take a game-theoretic approach to evaluating whether a brand name company (OEM, she) should outsource its purchasing activities to a contract manufacturer (CM, him), to constructing an optimal menu of contracts, and to analyzing incentives for information sharing. We also study the impact of various uncertainties on the outsourcing decisions to gain a better understanding of the value of procurement outsourcing strategy in offsetting supply chain risks. To the best of our knowledge, this is the first study to quantify the costs and benefits of such decisions at both a brand name company and its contract manufacturer, to analyze the impact of uncertainties on outsourcing decisions,
and to evaluate the mechanisms for ensuring effective procurement outsourcing.

One of our main findings is that procurement outsourcing improves the profitability of both the OEM and the CM. It also leads to a higher quality of purchased material and production system. Furthermore, we find that this conclusion holds even in situations where the CM’s bargaining power over the suppliers is lower than that of the OEM herself. In other words, our analysis suggests that, a CM’s bargaining power over the suppliers alone is not sufficient for an OEM to make the procurement outsourcing decisions. If given the procurement opportunity and properly motivated, a CM with relatively low bargaining power over the suppliers may also offer opportunities for boosting the OEM’s bottom line.

Another interesting finding is that if delegating the contracting of procurement outsourcing to the CM is a viable option for the OEM, it can further improve the supply chain efficiency, which may in turn increase the OEM’s profitability. That is, with the OEM ceding more functions (production, procurement, as well as contracting) to the CM, she may have motivated the CM to make decisions that will gradually improve the supply chain profit, benefiting both parties.

We also analyze how demand variability and the possibility of the CM using a material with lower quality than what is in the contract, two of the main concerns in procurement outsourcing, affect the OEM’s decision on procurement outsourcing. We find that demand variability may reduce the benefits from procurement outsourcing and make procurement outsourcing less attractive. To prevent the CM from contract violation, the OEM can design a mixed strategy for material inspection. However, such an approach incurs a cost and thus again makes procurement outsourcing less preferable.

The chapter is organized as follows. After reviewing relevant literature in Section 2.2, we introduce and analyze our model in Section 2.3. We analyze the decisions at both parties under the procurement in-house option in Section 2.3.1 and derive the optimal contract(s) under the procurement outsourcing option in Section 2.3.2, and compare the quality levels of purchased materials under the two options in Section 2.3.3. We devote Section 2.4 to various extensions to our basic model and conclude this chapter in Section 2.5.
2.2. Literature Review

Outsourcing procurement is a relatively recent industrial trend, and so far there have only been some empirical studies that address this issue. Quinn et al. (1994) study the impact and outcomes of strategic outsourcing by developing ways for determining a company’s core competencies and identifying activities that are better to be performed externally. They also investigate strategic risks associated with outsourcing. Ellram and Billington (2001) explore the management of potential losses of volume and purchasing leverage when material procurement is outsourced. Building on the transaction-cost literature, they develop a prescriptive framework to implement efficient management. Billington and Kuper (2003) analyze various aspects of procurement outsourcing and the potential challenges for firms that adopt this strategy, and examine the value of Hewlett-Packard’s procurement outsourcing strategy named “the buy-sell mode”. An article in Purchasing (2003) emphasizes the necessity of controlling strategic relationships with key component suppliers when buyers outsource their purchasing along with the manufacturing. Amaral et al. (2006) examine both the opportunities and hazards in procurement outsourcing by conducting a comprehensive industrial survey. They propose a perspective approach for a brand name company to determining the activities to outsource while maintaining its bargaining power and minimizing the risks involved. Our research differs from these studies in that we develop an analytical model to study whether the purchasing function should be outsourced along with production.

As we consider procurement contract design under private information, the literature that addresses supply chain contracting under asymmetric information is also relevant. Chen (2003) and Cachon (2003) provide extensive surveys of pertinent literature. One stream of research studies the screening problem, in which one firm holds private information and another uninformed firm offers a menu of contracts and thus induces the former to truthfully report the private information. In this stream, Corbett and Groote (2000), Ha (2001), and Corbett et al. (2004) analyze quantity-based contract menus, Corbett (2001) studies an optimal reorder policy under information asymmetry on the setup cost, Cachon and Zhang (2006) compare several simple mechanisms with the optimal one.
under a queuing framework, and Plambeck and Taylor (2007) analyze the impact of con-
tract renegotiation on firms’ profitability. Another stream of research relevant to our work
falls within a class of problems known as the signaling models. The stream dates back to
Spence (1973) who suggests that education credentials can be used as a signal to a firm,
indicating a certain level of ability of an employee; thereby narrowing the informational
gap. Related research in supply chain management has mainly focused on the signaling
and the sharing of demand information. In particular, Chu (1992) and Lariviere and Pad-
manabhan (1997) discuss demand information sharing in a setting where a channel faces
a deterministic demand function, while Cachon and Lariviere (2001) and Ozer and Wei
(2006) study a model with stochastic demand. Other papers address the incentives for in-
formation sharing include Li (2002), Zhang (2002), and Li and Zhang (2002, 2008). It is
also worth mentioning that there exists some literature (for instance, Desai and Srinivasan
1995) that addresses screening mechanism design with embedded signaling games. Our
work adopts some of the methodologies for analyzing signaling models to study decision
making in procurement outsourcing in the presence of two-sided information asymmetry.

Finally, we note that there has been an increasing amount of literature addressing
quality management in supply chains. Reyniers and Tapiero (1995) study the interaction
between a supplier’s quality level and a buyer’s inspection policy. They demonstrate the
existence of a Nash equilibrium and quantify the value of cooperation to both players.
Tagaras and Lee (1996) analyze the tradeoff between quality and costs in supplier selec-
tion as well as the interaction between the input quality and internal process quality. They
advocate the importance of improving process quality as it affects the economic value of
the inputs. Starbird (1997) examines how a buyer’s inspection policy affects its supplier’s
quality level. His analysis indicates that full quality conformance can be achieved either
by inspiring the supplier to deliver zero defects or by using a complete inspection pol-
cy. Lim (2001) considers a supplier that sells components to a buyer but holds private
information about its quality level and shows that the buyer can design a contract to op-
timize its profit while achieving truthful revelation of the supplier’s quality. Benjaafar et
al. (2007) compare two demand allocation schemes from a buyer’s perspective based on
the quality levels delivered by its suppliers. The first scheme allocates the demand to se-
veral suppliers, while the second scheme to only one supplier. The authors show that the

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quality advantage of the first scheme over the second depends on the fixed and variable post-allocation costs. Zhu et al. (2007) explore the roles of different supply chain parties in quality improvement. They find that a buyer’s proactive investment in its supplier’s quality improvement can have a significant impact on the profitability of both parties and of the supply chain as a whole. However, none of these studies considers whether and how a company should outsource procurement activities, which is the focus of the chapter.

2.3. Model and Analysis

We consider a brand name company or an original equipment manufacturer (OEM, she) who designs and owns a product but outsources the production of the product to a contract manufacturer (CM, him). To focus on decisions related to material outsourcing, we assume that the objectives at both the OEM and CM are unit cost minimization. This is true if the demand and selling price of the final product are quite stable. While outsourcing her production to the CM, the OEM has two options for purchasing a key raw material/component. She can purchase and deliver the material to the CM, referred to as the in-house procurement option, or delegate the purchasing activities along with the production to the CM, referred to as the procurement outsourcing option.

The buyer (the OEM under the in-house option and the CM under the outsourcing option) sources the material from a single supplier chosen from a pool of suppliers, each associated with a distinct quality level $\alpha$, $0 < \alpha < 1$. Thus, we will refer to the supplier with $\alpha$ as supplier $\alpha$. Here $\alpha$ may be the percentage of defective units supplied by the supplier and, the smaller the value of $\alpha$, the higher the quality level. Due to different bargaining powers of the OEM and the CM, the supplier may offer different prices to the two parties (unless otherwise specified, by “bargaining power”, we mean the bargaining power of the buyer, either the OEM or the CM, over the raw material suppliers). We assume that the CM pays at $\beta_{CM}(\alpha)$ and OEM at $\beta_{OEM}(\alpha)$ for each unit of the material from supplier $\alpha$ where the list price $c(\alpha)$ is decreasing convex in $\alpha$. Thus, the lower the value of $\beta_{CM}$ ($\beta_{OEM}$), the higher the bargaining power that the CM (OEM) has over the suppliers. Since the bargaining power is usually an important leverage in contract negotiation and kept private in reality, it is treated as private information. That is, the OEM only has an
estimate of $\beta_{CM}$, denoted by $\hat{\beta}_{CM}$ which is uniformly distributed over $[\beta_{CM}^L, \beta_{CM}^U]$ and the CM only has a prior knowledge on $\beta_{OEM}$, denoted by $\hat{\beta}_{OEM}$ uniformly distributed over $[\beta_{OEM}^L, \beta_{OEM}^U]$. We assume that the distributions are known to both parties. Due to information asymmetry, the OEM may not know for sure the material the CM will use if procurement is outsourced and her objective is to minimize her expected cost. Likewise, the CM may not know the material the OEM will use in advance if the OEM procures the material by herself.

Due to imperfection in the raw material, some of the final products sold will be non-conforming and will incur quality-related costs. For instance, the OEM as the brand owner suffers from the damage to her brand name and future market share for each nonconforming unit sold and we model the related cost as $R(\alpha)$, born only by the OEM. There are also repair/replacement costs associated with each nonconforming final product, denoted by $r(\alpha)$, out of which $r_1(\alpha)$ is born by the party who sources the material and $r_2(\alpha)$ by the CM. We assume that $R(\alpha)$, $r_1(\alpha)$, and $r_2(\alpha)$ are increasing convex, which are true in most real situations.

We are now ready to present our model and analysis. We analyze the material selection decision and the costs incurred by both parties under both options. Since procurement outsourcing is possible only if the parties can reach an agreement, there are two possible scenarios. The OEM (the CM) initiates a contract that specifies how the CM will be compensated for both production and component procurement. Procurement is outsourced if the CM (OEM) accepts the contract. We will focus on the case where the OEM offers a contract and compare the outcomes under both scenarios later in the chapter.

### 2.3.1 Decision Making under the In-House Option

If the OEM keeps procurement in-house, she needs to select a material $\alpha$ at $\beta_{OEM}(\alpha)$ and pays the CM a fixed wholesale price for each unit produced which covers the CM’s production cost plus a margin. We assume that this price is industry-specific and hence exogenous. Without loss of generality, we normalize this price as zero. Thus, the OEM’s unit cost is given by (where the superscript or subscript $I$ represents values associated...
with the in-house option)

$$\pi^I_{OEM}(\beta_{OEM}) = \min_{\alpha} \{\beta_{OEM}c(\alpha) + R(\alpha) + r_1(\alpha)\}$$

and she will select a material $$\alpha_I(\beta_{OEM})$$ such that

$$\beta_{OEM}c'(\alpha_I) + R'(\alpha_I) + r'_1(\alpha_I) = 0.$$  \hspace{1cm} (2.1)

We assume that it is a profitable business for the CM to produce the product for the OEM even if procurement is kept in-house. While the CM’s actual unit cost is

$$\pi^I_{CM}(\beta_{OEM}) = r_2(\alpha_I(\beta_{OEM})),$$

he only knows his expected cost based on his estimate of the OEM’s bargaining power, $$\hat{\beta}_{OEM},$$

$$\hat{\pi}^I_{CM} = E\left[\pi^I_{CM}(\hat{\beta}_{OEM})\right] = \frac{1}{\beta_{OEM} - \hat{\beta}_{OEM}} \int_{\hat{\beta}_{OEM}}^{\beta_{OEM}} r_2(\alpha_I(\beta))d\beta.$$  

2.3.2 Decision Making under the Outsourcing Option

If procurement is outsourced, the CM needs to select the material and be compensated for production as well as material procurement. The first question we need to answer is how the CM should be compensated for both material procurement and production. It is clear that the OEM should not offer the CM a fixed fee regardless of the material he uses as such a contract provides the CM with an incentive to use a less expensive material which is likely of lower quality. Thus, the OEM’s per unit payment should be a function of the material the CM actually uses, denoted by $$w_S(\alpha)$$ (where the subscript or later a superscript $$S$$ represents values associated with the outsourcing option). The CM can either reject it, in which case procurement will be kept in-house by the OEM, or accept it by selecting his supplier $$\alpha_S$$ and receiving a payment $$w_S(\alpha_S)$$. Throughout the chapter, we will assume that the CM will accept a contract as long as he is not worse off compared with the in-house option. If the CM accepts a contract, the OEM incurs the following unit
cost

\[ \pi^S_{OEM}(\beta_{CM}) = w_S(\alpha_S) + R(\alpha_S) \]

and the CM’s unit cost is given by

\[ \pi^S_{CM}(\beta_{CM}) = \beta_{CM} c(\alpha_S) + r(\alpha_S) - w_S(\alpha_S). \]

Below we first focus on the case where the CM has full information about \( \beta_{OEM} \) in Case 1 and then discuss the case where the CM only holds a prior distribution in Case 2.

**Case 1: The CM Has Full Information about the OEM’s Bargaining Power \( \beta_{OEM} \)**

When the OEM’s bargaining power \( \beta_{OEM} \) is public information, there are two scenarios depending on whether the CM’s bargaining power \( \beta_{CM} \) is public information.

**Case 1.1: The CM’s bargaining power \( \beta_{CM} \) is public information**

If the CM’s bargaining power \( \beta_{CM} \) is also known to the public, the OEM can simply force the CM to use material \( \alpha_S \) that achieves the lowest supply chain cost under the outsourcing option \( \beta_{CM} c(\alpha) + R(\alpha) + r(\alpha) \) at the wholesale price such that the CM earns the same as under the in-house option, i.e., \( w_S(\alpha_S) = \beta_{CM} c(\alpha_S) + r(\alpha_S) - \pi^I_{CM}(\beta_{OEM}) \). The CM will accept the contract and the OEM achieves the lowest cost. Of course, the OEM will offer such a contract only if \( \beta_{CM} \) is below a certain level at which the supply chain cost under the outsourcing option \( \beta_{CM} c(\alpha_S) + R(\alpha_S) + r(\alpha_S) \) equals the supply chain cost under the in-house option \( \beta_{OEM} c(\alpha_I) + R(\alpha_I) + r(\alpha_I) \). As a result, the CM earns exactly the same as that under the in-house option, while the OEM earns a higher profit than that under the in-house option and is the sole beneficiary of procurement outsourcing. Thus, a certain level of uncertainty about the CM’s bargaining power is necessary to prevent complete exploitation by the OEM.

**Case 1.2: The CM’s bargaining power \( \beta_{CM} \) is private information**

Now consider the case where the OEM only holds a prior distribution on \( \beta_{CM} \). With uncertainty in \( \beta_{CM} \), the OEM is not able to determine \( \alpha_S \) that achieves the lowest supply
chain cost under the outsourcing option. Instead, she has to offer a menu of contracts \( \{ \alpha_S, w_S(\alpha_S) \} \) and the CM may accept a contract or reject the offer. We will focus on contracts that induce the CM to reveal his true bargaining power and to use the material that minimizes the OEM’s expected cost.

By the revelation principle (Baron and Myerson 1982), we can formulate an equivalent menu of contracts as \( \{ \alpha_S(\beta_{CM}^R), w_S(\beta_{CM}^R) \} \) where \( \beta_{CM}^R \) is the CM’s reported bargaining power. If the CM reports \( \beta_{CM}^R \) as his bargaining power, he will choose a contract \( (\alpha_S(\beta_{CM}^R), w_S(\beta_{CM}^R)) \) with a resulting unit cost of \( \beta_{CM} c(\alpha_S(\beta_{CM}^R)) + S(\alpha_S(\beta_{CM}^R)) - w_S(\beta_{CM}^R) \). Thus, the CM will report \( \beta_{CM}^R \) such that

\[
\beta_{CM} c'(\alpha_S)\alpha_S'(\beta_{CM}) + r'(\alpha_S)\alpha_S'(\beta_{CM}) - w_S'(\beta_{CM}) = 0.
\]

Inducing truthful revelation is equivalent to ensuring that \( \beta_{CM}^R = \beta_{CM} \). Hence, we can formulate the incentive-compatibility constraint (IC) as

\[
\text{IC: } \beta_{CM} c'(\alpha_S)\alpha_S'(\beta_{CM}) + r'(\alpha_S)\alpha_S'(\beta_{CM}) - w_S'(\beta_{CM}) = 0. \tag{2.2}
\]

So, for any revealed choice of \( (\alpha_S, w_S) \) by the CM, the OEM can infer his true bargaining power \( \beta_{CM} \). Furthermore, the contract is acceptable to the CM only if his cost under the outsourcing option \( \pi_{CM}^S(\beta_{CM}) = \beta_{CM} c(\alpha_S(\beta_{CM})) + r(\alpha_S(\beta_{CM})) - w_S(\beta_{CM}) \) does not exceed his cost under the in-house option, which leads to the following participation constraint (PC).

\[
\text{PC: } \pi_{CM}^S(\beta_{CM}) \leq \pi_{CM}^I(\beta_{OEM}). \tag{2.3}
\]

Since the CM’s cost is increasing in \( \beta_{CM} \) under a contract satisfying the IC and PC, there exists a cutoff point \( \beta_{CM}^0 \) such that the CM will reject the contract if \( \beta_{CM} > \beta_{CM}^0 \) in which case procurement will remain in-house. Otherwise, the CM will accept the contract \( (\alpha_S(\beta_{CM}), w_S(\beta_{CM})) \). The resulting OEM’s cost is

\[
\pi_{OEM}^S(\beta_{CM}) = \begin{cases} 
  w_S(\beta_{CM}) + R(\alpha_S(\beta_{CM})), & \text{if } \beta_{CM} \leq \beta_{CM}^0, \\
  \pi_{OEM}^I(\beta_{OEM}), & \text{otherwise}.
\end{cases}
\]

Since the OEM only knows the distribution of the CM’s bargaining power while designing the contract, she only knows her expected unit cost \( \hat{\pi}_{OEM}^S \) by solving the following
optimal control problem:

\[ \hat{\pi}^S_{OEM} = E[\pi^S_{OEM}(\hat{\beta}_{CM})] \]

\[
= \min_{\alpha_S(\cdot), \, w_S(\cdot), \, \beta_{CM}} \frac{1}{\beta_{CM} - \beta_{CM}} \left[ \int_{\beta_{CM}}^{\beta_{CM}} \pi^S_{OEM}(\beta) d\beta + \int_{\beta_{CM}}^{\beta_{CM}} \pi^S_{OEM}(\beta_{OEM}) d\beta \right] \\
\text{s.t.} \quad \beta' \alpha_S(\beta) + r' \alpha_S(\beta) - w_S(\beta) = 0 \quad \text{for} \quad \beta \in [\beta_{CM}, \beta_{CM}^0], \\
\pi^S_{CM}(\beta_{CM}) = \pi^I_{CM}(\beta_{OEM}).
\]

To derive the OEM’s menu of contracts, we first establish the structure of the contracts \((\alpha_S(\beta), \, w_S(\beta))\) for a given cutoff point \(\beta_{CM}^0\) (Theorem 2.1) and then identify the optimal cutoff point \(\beta_{CM}^0\) (Theorem 2.2).

**Theorem 2.1.** Suppose that the OEM only holds a prior distribution of \(\beta_{CM}\) and decides to offer a menu of contracts, \((\alpha_S(\beta), \, w_S(\beta))\), to the CM. For any given cutoff point \(\beta_{CM}^0\), \(\alpha_S(\beta)\) is the unique solution to

\[ 2(\beta - k_v) \beta' \alpha_S(\beta) + R(\alpha_S(\beta)) + r(\alpha_S(\beta)) = 0, \]

and \(w_S(\beta)\) satisfies

\[ w_S(\beta) = k_v c(\alpha_S(\beta)) - \frac{R(\alpha_S(\beta))}{2} + \frac{r(\alpha_S(\beta))}{2} + k_f, \]

where \(k_v = \frac{\beta_{CM}}{2}\) and

\[ k_f = -\pi^I_{CM}(\beta_{OEM}) + (\beta_{CM}^0 - k_v) c(\alpha_S(\beta_{CM}^0)) + \frac{R(\alpha_S(\beta_{CM}^0)) + r(\alpha_S(\beta_{CM}^0))}{2}. \]

**Proof of Theorem 2.1:** For any given \(\beta_{CM}^0\), the OEM solves the following optimal control problem

\[
\min_{\alpha_S(\cdot), \, w_S(\cdot)} \int_{\beta_{CM}}^{\beta_{CM}^0} \left[ w_S(\beta) + R(\alpha_S(\beta)) \right] d\beta \\
\text{s.t.} \quad w_S(\beta) = \beta' \alpha_S(\beta) + r' \alpha_S(\beta) \quad \text{for any} \quad \beta \in [\beta_{CM}, \beta_{CM}^0] \\
\pi^S_{CM}(\beta_{CM}) = \pi^I_{CM}(\beta_{OEM}).
\]

Letting \(\alpha'_S(\beta) = u\), we construct the Hamiltonian for the optimal control problem as
\[ H(w_S, \alpha_s, u, \lambda_w, \lambda_\alpha) = w_S + R(\alpha_S) + \lambda_w[\beta c'(\alpha_S)u + r'(\alpha_S)u] + \lambda_\alpha u. \]

An optimal solution thus satisfies the following conditions.

\[
\frac{\partial H}{\partial u} = \lambda_w[\beta c'(\alpha_S) + r'(\alpha_S)] + \lambda_\alpha = 0, \tag{2.9}
\]
\[
\lambda'_w(\beta) = -\frac{\partial H}{\partial w_S} = -1, \tag{2.10}
\]
\[
\lambda'_\alpha(\beta) = -\frac{\partial H}{\partial \alpha_S}. \tag{2.11}
\]

**Structure of \(\alpha_S(\beta)\):** By (2.10), we have \(\lambda_w = -\beta + 2k_v\), where \(k_v\) is a constant, and \(\lambda_\alpha = (\beta - 2k_v)[\beta c'(\alpha_S) + r'(\alpha_S)] \) by (2.9). Since

\[
\lambda'(\beta) = \beta c'(\alpha_S) + r'(\alpha_S) + (\beta - 2k_v)[c'(\alpha_S) + \beta c''(\alpha_S)u + r''(\alpha_S)u],
\]
\[
\frac{\partial H}{\partial \alpha_S} = R'(\alpha_S) - (\beta - 2k_v)[\beta c''(\alpha_S)u + r''(\alpha_S)u],
\]

we have \(R'(\alpha_S) + r'(\alpha_S) = -2(\beta - k_v)c'(\alpha_S)\) or equivalently \(\beta = k_v - \frac{R'(\alpha_S) + r'(\alpha_S)}{2c'(\alpha_S)}\).

**Structure of \(w_S(\beta)\):** Substituting the above result into (2.4), we have

\[
w'_S(\beta) = r'(\alpha_S)a'_S(\beta) + \left[ k_vc'(\alpha_S) - \frac{R'(\alpha_S) + r'(\alpha_S)}{2} \right] a'_S(\beta).
\]

Solving the above differential equation, we obtain

\[
w_S(\beta) = r(\alpha_S(\beta)) + k_vc(\alpha_S(\beta)) - \frac{R(\alpha_S(\beta)) + r(\alpha_S(\beta))}{2} + k_f
\]
\[
= k_vc(\alpha_S(\beta)) - \frac{R(\alpha_S(\beta))}{2} + \frac{r(\alpha_S(\beta))}{2} + k_f
\]

where \(k_f\) is a constant. Letting \(\pi^S_{CM}(\beta^0_{CM}) = \pi^L_{CM}(\beta_{OEM})\), we obtain

\[
k_f = -\pi^L_{CM}(\beta_{OEM}) + (\beta^0_{CM} - k_v)c(\alpha_S(\beta^0_{CM})) + \frac{R(\alpha_S(\beta^0_{CM})) + r(\alpha_S(\beta^0_{CM}))}{2}.
\]

Substituting \(k_f\) into (2.8), we have the first order optimality condition

\[(2k_v - \beta_{CM})[c(\alpha_S(\beta_{CM})) - c(\alpha_S(\beta^0_{CM}))] = 0.\]
If $\beta_{CM} < \beta_{CM}^0$, $c(\alpha_S(\beta_{CM})) > c(\alpha_S(\beta_{CM}^0))$. The first order condition is satisfied at $k_v = \frac{\beta_{CM}}{2}$. It is easy to check that the second order condition is also satisfied and the OEM's expected cost is minimized at $k_v = \frac{\beta_{CM}}{2}$. Otherwise, $k_v = \frac{\beta_{CM}}{2}$ is also an optimal solution and the CM will accept a contract if and only if $\beta_{CM} = \underbar{\beta}_{CM}$.

Theorem 2.1 demonstrates that in the optimal menu of contracts $\{\alpha_S(\beta), w_S(\beta)\}$, the wholesale price function consists of three parts. (1) A fixed fee $k_f$ which guarantees the CM the same cost as under the in-house option if $\beta_{CM} = \beta_{CM}^0$, the cutoff point, and procurement is outsourced. (2) $\frac{\beta_{CM}}{2}$ of the list price $c(\alpha)$ for the material used. This variable cost depends on the OEM’s estimate of the CM’s bargaining power $\hat{\beta}_{CM}$ and increases in $\beta_{CM}$. (3) A half of the repair cost $r(\alpha_S)$ minus a half of the OEM’s reputation cost $R(\alpha_S)$, which results in an equal sharing of the total quality cost by the two parties.

With Theorem 2.1, we can determine the cutoff point $\beta_{CM}^0$ to minimize the OEM’s expected cost which is convex in $\beta_{CM}^0$. The cutoff point is either the solution to the first order condition $\pi_{OEM}(\beta_{OEM}) - \pi_{OEM}^S(\beta) = (\beta - \beta_{CM}^0)c(\alpha_S(\beta))$, denoted by $\beta_{CM}^1$ which is a function of $\beta_{CM}$, if it is between $[\beta_{CM}, \overline{\beta}_{CM}]$, or one of the boundaries otherwise. If $\beta_{CM}^1 \leq \beta_{CM}^0$ and the CM’s bargaining power is too low, there is no need for the OEM to offer any contracts because the OEM anticipates that the CM will reject the contracts with probability 1 even if they are offered and the OEM will simply keep the procurement in-house. Otherwise, she will offer a menu of contracts described in Theorem 2.1. The CM will accept or reject the contracts based on his $\beta_{CM}$. In the following theorem, $\beta_{CM}^1 \leq \beta_{CM}^0$ is equivalent to $\beta_{CM}^1 \leq \beta_{CM}$.

**Theorem 2.2.** There exists a threshold $\beta_{CM}^1$ such that if $\beta_{CM} \leq \beta_{CM}^1$, the OEM will keep the procurement in-house and select $\alpha_I(\beta_{OEM})$ given in (2.1). Otherwise, she will offer a menu of contracts $(\alpha_S(\beta), w_S(\beta))$ described in Theorem 2.1 with $\beta_{CM}^0 = \min\{\beta_{CM}^1, \overline{\beta}_{CM}\}$. If the CM accepts a contract, he will select $\alpha_S(\beta_{CM})$ given in (2.6).

**Proof of Theorem 2.2:** Substituting the results in Theorem 2.1 to (2.8) and taking the
derivative with respect to $\beta_{CM}^0$, we get the first order condition

$$
(2\beta_{CM}^0 - \beta_{CM}) c(\alpha_S(\beta_{CM}^0)) + R(\alpha_S(\beta_{CM}^0)) + r(\alpha_S(\beta_{CM}^0))
= \beta_{OEM} c(\alpha_I(\beta_{OEM})) + R(\alpha_I(\beta_{OEM})) + r(\alpha_I(\beta_{OEM}))
$$

and

$$
\pi_{OEM}^I(\beta_{OEM}) - \pi_{OEM}^S(\beta_{CM}^0) = (\beta_{CM}^0 - \beta_{CM}) c(\alpha_S(\beta_{CM}^0)). \quad (2.12)
$$

Let $\beta_{CM}^i$ be the solution to (2.12). Since $\frac{\partial \beta_{CM}^i}{\partial \beta_{CM}} = \frac{1}{2}$, there exists a unique solution $\beta_{CM}^c$ such that $\beta_{CM}^c \geq \beta_{CM}$ if $\beta_{CM} \leq \beta_{CM}^i$, and $\beta_{CM}^c < \beta_{CM}$ otherwise. Therefore, if $\beta_{CM} \leq \beta_{CM}^c$, the OEM offers a menu of contracts with a cutoff point $\beta_{CM}^0 = \min\{\beta_{CM}^c, \beta_{CM}^e\}$.

If $\beta_{CM} > \beta_{CM}^c$, the OEM simply keeps the procurement function in-house.

Below, we discuss how the OEM’s knowledge about $E(\hat{\beta}_{CM})$ and $\text{Var}(\hat{\beta}_{CM})$ impacts the costs of the two parities under procurement outsourcing, respectively. Note that the OEM’s belief about the CM’s bargaining power affects the wholesale prices she offers and hence the CM’s decision on whether to accept a contract. Here, we only consider the impact of $E(\hat{\beta}_{CM})$ and $\text{Var}(\hat{\beta}_{CM})$ on both parties when procurement is outsourced to the CM. The results are summarized in the proposition stated below.

**Proposition 2.1.** Suppose the CM accepts a contract and procurement is outsourced to the CM.

1. For a fixed $\text{Var}(\hat{\beta}_{CM})$, $\pi_{OEM}^S(\beta_{CM})$ increases in $E(\hat{\beta}_{CM})$, while $\pi_{OEM}^S(\beta_{CM})$ decreases in $E(\hat{\beta}_{CM})$.

2. For a fixed $E(\hat{\beta}_{CM})$, the total supply chain cost $\pi_{OEM}^S(\beta_{CM}) + \pi_{OEM}^S(\beta_{CM})$ increases in $\text{Var}(\hat{\beta}_{CM})$. For the CM, there exists $\sigma_1$ and $\sigma_2$, where $0 < \sigma_1 < \sigma_2$, such that $\pi_{OEM}^S(\beta_{CM})$ decreases in $\text{Var}(\hat{\beta}_{CM})$ if $\text{Var}(\hat{\beta}_{CM}) \leq \sigma_1^2$ and increases if $\text{Var}(\hat{\beta}_{CM}) \geq \sigma_2^2$.

**Proof of Proposition 2.1:** Let $\sigma = \frac{\beta_{CM} - \beta_{CM}^c}{2}$. Then $\sigma = \sqrt{3\text{Var}(\hat{\beta}_{CM})}$. When
\( \text{Var}(\hat{\beta}_{\text{CM}}) \) is large, i.e., \( \text{Var}(\hat{\beta}_{\text{CM}}) \geq \sigma_1^2 \), \( \beta_{\text{CM}}^0 = \beta_{\text{CM}}^\dagger \), the unconstrained solution to the first order condition. Therefore, \( \frac{\partial \pi_{\text{CM}}^S}{\partial \sigma} = \frac{c(\alpha_S)}{2} \geq 0 \) and \( \pi_{\text{CM}}^S \) is increasing in \( \text{Var}(\hat{\beta}_{\text{CM}}) \).

When \( \text{Var}(\hat{\beta}_{\text{CM}}) \) is small, \( \beta_{\text{CM}}^0 = \overline{\beta}_{\text{CM}} \). When \( \text{Var}(\hat{\beta}_{\text{CM}}) = 0 \), \( \frac{\partial \pi_{\text{CM}}^S}{\partial \sigma} = -\frac{c(\alpha_S)}{2} < 0 \). Since \( \frac{\partial \pi_{\text{CM}}^S}{\partial \sigma} \) is continuous in \( \sigma \), there exists \( \sigma_2 \) such that when \( \text{Var}(\hat{\beta}_{\text{CM}}) \leq \sigma_2^2 \), \( \frac{\partial \pi_{\text{CM}}^S}{\partial \sigma} < 0 \) and \( \pi_{\text{CM}}^S \) is decreasing in \( \text{Var}(\hat{\beta}_{\text{CM}}) \).

With a lower \( E(\hat{\beta}_{\text{CM}}) \), the OEM believes that the CM has a higher bargaining power over the suppliers on average and will offer a lower \( k_f \) and/or \( k_v \) and hence, lower wholesale prices in her menu of the contracts, benefiting herself while hurting the CM. Thus, while a reasonable bargaining power is necessary for a CM to obtain procurement opportunities from an OEM, he should not over advertise it.

With a higher \( \text{Var}(\hat{\beta}_{\text{CM}}) \), the OEM is less certain about the CM’s bargaining power. To hedge against the uncertainty, the OEM will reduce in her wholesale prices the portion of the material cost (i.e., a smaller \( k_v \)) and compensate the CM with a higher fixed cost \( k_f \). The overall effect is a higher supply chain cost. However, the impact on the CM is more intricate as the OEM’s uncertainty about \( \beta_{\text{CM}} \) has both a positive and negative effect on the CM. It prevents the OEM from exploiting the CM, yet it may force the CM to share a higher supply chain cost. When \( 0 < \text{Var}(\hat{\beta}_{\text{CM}}) \leq \sigma_1^2 \), the positive effect outweighs the negative one, benefiting the CM. When \( \text{Var}(\hat{\beta}_{\text{CM}}) \geq \sigma_2^2 \), the negative effect prevails, hurting the CM.

In summary, while the CM with on average higher bargaining power is preferred by the OEM, he can obtain higher wholesale prices from the OEM for taking on the procurement function and achieve lower costs if the OEM underestimates his bargaining power. In the meantime, while the OEM prefers a little or no uncertainty about the CM’s bargaining power so that she can benefit exclusively from procurement outsourcing, the CM needs to safeguard his true bargaining power and leave the OEM a certain level of uncertainty.
Case 2: The CM Has Partial Information about the OEM’s Bargaining Power $\beta_{OEM}$

When the CM only holds a prior distribution of $\beta_{OEM}$ over $[\beta_{OEM}, \tilde{\beta}_{OEM}]$, he can estimate his expected cost if procurement is done in-house $\hat{\pi}_{CM}^I$. This cost is then used by the CM as the reservation cost when deciding whether to accept a contract from the OEM and is taken into account by the OEM for determining the menu of wholesale price contracts. In such a case, the OEM may have an incentive to simply disclose her true bargaining power to the CM by presenting to the CM authentic receipts or price quotes from the suppliers. For instance, if $\pi_{CM}^I(\beta_{OEM}) > \hat{\pi}_{CM}^I$, the CM underestimates his cost under the in-house option without the true information $\beta_{OEM}$. It is for the best interest of the OEM to disclose her true bargaining power and the CM is more likely to accept a contract at a lower wholesale price.

Anticipating that the OEM may disclose her true bargaining power and will do so if it benefits her, the CM will update his belief about $\beta_{OEM}$ and adjust his expected cost $\hat{\pi}_{CM}^I$. In economics, such a game is referred to as a signaling game. In our setting, the OEM first decides whether to disclose her bargaining power, and then offers a contract if she has full information about $\beta_{CM}$ or a menu of contracts to the CM otherwise. The CM then updates his belief about $\beta_{OEM}$ and decides whether to accept a contract. We show below that information disclosure is preferred by the OEM following the well-known voluntary disclosure principle (Milgrom 1981).

**Proposition 2.2.** If the OEM decides to offer procurement contracts to the CM, she uses a strategy of full disclosure at every equilibrium. Thus, the expected costs at both parties are the same as those where the CM has full information about $\beta_{OEM}$.

The reason for full disclosure is as follows. If $\pi_{CM}^I(\beta_{OEM}) > \hat{\pi}_{CM}^I$, which implies that the OEM has relatively low bargaining power, the OEM will disclose her true bargaining power to encourage the CM to accept a contract at a lower wholesale price. If $\pi_{CM}^I(\beta_{OEM}) \leq \hat{\pi}_{CM}^I$, then the OEM will keep her bargaining power private. The CM will adjust downward his belief about $\beta_{OEM}$ as well as $\hat{\pi}_{CM}^I$. The process continues and full disclosure is a dominant strategy to offset the potential for adverse selection (Grossman 1981 and Milgrom 1981). From now on, we will only focus on the case where the CM has full information about the OEM’s bargaining power.
2.3.3 Comparisons between the In-House and Outsourcing Options

In this section, we compare the supplier or material selection decisions and the resulting costs under both options. We first establish that the OEM’s bargaining power \( \beta_{OEM} \leq \beta_{CM}^{\dagger} \), the threshold of \( \beta_{CM} \) below which the OEM will offer the CM a menu of contracts (see Theorem 2.2).

**Proposition 2.3.** \( \beta_{CM}^{\dagger} \geq \beta_{OEM} \).

**Proof of Proposition 2.3:** \( \beta_{CM}^{\dagger} \) is the cutoff point of \( \beta_{CM} \) above which \( \beta_{CM}^{\dagger} \leq \beta_{CM} \) and the OEM will keep the procurement in-house. Therefore,

\[
\beta_{CM}^{\dagger}c(\alpha_{S}(\beta_{CM}^{\dagger})) + R(\alpha_{S}(\beta_{CM}^{\dagger})) + r(\alpha_{S}(\beta_{CM}^{\dagger})) = \beta_{OEM}c(\alpha_{I}(\beta_{OEM})) + R(\alpha_{I}(\beta_{OEM})) + r(\alpha_{I}(\beta_{OEM})).
\]

Since \( \beta_{CM}^{\dagger}c(\alpha_{S}(\beta_{CM}^{\dagger})) + R(\alpha_{S}(\beta_{CM}^{\dagger})) + r(\alpha_{S}(\beta_{CM}^{\dagger})) = \min_{\alpha} \left\{ \beta_{CM}^{\dagger}c(\alpha) + R(\alpha) + r(\alpha) \right\} \), \( \beta_{CM}^{\dagger} \geq \beta_{OEM} \).

Thus, the OEM may outsource the procurement function to the CM even when she has higher bargaining power than the CM. Then, one natural question is whether procurement outsourcing may result in lower quality material and hence, a lower quality final product, as well as a higher actual overall cost due to information asymmetry. Fortunately, our analysis reveals that such concerns are unwarranted.

**Proposition 2.4.** Suppose the CM accepts a contract offered by the OEM, i.e., \( \beta_{CM} < \beta_{CM}^{0} \), and procurement is outsourced. Then, compared to the in-house option, procurement outsourcing results in

1. higher quality material, i.e., \( \alpha_{S}(\beta_{CM}) < \alpha_{I}(\beta_{OEM}) \), and
2. lower costs for both parties, i.e., \( \pi_{S,OEM}(\beta_{CM}) < \pi_{I,OEM}(\beta_{OEM}) \) and \( \pi_{S,CM}(\beta_{CM}) < \pi_{I,CM}(\beta_{OEM}) \).

**Proof of Proposition 2.4:** Consider the quality level at the unconstrained solution to the
first order condition, $\alpha_S(\beta_{CM}^\dagger)$. Since
\[
\min_{\alpha} \left\{ (2\beta_{CM}^\dagger - \beta_{CM}) c'(\alpha) + R(\alpha) + r(\alpha) \right\} = \beta_{OEM} c(\alpha_{I}(\beta_{OEM})) + R(\alpha_{I}(\beta_{OEM})) + r(\alpha_{I}(\beta_{OEM})),
\]
we have $2\beta_{CM}^\dagger - \beta_{CM} \geq \beta_{OEM}$. Furthermore, since
\[
\beta_{OEM} c(\alpha_{I}(\beta_{OEM})) + R(\alpha_{I}(\beta_{OEM})) + r(\alpha_{I}(\beta_{OEM})) = \min_{\alpha} \{ \beta_{OEM} c(\alpha) + R(\alpha) + r(\alpha) \},
\]
we obtain
\[
(2\beta_{CM}^\dagger - \beta_{CM}) c'(\alpha_{S}(\beta_{CM})) + R(\alpha_{S}(\beta_{CM})) + r(\alpha_{S}(\beta_{CM})) \\
\geq \beta_{OEM} c(\alpha_{I}(\beta_{OEM})) + R(\alpha_{I}(\beta_{OEM})) + r(\alpha_{I}(\beta_{OEM})).
\]
Therefore, $r_{2}(\alpha_{S}(\beta_{CM})) \leq r_{2}(\alpha_{I}(\beta_{OEM}))$, which implies $\alpha_{S}(\beta_{CM}) \leq \alpha_{I}(\beta_{OEM})$.

Also,
\[
\frac{\partial \pi_{OEM}^S}{\partial \beta_{CM}} = (\beta_{CM} - \beta_{CM}) c'(\alpha_{S}) \alpha_{S}'(\beta_{CM}) > 0,
\]
which leads to $\pi_{OEM}^I(\beta_{OEM}) - \pi_{OEM}^S(\beta_{CM}) \geq (\beta_{CM}^0 - \beta_{CM}) c(\alpha_{S}(\beta_{CM}^0)) > 0$. That is, the OEM’s cost is lower if the CM takes the contract. For the CM, he accepts the contract under the condition that $\pi_{CM}^S(\beta_{CM}) \leq \pi_{CM}^I(\beta_{OEM})$.

\[\square\]

This is indeed interesting and good news. Bargaining power over the suppliers alone is not sufficient for making outsourcing decisions. The OEM may outsource her procurement function to the CM even though his bargaining power is lower than her own’s, while the resulting material quality is always higher and both parties are better off. The reason is the balancing between delegation and control. Under the in-house option, the OEM selects the supplier by minimizing her own cost. Under the procurement outsourcing option, the OEM delegates the supplier selection decision to the CM, but she holds the control of contracting which forces the CM to pay a higher quality cost (i.e., one half
of the total quality cost). Thus, even with relatively low bargaining power, the CM is motivated to make a decision that takes into account the total quality cost and hence, selects higher quality material. As the CM is compensated with a well designed wholesale price, taking over the procurement function is still an attractive option to him. The OEM benefits from procurement outsourcing with higher quality material and a lower cost. Hence, procurement outsourcing, if successful, is indeed a win-win situation.

2.3.4 Numerical Example

In this section, we illustrate the main insight of procurement outsourcing decision using a simple example.

1. Material cost

We consider linear list material cost \( c(\alpha) = 1 - \alpha \). Let \( \beta_{OEM} = 0.8 \) and \( \beta_{CM} = 0.8005 \). Suppose the OEM only has an estimate of \( \beta_{CM} \), denoted by \( \bar{\beta}_{CM} \) which is uniformly distributed over \([\beta_{CM}, \bar{\beta}_{CM}]\). In this example, we let \([\beta_{CM}, \bar{\beta}_{CM}] = [0.7995, 0.8015] \) with a mean 0.8005 as the true value.

2. Quality cost

The OEM as the brand owner suffers from the damage to her brand name and future market share for each nonconforming unit sold and we model the related cost as \( R(\alpha) = 2\alpha^2 \).

There are also repair/replacement costs associated with each nonconforming final product, denoted by \( r(\alpha) \). Since the material quality and production system will jointly determine the quality of the final product, both the party who procures the materials and the manufacturer will bear part of \( r(\alpha) \). Suppose \( r_1(\alpha) = r_2(\alpha) = \frac{1}{2}\alpha^2 \), where \( r_1(\alpha) \) and \( r_2(\alpha) \) are the repair cost born by the party who sources the material and the CM, respectively.

In-house option

\[
\pi^I_{OEM}(\beta_{OEM}) = \min_{\alpha} \{ \beta_{OEM}c(\alpha) + R(\alpha) + r_1(\alpha) \} = \min_{\alpha} \left\{ 0.8(1 - \alpha) + 2\alpha^2 + \frac{1}{2}\alpha^2 \right\}
\]

Therefore the OEM will choose a material with \( \alpha_I = 0.16 \).
\[ \pi_{OEM}(\beta_{OEM}) = 0.736, \pi^I_{CM}(\beta_{OEM}) = 0.0128. \] Supply chain cost is 0.7488.

**Outsourcing option**

\[ \beta^I_{CM} \] is determined by

\[
\min_{\alpha} \left\{ \beta^I_{CM} c(\alpha) + R(\alpha) + r(\alpha) \right\} = \beta_{OEM} c(\alpha) + R(\alpha) + r(\alpha).\]

By solving the equation we obtain \( \beta^I_{CM} = 6 - 0.5 \times \sqrt{108.0576} = 0.8025 \) (which is strictly greater than \( \beta_{OEM} = 0.8 \)). Since \( \beta_{CM} = 0.7995 < \beta^I_{CM} \), the OEM will offer a menu of contracts to the CM.

\[
\beta^0_{CM} = \frac{\beta_{CM} + \beta^I_{CM}}{2} = 0.8010.
\]

\[
k_f = -\pi_{CM}(\beta_{OEM}) + (\beta^0_{CM} - \frac{\beta_{CM}}{2}) c(\alpha_s(\beta^0_{CM})) + \frac{R(\alpha_s(\beta^0_{CM})) + r(\alpha_s(\beta^0_{CM}))}{2}
\]

\[ = 0.3616.\]

The OEM will offer a menu of contracts as below:

\[
w_S = \frac{\beta_{CM}}{2} c(\alpha_s) - \frac{R(\alpha_s)}{2} + \frac{r(\alpha_s)}{2} + k_f
\]

\[ = 0.3995(1 - \alpha_s) - \frac{1}{2} \alpha_s^2 + 0.3616,\]

The menu of contracts is consisted of three parts: (1) \( 0.3995(1 - \alpha_s) \) as a proportion of the material cost which increases in the quality level, (2) a penalty \( \frac{1}{2} \alpha_s^2 \) which pushes the CM to share one half of the total quality cost and decreases in the quality level, and (3) a fixed payment that guarantees certain level of CM’s profitability.

CM with \( \beta_{CM} \in [0.7995, 0.8010] \) will accept a contract and will reject the offer otherwise.

The quality level in contract he accepts will be \( \alpha_s = \frac{2 \beta_{CM} - \beta_{CM}}{6}. \) Therefore, when \( \beta_{CM} \in [0.7995, 0.8010], \alpha_s \in [0.1333, 0.1337] < 0.16. \)

If the CM accepts a contract, total supply chain cost is

\[
\pi^S_{OEM}(\beta_{CM}) + \pi^S_{CM}(\beta_{CM}) = \beta_{CM} c(\alpha_s) + R(\alpha_s) + r(\alpha_s) \in [0.7462, 0.7476] < 0.7488
\]
The costs of the two parties are as below:

\[ \pi_{OEM}^S(\beta_{CM}) = w_S + R(\alpha) \in [0.7347, 0.7348] < 0.736. \]

\[ \pi_{CM}^S(\beta_{CM}) \in [0.0115, 0.0128] \leq 0.0128. \]

As we can see, both parties will have a lower realized cost and the quality level is improved under the in-house option.

2.4. Extensions and Discussions

In this section, we relax some of the assumptions in our model. Each extension builds on our basic model in Section 2.3 and is independent unless stated otherwise.

2.4.1 Preventive Mechanism against Contract Violation

So far we have implicitly assumed that the CM will accept the contract \( \{\alpha_S(\beta_{CM}), w_S(\beta_{CM})\} \) from a menu of contracts offered by the OEM given his true bargaining power \( \beta_{CM} \), or simply reject the contracts. If he accepts the contract, he will indeed select supplier \( \alpha_S(\beta_{CM}) \). In reality, it is difficult for the OEM to monitor the CM’s operations and, since the CM is compensated with a wholesale price associated with the quality of the material, the CM may have an incentive to use a different material from what is stated in the contract, referred to as contract violation, absent of an effective mechanism that validates the material actually used and punishes the CM in the event of contract violation. Furthermore, the CM may not necessarily accept the contract \( \{\alpha_S(\beta_{CM}), w_S(\beta_{CM})\} \)!

Suppose that the CM will accept a contract \( \{\alpha_S(\beta_{CM}^R), w_S(\beta_{CM}^R)\} \) and actually select supplier \( \tilde{\alpha}_S(\beta_{CM}) \). That is, the CM will select \( \beta_{CM}^R \) and \( \tilde{\alpha}_S(\beta_{CM}) \) to minimize his actual material and quality cost less his revenue. That is,

\[ \tilde{\pi}_{CM}^S(\beta_{CM}) = \beta_{CM}c(\tilde{\alpha}_S(\beta_{CM})) + r_2(\tilde{\alpha}_S(\beta_{CM})) - w_S(\beta_{CM}^R). \]

If \( R'(\alpha) < r'(\alpha) \) for some \( \alpha \), i.e., the marginal quality cost at the OEM may be lower than that of the CM if procurement is outsourced, then it is possible that \( \beta_{CM}^R > \beta_{CM} \) and \( \tilde{\alpha}_S(\beta_{CM}) < \alpha_S(\beta_{CM}^R) \). That is, in order to cover his high quality cost, the CM may prefer
to pretend to hold lower bargaining power over the suppliers in order to obtain a higher compensation (i.e., a wholesale price) from the OEM, even though he has to spend extra money to purchase better material than that on the contract later (which the OEM does not mind though).

In the rest of the section, we will focus on the case where \( R'(\alpha) \geq r'(\alpha) \), which holds in most real situations. In this case, the OEM desires high quality due to her high quality cost and is willing to pay more to motivate the CM to accept a contract if the CM has high bargaining power over the suppliers. Thus, with no penalty in the event of contract violation, the CM will for sure exaggerate his bargaining power as much as possible to obtain high wholesale prices from the OEM and then use a much worse material than that in the contract.

**Lemma 2.1.** For a given menu of contracts \( \{\alpha_S(\beta), w_S(\beta)\} \), it is for the best interest of the CM to accept the contract \( \{\alpha_S(\beta_{CM}), w_S(\beta_{CM})\} \) and use an inferior material \( \tilde{\alpha}_S(\beta_{CM}) > \alpha_S(\beta_{CM}) \). As a matter of fact, \( \tilde{\alpha}_S(\beta_{CM}) > \alpha_S(\beta_{CM}) \).

**Proof of Lemma 2.1:** If there is no inspection or penalty, the CM will select \( \{\tilde{\alpha}_S, \beta_{CM}^R\} \) to minimize his cost \( \beta_{CM}c(\tilde{\alpha}_S) + r(\tilde{\alpha}_S) - w_S(\beta_{CM}^R) \). Therefore, \( \beta_{CM} + r'(\tilde{\alpha}_S) = 0 \) and \( \beta_{CM}^R = \beta_{CM}^L \). Since \( (\beta_{CM} - \beta_{CM}^L)c'(\alpha_S(\beta_{CM})) + R'(\alpha_S(\beta_{CM})) > \beta c'(\alpha_S(\beta_{CM})) + r'(\alpha_S(\beta_{CM})) \), we have \( \beta_{CM}^Lc'(\alpha_S(\beta_{CM})) + r'(\alpha_S(\beta_{CM})) < 0 \). Therefore, \( \tilde{\alpha}_S(\beta_{CM}) > \alpha_S(\beta_{CM}) \geq \alpha_S(\beta_{CM}^R) \).

To prevent the CM from contract violation, the OEM may adopt some quality management procedures, e.g., inspect the source of material purchased by the CM before production begins. The OEM can trace the supplier from whom the materials are purchased, and thus can justify if the CM commits to the contract or not. We assume that, if the OEM inspects, she is able to find out exactly what material the CM uses. Once the CM is caught, he has to pay a fixed penalty \( M \) and use the material stated in the contract, \( \alpha_S(\beta_{CM}^R) \). However, performing inspection incurs a cost \( I \).

We first analyze the OEM’s inspection schedule for a given menu of contracts \( \{\alpha_S(\beta), w_S(\beta)\} \). Since inspection incurs extra costs, the OEM may need to redesign her
contracts which we will analyze later. We know from Lemma 2.1 that, if the CM knows that the OEM does not perform inspection, it is for the CM’s best interest to violate the contract by using an inferior material. It is easy to show that, if the CM knows that the OEM will inspect the material he uses for sure, the CM will comply with the contract. However, inspection is costly. Therefore, the OEM needs to adopt a mixed inspection strategy. Let $\gamma$ be the probability that the OEM will inspect the material purchased by the CM.

Suppose that the CM accepts a contract $\{\alpha_S(\beta_{CM}), w_S(\beta_{CM})\}$ and procurement is outsourced. If the OEM does not perform inspection (with probability $1 - \gamma$), the CM will use $\tilde{\alpha}_S(\beta_{CM})$ without being caught and incur a cost $r(\tilde{\alpha}_S(\beta_{CM}))+\beta_{CM}c(\tilde{\alpha}_S(\beta_{CM})) - w_S(\beta_{CM})$. Otherwise, the CM will comply with the contract using $\alpha_S(\beta_{CM})$ and incur a cost including a penalty $r(\alpha_S(\beta_{CM}))+\beta_{CM}c(\alpha_S(\beta_{CM}))+M - w_S(\beta_{CM})$. Thus, for a given OEM’s inspection schedule, the CM’s expected cost can be expressed as

$$\hat{\pi}_{CM}(\beta_{CM}) = (1 - \gamma)[r(\tilde{\alpha}_S(\beta_{CM}))+\beta_{CM}c(\tilde{\alpha}_S(\beta_{CM}))] + \gamma[r(\alpha_S(\beta_{CM}))+\beta_{CM}c(\alpha_S(\beta_{CM}))+M] - w_S(\beta_{CM}).$$

**Lemma 2.2.** There exists a threshold of $\gamma$, $\gamma(\beta_{CM})$, above which the CM will comply with the contract and accept the contract $\{\alpha_S(\beta_{CM}), w_S(\beta_{CM})\}$. Otherwise, the CM will accept a contract $\{\alpha_S(\beta_{CM}), w_S(\beta_{CM})\}$ with

1. $\beta_{CM}^R = \beta_{CM}$ if $\gamma \leq \frac{\beta_{CM}}{2\beta_{CM}}$ and

2. $\beta_{CM}^R$ being the unique solution to

$$\left(\gamma\beta_{CM} - \frac{\beta_{CM}}{2}\right)c'(\alpha_S(\beta_{CM}^R)) + \frac{R(\alpha_S(\beta_{CM}^R))}{2} - \left(\frac{1}{2} - \gamma\right)r'(\alpha_S(\beta_{CM}^R)) = 0$$

if $\frac{\beta_{CM}}{2\beta_{CM}} < \gamma \leq \gamma(\beta_{CM})$, while using a different material $\tilde{\alpha}_S(\beta_{CM})$ as described in Lemma 2.1.

**Proof of Lemma 2.2:** The CM will select $\beta_{CM}^R$ to minimize his expected cost

$$\hat{\pi}_{CM}(\beta_{CM}) = \left(\gamma\beta_{CM} - \frac{\beta_{CM}}{2}\right)c(\alpha_S(\beta_{CM})) + \frac{R(\alpha_S(\beta_{CM}))}{2} - \left(\frac{1}{2} - \gamma\right)r(\alpha(\beta_{CM})).$$
Since $R'(\alpha) > r'(\alpha)$, for any $\alpha$, $\frac{R(\alpha(\beta_{CM}^R))}{2} - \left(\frac{1}{2} - \gamma\right)r(\alpha(\beta_{CM}^R))$ is still increasingly convex in $\alpha$. It is obvious that the CM’s expected cost is increasing in $\gamma$. If $\gamma = 1$, he will comply with the contract. Therefore, there exists a threshold $\gamma(\beta_{CM})$ such that he will comply with the contract if $\gamma \geq \gamma(\beta_{CM})$.

If $\gamma \leq \frac{\beta_{CM}}{2\beta_{CM}}$, the objective function is increasing in $\beta_{CM}^R$ and $\beta_{CM}^R = \beta_{CM}^M$. If $\frac{\beta_{CM}}{2\beta_{CM}} < \gamma < \gamma(\beta)$, $\beta_{CM}^R$ is the solution to the first order condition

$$(\gamma \beta_{CM} - \frac{\beta_{CM}}{2})c'(\alpha_S(\beta_{CM}^R)) + \frac{R'(\alpha(\beta_{CM}^R))}{2} - \left(\frac{1}{2} - \gamma\right)r'(\alpha(\beta_{CM}^R)) = 0.$$  

Lemma 2.2 suggests that performing inspection may induce the CM not only to comply with the contract, but also to accept the right contract $\{\alpha_S(\beta_{CM}), w_S(\beta_{CM})\}$. Since the OEM only knows the distribution of $\hat{\beta}_{CM}$, the inspection schedule that guarantees truth revelation and contract compliance is $\gamma^* = \max_{\hat{\beta}_{CM}} \{\gamma(\hat{\beta}_{CM})\}$.

For a given inspection schedule, the OEM may need to redesign the contracts that incorporate her cost change. We show in the next proposition that, if the OEM adopts the inspection schedule $\gamma^*$ to completely eliminate possible contract violation, the contracts remain the same except with a lower fixed fee to compensate for her inspection cost, but only partially. As a result, both parties are worse off, and procurement outsourcing is less likely to happen.

**Proposition 2.5.** If the OEM performs inspection with probability $\gamma^*$, then the contracts $\{\alpha_S(\beta), w_S(\beta)\}$ remain the same as in Theorem 2.1 except with a lower $k_f$. This also implies that procurement outsourcing is less likely with possible contract violation.

**Proof of Proposition 2.5:** If the OEM inspects with $\gamma \geq \gamma^*$, the CM will not have any incentive to violate the contract and will select $\{\alpha_S(\beta_{CM}), w_S(\beta_{CM})\}$. The unconstrained first order condition (2.12) becomes

$$\frac{d\hat{\pi}_{OEM}}{d\beta_{CM}^0} + I \frac{d\gamma^*}{d\beta_{CM}^0} = 0.$$  

(2.13)
As $\beta_{CM}^0$ increases, $k_f$ increases and the CM is more likely to violate the contract. Therefore, $\gamma(\beta)$ increases in $\beta_{CM}^0$ and thus $\frac{d\gamma^*}{d\beta_{CM}^0} > 0$. Since $\frac{d\pi_{OEM}}{d\beta_{CM}^1} = 0$, for any $\beta \geq \beta_{CM}^1$, we have $\frac{d\hat{\pi}_{OEM}}{d\beta} + I \frac{d\gamma^*}{d\beta} > 0$. Therefore, the solution to (2.13) is less than $\beta_{CM}^1$, and the resulting $\beta_{OEM}^0$ will decrease. That is, the OEM is less likely to outsource the procurement function and both parties share the inspection cost.

$\square$

### 2.4.2 Manufacturing Process Uncertainty

So far, we have assumed that the quality costs, $R(\alpha)$ and $r(\alpha)$, are functions of the material used. In reality, both the material and the CM’s production system contribute to the quality of the final product and hence the quality costs. Let $\mu$ be the parameter representing the quality level of the CM’s production system which is public information. Suppose that defective material always results in defective final products, while an end product is defective with probability $\mu$ if the material is conforming. Then, the smaller $\mu$, the higher the quality level of the production system. Following the quality literature (see for instance Rosenblatt and Lee 1986 and Lee and Rosenblatt 1987), we can model the defective rate of the final product as $\alpha + (1 - \alpha)\mu$. For details, see the Appendix. Then, the reputation cost can be written as $R(\alpha + (1 - \alpha)\mu)$ and we restrict the repair cost to be linear as $r \times [\alpha + (1 - \alpha)\mu]$ and fully born by the CM for tractability.

Let $\mu_0$ be the quality level of the CM’s production system. The CM has the option to exert efforts to improve the quality of his production process from $\mu_0$ to $\mu$, $\mu \leq \mu_0$, at the cost $\nu\ln\left(\frac{\mu}{\mu_0}\right)$ (see Porteus 1986 for a justification of such a function). That is, the CM is given the option to improve the quality of his production system under both options. The next theorem states that the structure of the optimal menu of contracts offered by the OEM is similar to that in Section 2.3.

**Theorem 2.3.** The cutoff point $\beta_{CM}^0$ is either one of the two boundaries, $\beta_{CM}^\circ$ and $\beta_{CM}^\dagger$, or the unconstrained solution to the first order condition $\pi_{OEM}^I(\beta_{OEM}) - \pi_{OEM}^S(\beta) = (\beta - \beta_{CM}^\circ) c(\alpha_S(\beta))$. Furthermore, $\alpha_S(\beta)$ is the unique solution to

$$
\left(2\beta - \beta_{CM}^\dagger\right) c'(\alpha_S) + R' \left(\alpha_S + \frac{v}{r}\right) + r - \frac{v}{1 - \alpha_S} = 0
$$

(2.14)
and \( w_S(\beta) \) satisfies

\[
 w_S(\beta) = k_v c(\alpha_S(\beta)) + \frac{1}{2} \left[-R \left( \alpha_S + \frac{v}{r} \right) + r\alpha_S + vln(1 - \alpha_S) \right] + k_f, \tag{2.15}
\]

where \( k_v = \frac{\beta_CM}{2} \) and \( k_f \) satisfies \( \pi^S_{CM}(\beta^0_{CM}) = \pi^I_{CM}(\beta_{OEM}) \).

**Proof of Theorem 2.3:** Given any \( \alpha \), the optimal quality level of CM's production process is \( \mu = \frac{v}{r(1 - \alpha)} \). The rest of the proof is similar to that of Theorem 2.1 and thus omitted. \( \square \)

We now compare the quality levels and costs of the two parties under the in-house and the procurement outsourcing option. Let \( \mu_I(\mu_S) \) denote the quality improvement decision under the in-house option (procurement outsourcing option).

**Proposition 2.6.** If the CM accepts a contract and procurement is outsourced, compared to the in-house option, procurement outsourcing results in

1. both higher quality material and production system, i.e., \( \alpha_S(\beta) < \alpha_I(\beta_{OEM}) \) and \( \mu_S < \mu_I \), and

2. lower costs at both parties, i.e., \( \pi^S_{OEM}(\beta_{CM}) < \pi^I_{OEM}(\beta_{OEM}) \) and \( \pi^S_{CM}(\beta_{CM}) < \pi^I_{CM}(\beta_{OEM}) \).

**Proof of Proposition 2.6:** The unconstrained first order condition is given by

\[
 \beta_{OEM} c(\alpha_I) + R \left( \alpha_I + \frac{v}{r} \right) + r\alpha_I + vlnr + vln(1 - \alpha_I) \\
= \left( 2\beta - \frac{\beta_{CM}}{2} \right) c(\alpha_S) + R \left( \alpha_S + \frac{v}{r} \right) + r\alpha_S + vlnr + vln(1 - \alpha_S) \\
= \min_{\alpha} \left\{ \left( 2\beta - \frac{\beta_{CM}}{2} \right) c(\alpha) + R \left( \alpha + \frac{v}{r} \right) + r\alpha + vlnr + vln(1 - \alpha) \right\}.
\]

It is easy to show that \( 2\beta - \frac{\beta_{CM}}{2} > \beta_{OEM} \). Therefore, \( r\alpha_I + vln(1 - \alpha_I) > r\alpha_S + vln(1 - \alpha_S) \). Since \( \alpha + \frac{v}{r} < 1 \), \( r\alpha + vln(1 - \alpha) \) is increasing in \( \alpha \) and \( \alpha_S < \alpha_I \). Furthermore, \( \mu_I = \frac{v}{r(1 - \alpha_I)} > \frac{v}{r(1 - \alpha_S)} = \mu_S \).

\( \square \)
The above proposition reinforces the insights obtained in Section 2.3.3. Under the in-house option, supplier selection and process improvement decisions, both contributing to the quality of the final product, are made by different parties to minimize their own costs. When procurement is outsourced, the CM is the sole maker of both decisions and is responsible for half of the total quality costs. Therefore, he will select a better supplier and exert more effort to improve his production system. As a result, outsourcing, if successful, benefits both parties.

2.4.3 Demand Uncertainty

Suppose that demand for the final product $D$ is a random variable with a cumulative distribution function $G(D)$. Without loss of generality, we normalize the expected demand to be 1. With demand uncertainty, the OEM needs to decide the quantity that the CM should produce as well as the menu of contracts, if she intends to outsource the procurement function, at the beginning of the selling season. Unfulfilled demand is lost and excessive inventory of the final product has no salvage value.

If procurement is kept in-house, the OEM will select the quality of the material $\alpha_I$ and the order quantity $Q_I$ to maximize her expected profit. That is,

$$\Pi^I_{OEM}(\beta_{OEM}) = [p - R(\alpha_I) - r_1(\alpha_I)][Q_I - E(Q_I - D)^+] - \beta_{OEM}c(\alpha_I)Q_I,$$

where $p$ is the unit selling price of the final product and $Q_I - E(Q_I - D)^+$ is the expected quantity of the final product sold. We need to point out that we still normalize the wholesale price under the in-house option to be zero as it does not affect the structure of the contracts and the results in this section. The CM obtains a profit of

$$\Pi^I_{CM}(\beta_{OEM}) = -r_2(\alpha_I)[Q_I - E(Q_I - D)^+]].$$

If procurement is outsourced, the OEM will offer a menu of contracts $\{\alpha_S(\beta), w_S(\beta),$
Q_s(\beta)} to maximize her expected profit \( \hat{\Pi}^{S}_{OEM} = E[\Pi^{S}_{OEM}(\hat{\beta}_{CM})] \) where

\[
\Pi^{S}_{OEM}(\hat{\beta}_{CM}) = [p - R(\alpha_s(\hat{\beta}_{CM}))][Q_s(\hat{\beta}_{CM}) - E(Q_s(\hat{\beta}_{CM}) - D)^+] - w_s(\hat{\beta}_{CM})Q_s(\hat{\beta}_{CM}).
\]

If the CM accepts the contract \( \{\alpha_s(\beta_{CM}), w_s(\beta_{CM})\} \), his profit is given by

\[
\Pi^{S}_{CM}(\beta_{CM}) = [w_s(\beta_{CM}) - \beta_{CM}c(\alpha_s(\beta_{CM}))]Q_s(\beta_{CM}) - r(\alpha_s(\beta_{CM}))[Q_s(\beta_{CM}) - E(Q_s(\beta_{CM}) - D)^+].
\]

The next theorem presents the optimal menu of contracts offered by the OEM that satisfies the IC and PC constraints.

**Theorem 2.4.** In the optimal menu of contracts, \( \{\alpha_s(\beta), Q_s(\beta)\} \) is the unique solution to the following two equations

\[
\begin{align*}
2(\beta - k_v)c'(\alpha_s) + \left\{ 1 - \frac{E[Q_s(\beta) - D]^+}{Q_s(\beta)} \right\} [R'(\alpha_s) + r'(\alpha_s)] &= 0, \\
1 - G(Q_s(\beta)) &= \frac{2(\beta - k_v)c(\alpha_s(\beta))}{p - R(\alpha_s(\beta)) - r(\alpha_s(\beta))},
\end{align*}
\]

and

\[
q_s(\beta) = k_vc(\alpha_s(\beta)) - \frac{1}{2} \left\{ 1 - \frac{E[Q_s(\beta) - D]^+}{Q_s(\beta)} \right\} [R(\alpha_s(\beta)) - r(\alpha_s(\beta))] - \\
\frac{p}{Q_s(\beta)} \int_{Q_s(\beta)}^{Q_s(\beta)} G(Q) dQ + \frac{k_f}{Q_s(\beta)},
\]

where \( k_v = \frac{\beta_{CM}}{2} \), \( k_f \) is determined by \( \Pi^{S}_{CM}(\beta_{CM}) = \Pi^{I}_{CM}(\beta_{OEM}) \), and \( \beta_{CM} \) is either a solution to \( \Pi^{I}_{OEM}(\beta_{OEM}) - \Pi^{S}_{OEM}(\beta_{CM}) = -c(\alpha_s(\beta_{CM}))(\beta_{CM} - \beta_{CM})Q_s(\beta_{CM}), \) or one of the two boundaries, \( \beta_{CM} \) and \( \overline{\beta}_{CM}. \)

**Proof of Theorem 2.4:** Given the cutoff point and \( Q_s(\beta) \), and letting \( u = \alpha_s(\beta) \), we
construct the Hamiltonian

$$H = [p - w_S - R(\alpha_S)] - w_S E(Q_S - D)^+ - [p - w_S - R(\alpha_S)] E(D - Q_S)^+ + \lambda_w \left[ \beta c'(\alpha_S) u + r'(\alpha_S) u - \frac{(w - \beta c(\alpha_S) - r(\alpha_S)) Q_S}{Q_S} \right] - \frac{r'(\alpha_S) u E(Q_S - D)^+}{Q_S} - \frac{r(\alpha_S) F(Q_S) Q'_S(\beta)}{Q_S} + \lambda_u.$$ 

Taking the derivative, we have

$$\lambda(\beta) = -\frac{\partial H}{\partial \alpha_S} = R'(\alpha_S)[1 - E(D - Q_S)^+] - \lambda_w \left[ \beta c''(\alpha_S) u + r''(\alpha_S) u + \frac{Q'_S(\beta)}{Q} (\beta c'(\alpha_S) + r'(\alpha_S)) \right] - \frac{r''(\alpha_S) u E(Q_S - D)^+}{Q} - \frac{r'(\alpha_S) F(Q_S) Q'_S(\beta)}{Q}.$$ 

Since $\lambda(\beta) = -\frac{\partial H}{\partial w_S} = Q_S + \lambda_w \frac{Q'_S(\beta)}{Q}$ and $\lambda_w (\beta_{CM}) = 0$, we have $\lambda_w = Q_S (\beta - \beta_{CM})$. Therefore, $\frac{\partial H}{\partial u} = 0$ leads to

$$(2\beta - \beta_{CM}) c'(\alpha_S) + \left[ 1 - \frac{E(Q_S - D)^+}{Q} \right] [R'(\alpha_S) + r'(\alpha_S)] = 0.$$ 

Similarly, for a given $\alpha_S(\beta)$, we can construct a Hamiltonian and obtain

$$1 - G(Q_S) = \frac{(2\beta - \beta_{CM}) c(\alpha_S)}{p - R(\alpha_S) - r(\alpha_S)}.$$ 

Therefore, $\{\alpha_S(\beta), Q_S(\beta)\}$ is the unique solution to the following optimization problem

$$\max_{\alpha_S, Q_S} \{[p - R(\alpha_S) - r(\alpha_S)] [Q_S - E(Q_S - D)^+] - (2\beta - \beta_{CM}) c(\alpha_S) Q_S \}. \quad (2.16)$$ 

Letting $w_S(\beta) = \frac{\beta_{CM}}{2} c(\alpha_S) - \frac{1}{2} \left(1 - \frac{E(Q_S - D)^+}{Q_S}\right) [R(\alpha_S) - r(\alpha_S)] + \phi(\beta)$, we
obtain

\[ w'_S(\beta) = \beta c'(\alpha S)u + r'(\alpha S)u \left( 1 - \frac{E(Q_S - D)^+}{Q_S} \right) - \frac{r(\alpha S)G(Q_S)Q'_S(\beta)}{Q_S} - \left[ \frac{c(\alpha S)}{2} - \frac{1}{2} \left( 1 - \frac{E(Q_S - D)^+}{Q_S} \right) (R(\alpha S) - r(\alpha S)) \right] \frac{Q'_S(\beta)}{Q_S} + \left[ \phi(\beta) - \beta c(\alpha S) - r(\alpha S) \right] \frac{Q'_S(\beta)}{Q_S}. \]

Substituting the above equation to the IC constraint, we have \( \phi'(\beta) + \phi \frac{Q'_S(\beta)}{Q_S} = h(\beta) \), where

\[ h(\beta) = \frac{1}{2} \left[ (2\beta - \beta_{CM})c(\alpha S) + (1 - G(Q_S))(R(\alpha S) + r(\alpha S)) \right] \frac{Q'_S(\beta)}{Q_S} = \frac{p (1 - G(Q_S))}{Q_S} Q'_S(\beta). \]

Solving the above differential equation, we have

\[ \phi(\beta) = \frac{1}{Q_S} \left( \int^\beta h(t)Q_S(t)Dt + k'_f \right) = -\frac{p}{Q_S} \left( \int^Q G(t)Dt + k_f \right) \]

and

\[ w_S(\beta) = k_c c(\alpha S) - \frac{1}{2} \left[ 1 - \frac{E(Q_S - D)^+}{Q_S} \right] [R(\alpha S) - r(\alpha S)] - \frac{p}{Q_S} \int^Q G(Q)dQ + \frac{k_f}{Q_S} \]

where \( k_f \) is the solution to \( \Pi^S_{CM}(\beta_{CM}) = \Pi^F_{CM}(\beta_{OEM}) \). The cutoff point is either the unconstrained solution to the first order condition, or at one of the boundaries.

\[ \square \]

Again, if the CM accepts a contract and procurement is outsourced, both parties will obtain a higher profit than that under the in-house option. Moreover, as long as demand variability is not too high, we have the following result.

**Proposition 2.7.** As \( \text{Var}(D) \) increases, \( \beta_{CM}^0 \) decreases and procurement outsourcing is less likely to happen.

**Proof of Proposition 2.7:** The first order condition is given by
$$\max_{\alpha, Q} \left\{ [p - R(\alpha) - r(\alpha)][Q - E(Q - D)^+] - (2\beta^*_CM - \beta_CM)c(\alpha)Q \right\}$$

$$= [p - R(\alpha_I) - r(\alpha_I)][Q_I - E(Q_I - D)^+] - \beta_OEMc(\alpha_I)Q_I.$$  

Fixing $\beta^*_CM$, we consider the derivative of the left and right side of the above equation with respect to $Var(D)$

$$\frac{d}{d Var(D)} \max_{\alpha, Q} \left\{ [p - R(\alpha) - r(\alpha)][Q - E(Q - D)^+] - (2\beta^*_CM - \beta_CM)c(\alpha)Q \right\}$$

$$= [p - R(\alpha_S) - r(\alpha_S)] \frac{\partial}{\partial Var(D)} [Q_S - E(Q_S - D)^+]$$

and

$$\frac{d}{d Var(D)} \left\{ [p - R(\alpha_I) - r(\alpha_I)][Q_I - E(Q_I - D)^+] - \beta_OEMc(\alpha_I)Q_I \right\}$$

$$= [p - R(\alpha_I) - r(\alpha_I)] \frac{\partial}{\partial Var(D)} [Q_I - E(Q_I - D)^+] -$$

$$r_2(\alpha_I) \frac{\partial}{\partial Q_I} [Q_I - E(Q_I - D)^+] \frac{dQ_I}{d Var(D)} - r'_2(\alpha_I) \frac{d\alpha_I}{d Var(D)} [Q_I - E(Q_I - D)^+]$$.

When $Var(D)$ approaches to zero, we have $Q_S = Q_I$, $\alpha_S \geq \alpha_I$, $\frac{d\alpha_I}{d Var(D)} = 0$, $\frac{dQ_I}{d Var(D)} = 0$, and hence,

$$-r(\alpha_S) \frac{\partial}{\partial Var(D)} [Q_S - E(Q_S - D)^+]$$

$$\leq -r(\alpha_I) \frac{\partial}{\partial Var(D)} [Q_I - E(Q_I - D)^+] - r'_2(\alpha_I) \frac{d\alpha_I}{d Var(D)} [Q_I - E(Q_I - D)^+]$$

$$-r_2(\alpha_I) \frac{\partial}{\partial Q_I} [Q_I - E(Q_I - D)^+] \frac{dQ_I}{d Var(D)}.$$

Therefore, there exists a threshold of $Var(D)$, below which the left hand side of the first order condition is less than or equal to the right hand side as $Var(D)$ increases. Thus, $\beta^*_CM$ is decreasing, which implies that, as $Var(D)$ increases, procurement outsourcing is less likely to happen.

$$\square$$

This is because the production quantity as well as the unsold inventory at the end of the
selling season increases as demand uncertainty increases up to a point. Since any quality related effort has no impact on those unsold units, both parties have less incentive to invest in material and quality improvement. Therefore, the OEM will offer lower wholesale prices. As a result, the CM will select relatively low quality suppliers and procurement outsourcing is less attractive. When demand uncertainty reaches a certain level, the OEM may reduce her production quantity to offset the risk of unsold inventory and procurement outsourcing may still be a preferred option for both parties.

2.4.4 Ceding Contracting Control to the CM

In previous discussions, the OEM is the contract initiator, while the CM can either accept a contract or reject the offer. Such a game sequence with the OEM being the leader and the CM follower reflects the channel power of the two parties in many practical situations.

In this section, we reverse the sequence and consider the situation where the CM is the leader and initiates the contracts. Such a situation arises when the CM holds higher channel power or the OEM on purpose delegates contracting to the CM. Note that, for any given contract offered by the CM, \((\alpha_S, w_S)\), the OEM’s cost \(w_S + R(\alpha_S)\) is independent of her bargaining power. Therefore, the CM only needs to offer a single contract \((\alpha^*, w^*)\) described in the next theorem if he is interested in taking over material procurement. The CM will not offer a contract if the OEM’s bargaining power is high enough. Note also that the CM does not have full information about the OEM’s bargaining power when offering the contract and the OEM will accept it only if her bargaining power is low.

**Theorem 2.5.** There exists a threshold of \(\beta_OEM\), below which the CM will not offer a contract and procurement is kept in-house. Otherwise, he offers a contract \((\alpha^*, w^*)\) such that \(\alpha^*\) is the unique solution to

\[
\beta_{CM} c'(\alpha) + R'(\alpha) + r'(\alpha) = 0
\]

and

\[
w^* = \beta_{OEM}^0 c(\alpha_I(\beta_{OEM}^0)) + R(\alpha_I(\beta_{OEM}^0)) + r_I(\alpha_I(\beta_{OEM}^0)) - R(\alpha^*)
\]
where $\beta_{OEM}^0 = \max\{\beta_{OEM}^\dagger, \beta_{OEM}\}$ and $\beta_{OEM}^\dagger$ is the unique solution to

$$(2\beta - \beta_{OEM})c(\alpha_I(\beta)) + R(\alpha_I(\beta)) + r(\alpha_I(\beta)) = \beta_{CM}c(\alpha^*) + R(\alpha^*) + r(\alpha^*).$$

The OEM will accept the contract if $\beta_{OEM} > \beta_{OEM}^0$ and reject it otherwise.

**Proof of Theorem 2.5:** If the CM offers a contract, there exists a cutoff point $\beta_{OEM}^0$ such that the OEM will accept the contract if $\beta_{OEM} \geq \beta_{OEM}^0$ and will reject it otherwise.

Given any cutoff point $\beta_{OEM}^0$, the CM chooses $(\alpha^*, w^*)$ to minimize his expected cost

$$\frac{1}{\beta_{OEM}^0 - \beta_{OEM}} \left\{ \int_{\beta_{OEM}^0}^{\beta_{OEM}} \pi_{CM}(\beta) d\beta + \int_{\beta_{OEM}}^{\beta_{OEM}^0} [\beta_{CM}c(\alpha^*) + r(\alpha^*) - w^*] d\beta \right\}. (2.17)$$

Since $w^* + R(\alpha^*) = \pi_{OEM}(\beta_{OEM}^0)$, $\alpha^*$ minimizes $\beta_{CM}c(\alpha) + r(\alpha) + R(\alpha)$, the supply chain cost and achieves the first best solution.

Taking the derivative of (2.17) with respect to $\beta_{OEM}^0$ we have the unconstrained solution $\beta_{OEM}^\dagger$ which satisfies the following equation

$$\begin{align*}
(2\beta_{OEM}^\dagger - \beta_{OEM})c(\alpha_I(\beta_{OEM}^\dagger)) + R(\alpha_I(\beta_{OEM}^\dagger)) + r(\alpha_I(\beta_{OEM}^\dagger))) &= \beta_{CM}c(\alpha^*) + R(\alpha^*) + r(\alpha^*),
\end{align*}$$

and $\beta_{OEM}^0$ is either $\beta_{OEM}^\dagger$ or one of the boundaries. □

Since the CM can decide the wholesale price, he has the incentive to select the supplier $\alpha^*$ that minimizes the total supply chain cost $\beta_{CM}c(\alpha) + R(\alpha) + r(\alpha)$! That is, he will select the supply chain first best solution, while making sure that he is properly rewarded. If the OEM accepts the contract, procurement outsourcing must benefit both parties compared with the in-house option. Furthermore, ceding contracting control to the CM may benefit the OEM or/and the CM as the parties are sharing a bigger pie if outsourcing is successful.

The OEM now has three options: (1) keep procurement in-house, (2) outsource procurement, and (3) cede contracting control to the CM. With each evolution from (1) to (2)
and then to (3), the OEM delegates more functions to the CM, from production to material procurement and to contracting control, yet the supply chain becomes more and more efficient when outsourcing is successful. In Figure 2.1, we summarize the final outcomes in the presence of the three options based on the bargaining power of the parties.

As shown in Figure 2.1, when the OEM’s (CM’s) bargaining power is high but the CM’s (OEM’s) is low, i.e., in region $\Omega_1$ (region $\Omega_5$), procurement is kept in-house (outsourced) regardless who is the contract initiator. When both parties have low bargaining power (in region $\Omega_2$), the OEM prefers to delegate contracting control to the CM because the in-house option is too costly and ceding contracting control provides an incentive for the CM with low bargaining power to minimize the supply chain cost. In regions $\Omega_3$ and $\Omega_4$, the CM’s high bargaining power makes procurement outsourcing a preferred option. However, if the OEM’s bargaining power is not high enough (in region $\Omega_4$), she has to
increase the wholesale prices in her contracts to encourage the CM to accept a contract if she is the contract initiator. Thus, it is for her best interest to delegate contracting control to the CM and to share a bigger pie.

We need to point out that, in reality, ceding contracting control may not be a viable option for the OEM because it may make it difficult for the OEM in contract negotiations with other supply chain partners. However, such an option, if exists either because it is viable to the OEM or because the CM indeed has higher channel power, can benefit both the OEM and the CM when it leads to procurement outsourcing.

### 2.4.5 Fixed Costs for Material Procurement

Let $K_{OEM}$ and $K_{CM}$ be the fixed cost at the OEM and CM, respectively. As one can see, as long as the fixed costs at the parties are the same, all results and insights in Section 2.3 carry over. However, procurement outsourcing is more likely to happen when $K_{OEM} > K_{CM}$ even though it is possible that outsourcing leads to lower quality material, which happens when $K_{OEM}$ is significantly higher than $K_{CM}$.

**Proposition 2.8.** With fixed costs for material procurement, the structure of $\alpha_S(\beta)$ remains the same as in Theorem 2.1. Compared with the case without the fixed costs,

1. if $K_{OEM} < K_{CM}$, the wholesale price $w_S(\beta)$ is lower and hence, procurement outsourcing is less likely to happen. Furthermore, $\alpha_S(\beta) \leq \alpha_I$ if procurement outsourcing is successful and outsourcing results in better quality.

2. If $K_{OEM} = K_{CM}$, all the results in Section 2.3 hold.

3. If $K_{OEM} > K_{CM}$, the wholesale price $w_S$ is higher and procurement outsourcing is more likely to happen. Furthermore, $\alpha_S(\beta) \leq \alpha_I$ if procurement outsourcing is successful, unless $K_{OEM}$ is significantly higher than $K_{CM}$.

**Proof of Proposition 2.8:** If there are fixed costs, (2.13) in the proof of Proposition 2.4 becomes

\[
\min_{\alpha} \left\{ (2\beta_{CM}^1 - \beta_{CM})c(\alpha) + R(\alpha) + r(\alpha) \right\} = \beta_{OEM}c(\alpha_I(\beta_{OEM})) + R(\alpha_I(\beta_{OEM})) + r(\alpha_I(\beta_{OEM})) + K_{OEM} - K_{CM}.
\]
As $K_{OEM} - K_{CM}$ increases, $\beta_{CM}^0$ increases. When $K_{OEM} = K_{CM}$, the problem reduces to that in Section 2.3.

1. If $K_{OEM} < K_{CM}$, the value of $\beta_{CM}^0$ is lower than that in Section 2.3 and procurement outsourcing is less likely to happen and $\alpha_S(\beta_{CM}^0) < \alpha_I(\beta_{OEM})$ still holds. Therefore, $\alpha_S(\beta_{CM}) < \alpha_I(\beta_{OEM})$ for any $\beta_{CM} < \beta_{CM}^0$.

2. If $K_{OEM} > K_{CM}$, the value of $\beta_{CM}^0$ is higher than that in Section 2.3 and procurement outsourcing is more likely to happen. However, $\alpha_S(\beta_{CM}^0) < \alpha_I(\beta_{OEM})$ may not hold. As $\alpha_S(\beta_{CM}^0)$ increases in $\beta_{CM}^0$ and $\beta_{CM}^0$ increases in $K_{OEM} - K_{CM}$, there exists a threshold of $K_{OEM} - K_{CM}$, below which $\alpha_S(\beta_{CM}^0) \leq \alpha_I(\beta_{OEM})$ and $\alpha_S(\beta_{CM}^0) > \alpha_I(\beta_{OEM})$ otherwise. Therefore, unless $K_{OEM}$ is significantly higher than $K_{CM}$, $\alpha_S(\beta_{CM}) < \alpha_I(\beta_{OEM})$ still holds if $\beta_{CM} < \beta_{CM}^0$.

\[\square\]

2.5. Conclusion

After years of cost cutting on factory floors, and in back offices and warehouses, many brand-name companies now turn to procurement departments. Rather than keeping a fully staffed procurement department in-house, they start to invite their contract manufacturers to get involved in or even be fully responsible for material procurement. One important question is how to construct the contracts between an OEM and CM and structure quality assurance activities. Our research is the first attempt to study analytically the cost and benefit of procurement outsourcing along with production.

In the chapter, we consider an OEM who outsources the production to a CM. There are a set of qualified material suppliers over which the OEM and CM hold different bargaining power, private information. The OEM may keep the procurement function in-house or choose to offer a menu of contracts to the CM for taking over the procurement function. Procurement is outsourced if the CM accepts a contract. Our main finding is that procurement outsourcing, if successful, improves quality of the final product as well as the profitability of both parties, and is indeed a win-win option. This is true even when the OEM holds higher bargaining power over the suppliers than the CM, which implies
that bargaining power alone is not sufficient to determine whether procurement should be outsourced or not. For an OEM with relatively low power, a well-designed menu of contracts can motivate a CM to take over procurement as well as to improve quality.

Our analysis also indicates that, if the OEM is willing to delegate the contracting of procurement outsourcing to the CM, the CM may have the incentive to make decisions that achieve supply chain efficiency which may in turn benefit the OEM. However, procurement outsourcing is less likely to happen in the presence of demand variability and that a mixed inspection strategy can be deployed to prevent the CM from contract violation.

Future study will extend our analysis from single round decision making to multi-round ones. With multi-rounds and information asymmetry, the incentives of the parties may change and truth revelation may not necessarily be possible. It is likely that the OEM may not have an incentive to reveal her true bargaining power and the problem can be quite complex.
Chapter 3

Dynamic Mechanism Design in Outsourcing Management

3.1. Overview

Today’s dynamic market forces put tremendous pressure on brand name companies to leverage resources and maximize their own core competencies. To do so, more and more companies outsource their production to maintain sustainable competitive advantages in pursuit of long-term profit. Outsourcing may enable companies to achieve lower labor and material cost, to focus on building their core competencies, to get access to world class manufacturing facilities and technology advantage, to reduce delivery time, and to enhance supply chain flexibility and efficiency. In order to make continuous improvements in a long period of time, many companies intend to maintain long-term partnerships with their service providers.

Long-term relationships enable both parties to obtain better knowledge of each other’s operations, policies, standards, strengths as well as weaknesses, and to maintain mutual trust between the two parties. Outsourcing strategy, known as “synergy between all partners with tight interaction, combined with knowledge exchange and mutual support”, helps integrate the two parties as a whole through intense communication and cooperation, and is therefore a long-term commitment rather than a temporary contractual arrangement. For example, Hon Hai Precision Industry Co., the biggest contract manufacturer of electronics in the world, has been keeping long-term partnerships with many brand name companies such as Apple Inc., Sony, HP and Intel. Hon Hai has built in a relationship
with Apple that dates back to 1997, with roots in desktop manufacturing, and now is producing Mac mini, iPod, iPad, and iPhone for Apple. Hon Hai signed on an important client in 2000, when it agreed to produce components for Sony’s Playstation 2 console. The two parties have been keeping partnership to this day, when Hon Hai is producing the latest generation of Playstation for Sony. By engaging in long-term partnerships with contract manufacturers who are world-class experts such as Hon Hai, leading companies instill high-performance operations that guarantee competitive pricing and on-going cost reduction.

While keeping long-term partnerships with her contract manufacturer, a brand name company often avoids signing a single long-term contract with him, as business is dynamic and markets change everyday, including customers, prices, products and so on. The partnership is sure to sour if no process for adjusting to these dynamics exists to guarantee on-going improvements. Instead, a brand name company will intend to offer contracts by continuous assessment based on past performances, and may revise contract terms with the changing economy. That way, she is able to maintain flexibilities when market changes, to share on-going cost savings, and to motivate the contract manufacturer to improve services. In particular, as production cost decreases, brand name companies and manufacturers will negotiate to agree on a new contract that divides cost savings between the two parties. Production cost reduction is achieved in a learn-by-doing process, when workers are capable of improving productivity through practice, self-perfection and minor innovations and therefore reduce the cost of producing each unit of the final product. An example is shown in the figure below with cost reduction trend of LED backlights in a five year period. A quarterly report of DisplaySearch, a leading global market research and consulting firm specializing in display supply chains, analyzes and forecasts LED backlight costs and discloses that the average cost of 40-inch edge LED backlight unit that is $118 in Quarter 1, 2010 is expected to fall to $100 by Quarter 4, 2010 (see Figure 3.1). According to DisplaySearch, “cost reduction is driven by increasing production volumes, which affects LED and material costs, and by improvement in LED luminous intensity, which enables the use of fewer LED chips”. The forecast supports the fact that the industry is seeing a cost reduction trend through cumulative experience and production volume, which also causes price compression.
However, designing contracts in a long period of time is a tough task rather than a simple static optimization problem, due to the existence of information asymmetry and conflicting incentives. While keeping its downtrend with cumulative production volume, the cost also fluctuates with market change such as an surging cost for materials and labors during a period. However, a brand name company may not have an access to real-time market change and thus may not have accurate information about the production cost. Instead of sharing the hidden cost with the brand name company, the manufacturer has an incentive to keep it privately to maintain his bargaining power and to negotiate for a higher profit margin, which can cause supply chain inefficiency. Therefore, when outsourcing production functions in a long period of time, a brand name company needs to put in place effective dynamic mechanisms to align the interest of the manufacturer with the long-term profit of the OEM.

In this chapter, we propose a stylized model to study the dynamic outsourcing mechanism that maximizes the long-term profit of the brand name company. We derive the optimal quantity-based incentive contracts in each period, and examine the sharing of cost savings among the two parties. We study the impact of the learning rate on outsourcing decisions, and also compare the difference between a single-period game and a multi-period game, as well as that between a static deterministic game and a dynamic
mechanism design game using the static and single-period decisions as benchmarks. We find that in a dynamic mechanism design game, the order quantity in each period is positively correlated with the cumulative order quantities, and the sales of final products are increasing in the learning rate of the manufacturer.

The rest of this chapter is organized as follows. We review relevant literature in the next section and present the model and assumptions in Section 3.3. After deriving the optimal dynamic mechanism in Section 3.4, we explore the impact of various factors in Section 3.5, followed by concluding remarks in Section 3.6.

3.2. Literature Review

There is a fast-growing literature on dynamic mechanism design. Dynamic mechanism design differs from static mechanism design in the sense that in the former, an agent obtains the information of future deviations, and thus will make misrepresentation conditional on historical information, which is his past type, reports and decisions, rather than a simple misrepresentation of his current type as in a static game. Baron and Besanko (1984) start their pioneering work in this field by analyzing the regulation of a natural monopoly in a two-period model. They study a model in which a principal is able to audit at an agent’s cost, and therefore can order a refund to customers afterwards if a misrepresentation is discovered. The authors derive the optimal pricing and auditing strategy in each stage and show that the auditing strategy in the second period depends on the initial pricing decision. Recent studies in Economics focus on incentive contract design in a general dynamic setting with multiple periods or on the infinite time horizon. Athey and Segal (2007a) construct an incentive-compatible mechanism for a general dynamic model over the infinite horizon, in which decisions are made to allocate a good between two players. The players’ valuations are private information, and an efficient mechanism as well as its properties are throughly studied. Athey and Segal (2007b) extend the model to a game with multiple agents and each of them privately observes a cost signal in every period. Decisions on money transfer are made after the agents report their types. The authors construct an efficient incentive-compatible mechanism with both non-balanced and balanced budgets. Pavan et al. (2009) study an incentive compatible screening model.
in a multi-period game, in which an agent’s type follows a stochastic process and is only observed by the agent himself. The authors propose a formula summarizing conditions for incentive compatibility, explore the property of the optimal mechanisms, and apply the result to some novel settings.

There also exist several papers that apply dynamic mechanism design in practical business models. For example, Jehiel and Moldovanu (1999) examine a resale market in which the trading procedure of an indivisible good is determined by its current owner. The authors propose a dynamic trading mechanism and prove an irrelevance of the initial ownership with the resale process. Courty and Li (2000) study a model on advanced ticket sales, when customers subsequently learn their actual valuations. An optimal refund mechanism is proposed in a two-period model. Bajari and Tadelis (2001) develop a model to study long-term procurement contracts. They analyze a tradeoff between providing incentives and reducing transaction costs due to renegotiation. Albanesi and Sleet (2006) apply a dynamic mechanism design model to examine an optimal taxation problem with information asymmetry. In our study, we adopt some of the methodologies and use dynamic incentive contracts as an instrument to manage long-term outsourcing relationships.

As we consider outsourcing management with on-going cost savings due to a learning effect, the literature that addresses dynamic learning is also relevant. Fine (1986) introduces the idea of a quality-based learning curve to show that production cost is decreasing in experience in a learn-by-doing process. He explores a volume-based learning model, where experience is gained through cumulative production volume, and extends it to a quality-based learning model, in which experience is increasing in cumulative conforming production volume. To study the decision of continuous investment in quality improvement, he presents a deterministic continuous-time model and provides conditions under which it is optimal to continue investing to achieve perfect quality. Fine (1988) and Fine and Porteus (1989) apply the learning effect to a problem associated with a quality control process. Fine (1988) characterizes the optimal inspection policies through stochastic dynamic programming, while Fine and Porteus (1989) formulate a Markov decision process to explore the optimal investment decision in process improvement.
Finally, there has been an increasing amount of literature addressing dynamic outsourcing decisions. Kouvelis and Milner (2002) study the interplay of demand and supply uncertainty and the outsourcing decision in a multi-period supply chain. They derive the optimal capacity investment decisions for both single- and multi-period models and focus on how outsourcing is affected by changes in supply and demand uncertainty. Kim (2003) investigates a supply chain consisting of a manufacturing company and two contract manufacturers, each with distinct price and improvement capability. He studies dynamic decisions on allocating the production volumes among the contract manufacturers. Opp et. al. (2005) discuss a problem of warranty repair outsourcing. The authors proposed two approaches to develop the optimal allocation of tasks among multiple service vendors via dynamic programming. However, none of the papers has examined information asymmetry in dynamic outsourcing management. Our study differs from these studies in that we focus on games with private information, incorporate dynamic mechanism design in outsourcing management and propose a stylized model to construct optimal incentive contracts in multiple periods.

3.3. Problem formulation

We consider a supply chain consisting of one brand name company or an Original Equipment Manufacturer (OEM, she) and one contract manufacturer (CM, he) in a $T$-period game. In each period, the OEM outsources production of final products to the CM through a quantity-based contract associating a total payment $R_t$ with an order quantity $Q_t$.

The unit production cost in Period $t$, $c_t = \varepsilon_t - Z_t$, consists of two components. First, there exits a random shock $\varepsilon_t$ representing the impact of market change on material and labor cost. For simplicity and tractability, we assume that $\varepsilon_t$ are i.i.d. random variables uniformly distributed over $[\varepsilon, \bar{\varepsilon}]$. In each period, production cost is affected by the changing market, e.g. labor or material costs may vary from period to period, and companies may have some common knowledge about the impact of the changing market in every period. In Period $t$, the CM is able to learn the realization of the random cost $\varepsilon_t$, while the OEM only holds a prior distribution as $\varepsilon_t$ is often kept as she may not have an access to its
true value. However, both parties only hold a prior distribution about $\varepsilon_i$, $t + 1 \leq i \leq T$, future random shocks. We denote the mean of each random shock by $\mu = \frac{\varepsilon + \varepsilon}{2}$, and the variability by $\sigma = \frac{\varepsilon - \varepsilon}{2}$. Second, there is also a deterministic cost representing the learning effect, which is decreasing in the cumulative production volume. We model the cumulative experience gained from production up to Period $t$ as $Z_t = \sum_{i=1}^{t-1} \beta^i Q_i$, which is increasing in the cumulative production volume, and $Z_0 = 0$. As the CM produces more products, he is able to accumulate more valuable experience from the learning-by-doing process and can thus reduce production cost in the future. Therefore, unit production cost in each period $c_t$ decreases in the cumulative production volume of end products.

Finally, the price of the final products is modeled as the inverse demand $p(Q_t) = p_0 - Q_t$, where $p_0 > \mu + \sigma$.

The sequence of events in each period is described as below:

1. The CM learns the true value of $\varepsilon_t$.
2. The OEM offers a menu of contracts $\{Q_t(\varepsilon, Z_t), R_t(\varepsilon, Z_t)\}$ to the CM.
3. The CM reports his type as $\hat{\varepsilon}_t$ by committing to a contract $\{Q_t(\hat{\varepsilon}_t, Z_t), R_t(\hat{\varepsilon}_t, Z_t)\}$.
4. The CM produces the products and the OEM sells them in the market.

### 3.4. Optimal Dynamic Mechanism Design

In each period, the OEM will offer an optimal menu of contracts $\{Q_t(\varepsilon, Z_t), R_t(\varepsilon, Z_t)\}$ to maximize her present value of profit. By the revelation principle, we restrict attention to direct mechanisms for which a truthful strategy is optimal for the CM. That is, the CM will report $\hat{\varepsilon}_t = \varepsilon_t$ and the contract $(Q_t(\varepsilon_t, Z_t), R_t(\varepsilon_t, Z_t))$ will maximize his present value of profit. Therefore, the profits for both the OEM and the CM in period $t$ can be determined as below:

$$
\pi_{OEM}^t(Q_t, R_t) = p(Q_t(\varepsilon_t, Z_t)) Q_t(\varepsilon_t, Z_t) - R_t(\varepsilon_t, Z_t),
$$
$$
\pi_{CM}^t(Q_t, R_t, \varepsilon_t) = R_t(\varepsilon_t, Z_t) - c_t(\varepsilon_t) Q_t(\varepsilon_t, Z_t),
$$
And the present values of profits for both parties in Period \( t \) are

\[
V_{OEM}^t(Q_t, R_t, Z_t) = \pi_{OEM}^t(Q_t, R_t, Z_t) + \rho E_{t+1} V_{OEM}^{t+1}(Q_{t+1}(\varepsilon_{t+1}, Z_{t+1} + \beta^t Q_t), R_t(\varepsilon_{t+1}, Z_{t+1} + \beta^t Q_t), Z_t + \beta^t Q_t),
\]

\[
V_{CM}^t(Q_t, R_t, Z_t, \varepsilon_t) = \pi_{CM}^t(Q_t, R_t, Z_t, \varepsilon_t) + \rho E_{t+1} V_{CM}^{t+1}(Q_{t+1}(\varepsilon_{t+1}, Z_{t+1} + \beta^t Q_t), R_t(\varepsilon_{t+1}, Z_{t+1} + \beta^t Q_t), Z_t + \beta^t Q_t, \varepsilon_{t+1}),
\]

where \( \rho \) is the discount factor. Suppose the contract manufacturer will only accept an offer if he can earn a non-negative present value of profit \( V_{CM}^t \geq 0 \) in each period.

We will use backward induction to characterize the structure of the optimal dynamic incentive contracts. First, we derive the optimal menu of contracts offered in the last period.

### 3.4.1 Mechanism Design in Period \( T \)

When there is only one period remaining, the OEM will offer a menu of contracts \( \{ Q_T(\varepsilon, Z_T), R_T(\varepsilon, Z_T) \} \) to maximize her expected profit by solving the following problem, given the Incentive-Compatibility Constraint that induces truthful revelation, and Participation Constraint that guarantees a non-negative present value of profit for the CM. Given \( Z_T \), the OEM will determine the menu of contracts as functions of \( \varepsilon \) to solve the following problem:

\[
\hat{V}_{OEM}^T(Z_T) = \max_{Q_T(\varepsilon, Z_T), R_T(\varepsilon, Z_T)} \left\{ E_{\varepsilon_T} [\pi_{OEM}^T(Q_T(\varepsilon_T, Z_T), R_T(\varepsilon_T, Z_T))] \right\}
\]

s.t.

\[
\frac{\partial R_T}{\partial \varepsilon} - c_T \frac{\partial R_T}{\partial \varepsilon} = 0 \quad \text{for all} \quad \varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}],
\]

\[
\pi_{CM}^T(Q_T(\overline{\varepsilon}), R_T(\overline{\varepsilon}), \overline{\varepsilon}) = 0.
\]

This is the one-period incentive contract design problem and the optimal decision is described as below:
Theorem 3.1. In Period $T$, the optimal menu of contracts is given by:

$$Q_T(\varepsilon, Z_T) = \frac{1}{2} (p_0 + Z_T + \mu - \sigma) - \varepsilon,$$

$$R_T(\varepsilon, Z_T) = \frac{1}{2} [p(Q_T) - Z_T + \mu - \sigma] Q_T + R_0^T(Z_T),$$

where $R_0^T(Z_T)$ is a constant independent of $\varepsilon$ such that $\pi_{CM}^T(Q_T(\varepsilon, Z_T), R_T(\varepsilon, Z_T), \bar{\varepsilon}) = 0$.

Proof of Theorem 3.1: The IC constraint is determined by $\frac{\partial R_T}{\partial \varepsilon} - c_T \frac{\partial R_T}{\partial \varepsilon} = 0$. Let $u = \frac{\partial Q_T}{\partial \varepsilon}$ and $v = \frac{\partial R_T}{\partial \varepsilon}$.

The Hamiltonian is $H = p(Q_T)Q_T - R_T + \lambda_R(\varepsilon - Z_T)u + \lambda_Q u$. An optimal solution thus satisfies the following conditions.

$$\frac{\partial H}{\partial u} = \lambda_R(\varepsilon - Z_T) + \lambda_Q = 0,$$  \hspace{1cm} (3.3)

$$\lambda_R'(\varepsilon) = -\frac{\partial H}{\partial R_T} = 1,$$  \hspace{1cm} (3.4)

$$\lambda_Q'(\varepsilon) = -\frac{\partial H}{\partial Q_T}.$$  \hspace{1cm} (3.5)

From (3.4), we have $\lambda_R = \varepsilon - \bar{\varepsilon}$.

Substituting (3.4.1) into (3.3) and taking derivative on both sides, we obtain $\lambda_Q'(\varepsilon) = -(2\varepsilon - Z_T - \bar{\varepsilon})$. Substituting the results into (3.5) we have

$$Q_T(\varepsilon, Z_T) = \frac{1}{2} (p_0 + Z_T + \bar{\varepsilon}) - \varepsilon$$

Therefore,

$$\varepsilon = \frac{1}{2} [Z_T + \bar{\varepsilon} + p'(Q_T)Q_T + p(Q_T)].$$  \hspace{1cm} (3.6)

Substituting (3.6) to the IC constraint and solve the equation we have

$$R_T(\varepsilon, Z_T) = \frac{1}{2} [p(Q_T) - Z_T + \bar{\varepsilon}] Q_T + R_0^T(Z_T),$$

where $R_0^T(Z_T)$ is a constant independent of $\varepsilon$ such that $\pi_{CM}^T(Q_T(\varepsilon, Z_T), R_T(\varepsilon, Z_T), \bar{\varepsilon}) = 0.$
Theorem 3.1 demonstrates that in the optimal menu of contracts, the payment \( R_T \) contains two parts, (1) a per-unit charge \( \frac{1}{2} [\mu(Q_T) - Z_T + \mu - \sigma] \), and (2) a lump-sum fee \( R_{0T}^t(Z_T) \) that guarantees certain profitability of the CM. Through the contract, the OEM shares one half of the sales income of final products, as well as compensating the CM for one half of the lowest production cost. This way, she is able to impose the CM certain obligations that align the CM’s incentive with the interest of her own under information asymmetry. Furthermore, when variability \( \sigma \) increases, the OEM will lower the per-unit charge to offset supply chain uncertainty, as a result of which the order quantity will also decrease.

3.4.2 Mechanism Design in Period \( t \)

Using backward induction, we can derive the structure of the optimal contracts. The OEM will design a mechanism to maximize her present value of profit. Therefore, in Period \( t \), she will offer a menu of contracts to solve the following problem, given that \( \{Q_i(\varepsilon, Z_i), R_i(\varepsilon, Z_i)\} \) will be the optimal menu of contracts offered in Period \( i \), for any \( t + 1 \leq i \leq T \):

\[
\hat{V}_{OEM}^t(Z_t) = \max_{Q_t(\varepsilon, Z_t), R_t(\varepsilon, Z_t)} E_{\varepsilon_t} \left[ \pi_{OEM}^t(Q_t(\varepsilon_t, Z_t), R_t(\varepsilon_t, Z_t)) + \rho \hat{V}_{OEM}^{t+1}(Z_t + \beta^t Q_t(\varepsilon_t, Z_t)) \right]
\]

\[
\text{s.t. } \varepsilon = \arg\max_{\varepsilon} V_{CM}^t(Q_t(\bar{\varepsilon}, Z_t), R_t(\bar{\varepsilon}, Z_t), Z_t), \quad V_{CM}^t(Q_t(\bar{\varepsilon}, Z_t), R_t(\bar{\varepsilon}, Z_t), Z_t, \bar{\varepsilon}) = 0..
\]

The profits for both parties in period \( t \) depend on the sequence of decisions \( Q^{t-1} = (Q_i)_{i=1}^{t-1} \). Let

\[
h_t(Q^{t-1}) = \frac{d}{dQ_t} E_{\varepsilon_t} \left[ V_{OEM}^t(Q_t(\varepsilon_t, Z_t), R_t(\varepsilon_t, Z_t), Z_t) \right], \quad \text{and}
\]

\[
g_t(Q^{t-1}) = \frac{d}{dQ_t} E_{\varepsilon_t} \left[ V_{CM}^t(Q_t(\varepsilon_t, Z_t), R_t(\varepsilon_t, Z_t), Z_t, \varepsilon_t) \right],
\]

respectively. Lemma 3.1 characterizes the structure of optimal contracts in each period, and Theorem 3.2 derives the recursive equations for the parameters in the optimal contracts.
Lemma 3.1. In each period, \( h_t \) and \( g_t \) have the following linear structures:

\[
\begin{align*}
    h_t(Q^{t-1}) &= h_t^p p_0 + h_t^s Z_t + h_t^u \mu - h_t^v \sigma, \\
    g_t(Q^{t-1}) &= g_t^p p_0 + g_t^s Z_t + g_t^u \mu - g_t^v \sigma,
\end{align*}
\]

where \( h_t^i \) and \( g_t^i \) are constants for any \( i \in \{p, s, u, v\} \).

The optimal menu of contracts \( \{Q_t(\varepsilon, Z_t), R_t(\varepsilon, Z_t)\} \) has the following structure:

\[
\begin{align*}
    Q_t(\varepsilon, Z_t) &= a_t^p p_0 + a_t^s Z_t + a_t^u \mu - a_t^v \varepsilon, \\
    R_t(\varepsilon, Z_t) &= (b_t^p p_0 + b_t^s Z_t + b_t^u \mu - b_t^v \varepsilon)Q_t - b_t^q Q_t^2 + R_t^0(Z_t),
\end{align*}
\]

where \( a_t^i \) and \( b_t^j \) are constants, for any \( i \in \{p, s, u, v\} \) and \( j \in \{p, s, u, v, q\} \). \( R_t^0(Z_t) \) is a constant independent of \( \varepsilon \) such that \( V_{CM}^t(Q_t(\varepsilon, Z_t), R_t(\varepsilon, Z_t), Z_t, \varepsilon) = 0 \).

Proof of Lemma 3.1 is given in the appendix.

Furthermore, we can simplify the contract through the following lemma:

Lemma 3.2. \( a_t^p + a_t^u = a_t^e \) for every \( 1 \leq t \leq T \).

Proof of Lemma 3.2: We will show that \( g_t^p + h_t^p + g_t^u + h_t^u = 0 \) for every \( 2 \leq t \leq T + 1 \).

Initial condition is satisfied since the result hold when \( t = T + 1 \).

Suppose the above result is true for \( i + 1 \leq t \leq T \). For \( t = i \), we have

\[
E(V_{OEM}^i + V_{CM}^i) = E_\varepsilon \left[ (p_0 - Q_i)Q_i - c_i Q_i + \rho E_{\varepsilon_{i+1}}(V_{OEM}^{i+1} + V_{CM}^{i+1}) \right].
\]
Therefore

\[
    h_i(Q_{i-1}^t) + g_i(Q_{i-1}^t) = E_{\varepsilon_i} \left[ (-a_i^* \beta^{i-1} + \beta^{i-1})Q_i + a_i^* \beta^{i-1}(p_0 - Q_i - \varepsilon_i + Z_i) \right] + \frac{\rho \beta}{1 + \beta a_i^*} E_{\varepsilon_i} \left[ (g_{i+1}(Q^t) + h_{i+1}(Q^t)) \right] \\
    = (\beta^{i-1} - 2a_i^* \beta^{i-1})(a_i^* p_0 + a_i^* Z_i + a_i^* \mu - a_i^* \sigma - a_i^* \mu) + a_i^* (p_0 - \mu + Z_i) + \rho (g_{i+1}^* + h_{i+1}^*) \left( \sum_{j=1}^{i-1} Q_j + E Q_i \right) \\
    = \frac{\rho \beta}{1 + \beta a_i^*} \left[ (g_{i+1}^* + h_{i+1}^*)p_0 + (g_{i+1}^{\mu} + h_{i+1}^{\mu})\mu - (g_{i+1}^{\nu} + h_{i+1}^{\nu})\sigma \right]
\]

Since \( g_i^p + h_i^p + g_i^\mu + h_i^\mu = 0 \) for every \( i + 1 \leq t \leq T \), \( a_i^p + a_i^\mu = a_i^\varepsilon \) for every \( i \leq t \leq T \).

Therefore, it is straightforward that \( g_{i+1}^p + h_{i+1}^p + g_{i+1}^\mu + h_{i+1}^\mu = 0 \).

By induction, we can prove that \( g_t^p + h_t^p + g_t^\mu + h_t^\mu = 0 \) for all \( 2 \leq t \leq T + 1 \). Therefore \( a_t^p + a_t^\mu = a_t^\varepsilon \) for all \( 1 \leq t \leq T \).

\[\square\]

Based on the above results, we can derive the optimal menu of contracts in each period using recursions. Next theorem characterizes the dynamic mechanism offered by the brand name company that can maximize her long-term discounted profit.

**Theorem 3.2.** The optimal menu of contracts in Period \( t \), \( \{Q_t(\varepsilon, Z_t), R_t(\varepsilon, Z_t)\} \), is dynamically determined by the following recursive equations:

\[
    Q_t(\varepsilon, Z_t) = a_t^p p_0 + a_t^\mu Z_t + a_t^\mu \mu - a_t^\sigma \sigma - a_t^\varepsilon \varepsilon, \text{ where} \\
    a_t^p = \frac{\beta - \rho + \rho (2 + \beta^{t+1})a_{t+1}^s}{(2\beta + \rho \beta^t) - \rho \beta^t (2 + \beta^{t+1})a_{t+1}^s}, \\
    a_t^\mu = \frac{\beta - \rho + \rho (2 + \beta^{t+1})a_{t+1}^{\nu}}{(2\beta + \rho \beta^t) - \rho \beta^t (2 + \beta^{t+1})a_{t+1}^{\nu}}, \\
    a_t^\varepsilon = \frac{2}{(2\beta + \rho \beta^t) - \rho \beta^t (2 + \beta^{t+1})a_{t+1}^s}, \\
    a_t^p = a_t^s, \\
    a_t^\mu = a_t^\nu = a_t^\varepsilon = a_t^T = a_T^p = \frac{1}{2} \text{ and } a_T^\varepsilon = 1.
\]

With initial values as \( a_T^p = a_T^s = a_T^\mu = a_T^\nu = a_T^\varepsilon = \frac{1}{2} \) and \( a_T^\varepsilon = 1 \).
The payment $R_t$ associated with $Q_t$ is

$$R_t(\varepsilon, Z_t) = [b_t^p \rho_0 + b_t^Z Z_t + b_t^\mu \mu - b_t^\sigma \sigma] Q_t - b_t^Q Q_t^2 + R_t^0(Z_t),$$

$$b_t^q = \frac{a_t^q}{a_t^q} - \rho \beta a_{t+1},$$

$$b_t^s = \frac{a_t^s}{a_t^s} - 1 - 2 \rho \beta a_{t+1},$$

$$b_t^u = \frac{a_t^u}{a_t^u} + \rho \beta (a_{t+1}^p + a_{t+1}^s),$$

$$b_t^v = \frac{a_t^v}{a_t^v} - \rho \beta (a_{t+1}^s - a_{t+1}^s - a_{t+1}^v),$$

$$b_t^g = \frac{1}{2} \left( \frac{1}{a_t^s} + 2 \rho \beta a_{t+1}^s \right),$$

with initial values as $b_T^p = b_T^u = b_T^v = \frac{1}{2}$ and $b_T^s = -\frac{1}{2}$. $R_t^0(Z_t)$ is a constant such that

$$V_{CM}(Q_t(\varepsilon, Z_t), R_t(\varepsilon, Z_t), Z_t, \varepsilon) = 0.$$

**Proof of Theorem 3.2:**

$$g_t^p + h_t^p = a_t^p[1 - 2 a_t^s + \rho(1 + \beta a_t^s)](g_{t+1}^p + h_{t+1}^p) + \left[\beta a_t^s + \rho \frac{1}{\beta}(1 + \beta a_t^s)(g_{t+1}^p + h_{t+1}^p)\right],$$

$$g_t^s + h_t^s = a_t^s[1 - 2 a_t^s + \rho(1 + \beta a_t^s)](g_{t+1}^s + h_{t+1}^s) + \left[\beta a_t^s + \rho \frac{1}{\beta}(1 + \beta a_t^s)(g_{t+1}^s + h_{t+1}^s)\right],$$

$$g_t^u + h_t^u = (a_t^u - a_t^s)[1 - 2 a_t^s + \rho(1 + \beta a_t^s)](g_{t+1}^u + h_{t+1}^u) + \left[-\beta a_t^s + \rho \frac{1}{\beta}(1 + \beta a_t^s)(g_{t+1}^u + h_{t+1}^u)\right],$$

$$g_t^v + h_t^v = a_t^v[1 - 2 a_t^s + \rho(1 + \beta a_t^s)](g_{t+1}^v + h_{t+1}^v) + \rho \frac{1}{\beta}(1 + \beta a_t^s)(g_{t+1}^v + h_{t+1}^v).$$

Since $a_t^s = \frac{1}{2 - \rho \beta a_t^s}[1 + \rho(h_{t+1}^s + g_{t+1}^u)]$, we have

$$(1 + \beta a_t^s)(h_{t+1}^s + g_{t+1}^u) = 2 a_t^s - 1.$$

Therefore, $g_t^v + h_t^v = \frac{1}{\beta}[(2 + \beta a_t^s) a_t^s - 1].$
Substituting the above result to the formulation of $a_{t-1}^s$, we obtain

$$
\begin{align*}
    a_{t-1}^s &= \frac{\beta - \rho + \rho(2 + \beta^t)a_t^s}{(2\beta + \rho\beta^{t-1}) - \rho\beta^{t-1}(2 + \beta^t)a_t^s}, \\
    a_{t-1}^e &= \frac{2}{(2\beta + \rho\beta^{t-1}) - \rho\beta^{t-1}(2 + \beta^t)a_t^s}.
\end{align*}
$$

Since

$$
2 - \rho\beta^t(h_{t+1}^s + g_{t+1}^s) = 2 - \beta^t \frac{2a_t^s - 1}{1 + \beta^t a_t^s} = \frac{2 + \beta^t}{1 + \beta^t a_t^s},
$$

we obtain

$$
h_t^p + g_t^p = \frac{1}{\beta}[(2 + \beta^t)a_t^p - 1].
$$

Substituting the above result to the formulation of $a_{t-1}^p$, we obtain

$$
a_{t-1}^p = \frac{(\beta - \rho) + \rho(2 + \beta^t)a_t^p}{(2\beta + \rho\beta^{t-1}) - \rho\beta^{t-1}(2 + \beta^t)a_t^p}.
$$

Similarly, we have

$$
h_t^v + g_t^v = \frac{1}{\beta}[(2 + \beta^t)a_t^v - \beta^t a_t^s - 1].
$$

Substituting the above result to the formulation of $a_{t-1}^v$, we obtain

$$
a_{t-1}^v = \frac{(\beta - \rho) + \rho(2 + \beta^t)a_t^v - \rho a_t^s}{(2\beta + \rho\beta^{t-1}) - \rho\beta^{t-1}(2 + \beta^t)a_t^s}.
$$

Therefore the recursive formulation of $Q_{t-1}$ is obtained. The recursive formulation of $R_{t-1}$ is shown in the proof of Lemma 3.1.

$\square$

### 3.4.3 Dynamic Design in Infinite Time Horizon

From Theorem 3.2, the optimal menu of contracts in each stage is well defined based on recursive equations. To extend the results to the infinite time horizon, we will derive conditions under which the optimal decisions in the initial stage will converge as $T \to \infty$. Assume that $\beta < \rho$. 

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Lemma 3.3. If $a_t^s > 0$ for any $t$, $a_t^s$ is decreasing in $t$ for $t = 1, \cdots, T$.

Proof of Lemma 3.3: Suppose that the result is true for $i \geq t$. That is, $a_t^s \geq a_{t+1}^s \geq \cdots \geq a_T^s = \frac{1}{2}$. Consider the case when $i = t - 1$. Then

\[
\frac{a_{t-1}^s}{a_t^s} = \frac{\beta - \rho}{a_t^s} + \rho(2 + \beta^t) > \frac{2(\beta - \rho) + \rho(2 + \beta^t)}{2\beta + \rho\beta^{t-1} - \frac{1}{2}\rho\beta^{t-1}(2 + \beta^t)}
\]

\[
= \frac{2\beta + \rho\beta^t}{2\beta - \frac{1}{2}\rho\beta^{t-1}} > 1,
\]

the result holds for $i = t - 1$. By induction, we can prove that $a_t^s$ is decreasing in $t$ for $t = 1, \cdots, T$.

\[\square\]

Lemma 3.4. If $a_t^s$ is decreasing in $t$ for $t = 1, \cdots, T$, $a_t^p$, $a_t^v$ and $a_t^e$ are decreasing in $t$ for $t = i, \cdots, T$.

Proof of Lemma 3.4: We define $a_{T+1}^s = \frac{1}{2 + \beta^{T+1}}$. Then for $t = 2, \cdots, T$, we have

\[
a_{t-1}^s = \frac{\beta - \rho + \rho(2 + \beta^t)a_t^s}{(2\beta + \rho\beta^{t-1} - \rho\beta^{t-1}(2 + \beta^t)a_t^s)},
\]

\[
a_{t-1}^e = \frac{2}{(2\beta + \rho\beta^{t-1} - \rho\beta^{t-1}(2 + \beta^t)a_t^s)}.
\]

Since $a_t^s$ is decreasing in $t$, it is obvious that $a_t^e$ is also decreasing in $t$.

Since $a_t^p = a_t^s$, $a_t^p$ is also decreasing in $t$.

Define $a_{T+1}^v = \frac{2}{(2 + \beta^{T+1})^2}$. Then for $t = 2, \cdots, T$,

\[
a_{t-1}^v = \frac{(\beta - \rho) + \rho(2 + \beta^t)a_t^v - \rho a_t^s}{(2\beta + \rho\beta^{t-1} - \rho\beta^{t-1}(2 + \beta^t)a_t^s)}.
\]
Since \( \frac{\partial a'_v}{\partial a'_t} > 0 \), \( a'_v > a'_{v+1} \) and \( a'_s < a'_{s+1} \), we have

\[
a'_{T-1} = \frac{(\beta - \rho) + \rho(2 + \beta^T)a'_T - \rho a'_s}{(2\beta + \rho \beta^{T-1}) - \rho \beta^{T-1}(2 + \beta^T)a'_T} \>
\]

\[
> \frac{(\beta - \rho) + \rho(2 + \beta^{T+1})a'_{T+1} - \rho a'_{s+1}}{(2\beta + \rho \beta^T) - \rho \beta^T(2 + \beta^{T+1})a'_{T+1}} = a'_v.
\]

Thus \( a'_v \) is decreasing in \( t \).

\( \square \)

From the above lemmas, we can derive a sufficient condition under which the recursive outsourcing contract converges as \( T \to \infty \).

**Theorem 3.3.** The initial menu of contracts \( \{Q_1(\varepsilon, Z_1), R_1(\varepsilon, Z_1)\} \) converges as \( T \to \infty \) if there exist positive \( k_i, i = 1, 2, 3 \) such that

\[
\rho < \frac{2 - k_3 - k_2 \beta}{k_1},
\]

\[
\beta < \frac{k_1 k_2}{1 + 4k_2}.
\]

**Proof of Theorem 3.3:** We first prove that \( a'_s \) will converge when the convergence condition holds. Then \( a'_p, a'_1, a'_i \) and \( a'_1 \) will converge. Let \( \bar{p} = \frac{\rho}{\beta} \) and \( A_t = -\bar{p} + \bar{p}(2 + \beta^t)a'_s \).

Then, we have

\[
A_{t-1} = -\bar{p} + \bar{p}(2 + \beta^{t-1})\frac{1 + A_t}{2 - \beta^{t-1}A_t}
\]

\[
= \bar{p} \cdot \frac{\beta^{t-1} + 2(1 + \beta^{t-1})A_t}{2 - \beta^{t-1}A_t}.
\]

Let \( B_t = 2 - \beta^{t-1}A_t \). Therefore

\[
A_{t-1} = \frac{\beta^{2t-2} + 2(1 + \beta^{t-1})(2 - B_t)}{\beta^{t-1}B_t}.
\]
Since

\[ B_{t-1} = 2 - \beta^{t-2} A_{t-1} \]
\[ = 2 - \rho \cdot \frac{\beta^{2t-2} + 2(1 + \beta^{t-1})(2 - B_t)}{\beta B_t} , \]

we next show that if there exist \( k_1 > 0, k_2 > 0 \) and \( k_3 > 0 \) such that

\[ k_1 \rho < 2 - k_3 \left[ \frac{\beta k_1}{2(1 + \beta)} \right]^t - k_2 \beta^2(t-1) < B_t < 2, \]

where

\[ \rho < \frac{2 - k_3 + k_2}{k_1} \text{ and } \beta < \frac{k_2}{1 + 4k_2}. \]

We then prove that

\[ k_1 \rho < 2 - k_3 \left[ \frac{\beta k_1}{2(1 + \beta)} \right]^{t-1} - k_2 \beta^2(t-2) < B_{t-1} < 2. \]

Suppose it is true for \( t \). Then it is straightforward to show that \( B_{t-1} < 2 \). Furthermore, since \( B_t > k_1 \rho \), we have

\[ 2 - B_{t-1} = \rho \cdot \frac{\beta^{2t-2} + 2(1 + \beta^{t-1})(2 - B_t)}{\beta B_t} \]
\[ < \frac{1}{k_1} \beta^{2t-2} + 2(1 + \beta^{t-1})(2 - B_t) \]
\[ < \frac{1}{k_1} \left\{ \beta^{2t-3} + \frac{1}{\beta} 2(1 + \beta) k_3 \left[ \frac{\beta k_1}{2(1 + \beta)} \right]^t + \frac{1}{\beta} 2(1 + \beta) k_2 \beta^2(t-1) \right\} . \]

Combined with the facts that \( \rho < \frac{2 - k_3 - k_2}{k_1} \) and \( \beta < \frac{k_1 k_2}{1 + 4k_2} \), we can show that

\[ k_1 \rho < 2 - k_3 \left[ \frac{\beta k_1}{2(1 + \beta)} \right]^{t-1} - k_2 \beta^2(t-2). \]

This leads to

\[ \frac{1}{k_1} \beta^{2t-3} + \frac{1}{k_1 \beta} 2(1 + \beta) \left\{ k_3 \left[ \frac{\beta k_1}{2(1 + \beta)} \right]^{t-1} + k_2 \beta^2(t-2) \right\} < k_3 \left[ \frac{\beta k_1}{2(1 + \beta)} \right]^{t-1} - \]
\[ k_2 \beta^2(t-2) . \]
Therefore, the result is true for $t - 1$.

Therefore, a sufficient condition under which $a_i^t$ converges will $T \rightarrow \infty$ is that there exist positive $k_i$, $i = 1, 2, 3$ such that

\[
\rho < \frac{2 - k_3 - k_2}{k_1} \beta,
\]
\[
\beta < \frac{k_1 k_2}{1 + 4 k_2}.
\]

That is, when the discount factor $\rho$ and/or learning rate $\beta$ are small, the decision process will converge as $T \rightarrow \infty$. Otherwise, the initial decision may not converge either because the interest rate is too low ($\rho$ is large), or the production cost decreases too fast ($\beta$ is large), under which the initial order quantity may be infinity.

Figure 3.2 shows the convergence and non-convergence regions for $\rho \in [0.75, 0.95]$.

![Figure 3.2: Convergence Region.](image-url)
3.5. Discussions

In this section, we will explore the properties of the optimal menu of contracts. Assuming that the convergence condition holds.

3.5.1 Impact of the Learning Rate

Without the learning effect ($\beta = 0$), the contracts offered in each period will be identical in each period characterized as below:

$$Q_t(\varepsilon) = \frac{1}{2} (p_0 + \mu - \sigma) - \varepsilon,$$

$$R_t(\varepsilon) = \frac{1}{2} (p_0 - Q_t + \mu - \sigma) Q_t + R_0,$$

where $R_0$ is a constant such that $\pi_{CM}^t(Q_t(\bar{\varepsilon}), R_t(\bar{\varepsilon})) = 0$. Therefore, the expected sales in each period will be the same. However, when $\beta > 0$, the order quantity in each period will depend on the production volumes in the previous periods. We show in the next proposition that $Q_t$ is increasing in the historical order quantities.

**Proposition 3.1.** $a^p_t$, $a^s_t$, $a^v_t$ and $a^e_t$ are positive for any $1 \leq t \leq T$.

The above result implies that the OEM will place more orders when the price for a given quantity is higher in the market and the random cost is low. It also shows that the order quantity is positively correlated with past orders. Moreover, when the OEM is more uncertain about the production cost, she will be conservative and order less to avoid supply chain risks.

Next we explore the impact of the learning rate on sales in each period.

**Proposition 3.2.** The average order quantity in each period $E(Q_t)$ is increasing in $\beta$ for all $1 \leq t \leq T$.

**Proof of Proposition 3.2:** First we prove that $a^p_t$, $a^s_t$ and $a^e_t$ are increasing in $\beta$ for all $1 \leq t \leq T$. Since

$$\frac{da^s_t}{d\beta} = \frac{\partial a^s_t}{\partial \beta} + \frac{\partial a^s_t}{\partial a^s_{t+1}} \frac{da^{s+1}_{t+1}}{d\beta}, \hspace{1cm} \frac{\partial a^s_t}{\partial \beta} = 2(1 - \rho) + \rho (2 + \beta)a^s_{t+1} > 0 \text{ and } \frac{\partial a^s_t}{\partial a^s_{t+1}} > 0,$$

we only need $\frac{da^{s+1}_{t+1}}{d\beta} \geq 0$. 

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If \( \frac{d\alpha_i^s}{d\beta} \geq 0 \) for \( i + 1 \leq t \leq T \), then \( \frac{d\alpha_i^s}{d\beta} > 0 \). Since \( \frac{d\alpha_i^s}{d\beta} = 0 \), by induction we can prove that \( \frac{d\alpha_i^s}{d\beta} \geq 0 \) for all \( 1 \leq t \leq T \). Similarly, we have \( a^p_i \) and \( a^v_i \) are also increasing in \( \beta \) for all \( 1 \leq t \leq T \).

By induction, we can prove that \( \frac{d\alpha_t}{d\beta} \geq \frac{d\alpha_i}{d\beta} \) for any \( 1 \leq t \leq T \) (equality only holds when \( t = T \)). By Lemma 3.2, we have:

\[
E(Q_t) = a^p_t(p_0 - \mu) + a^v_t E \left( \sum_{i=1}^{t-1} Q_i \right) - a^v_t \sigma.
\]

Suppose \( E(Q_t) \) is increasing in \( \beta \) for \( 1 \leq t \leq i - 1 \). \( E(Q_1) \) increases in \( \beta \), since

\[
\frac{dQ_1}{d\beta} = \frac{d\alpha_1^p}{d\beta}(p_0 - \mu) - \frac{d\alpha_1^v}{d\beta}\sigma \geq \frac{d\alpha_1^p}{d\beta}(p_0 - \mu - \sigma) > 0.
\]

Consider the case when \( t = i \). As \( a^p_i, a^v_i \) and \( E(\sum_{i=1}^{t-1} Q_i) \) are increasing in \( \beta \), and \( \frac{d\alpha_i^p}{d\beta} \geq \frac{d\alpha_i^v}{d\beta} \), it is straightforward that \( E(Q_t) \) is increasing in \( \beta \).

By induction, we can prove that \( E(Q_t) \) is increasing in \( \beta \) for all \( 1 \leq t \leq T \).

\[\square\]

That is, when the learning effect is enhanced, the unit production cost decreases which leads to a lower selling price and more sales. Thus, the OEM has an incentive to order larger quantities in order to achieve a lower cost for the future. On one hand, the learning effect is caused by economies of scale, as unit production cost decreases in the cumulative production volume. On the other, a high learning ability also boosts sales in return.

### 3.5.2 Comparison with and without Information Asymmetry

Without information asymmetry, the random shock \( \varepsilon_t \) is accessible to both parties after its realization, rather than only observable to the contract manufacturer. In that case, the brand name company will completely exploit the profit margin from the contract manufacturer, and her decisions in each period become the first best solution of the supply chain. We can derive the optimal contract offered in each period as below:
Theorem 3.4. In each period, the OEM will offer a contract for any realized \( \varepsilon_t \) as below:

\[
Q^*_t(\varepsilon_t, Z_t) = a^p_t p_0 + a^{**}_t Z_t + a^{u*}_t \mu - a^{e*}_t \varepsilon_t,
\]

\[
R^*_t(\varepsilon_t, Z_t) = (\varepsilon_t - Z_t)Q^*_t,
\]

where \( a^{p*}_t = a^p_t \) and \( a^{**}_t = a^*_t \). \( a^{u*}_t \) and \( a^{e*}_t \) are determined by the following recursive formulation:

\[
a^{e*}_{t-1} = \frac{1}{(2\beta + \rho \beta^t) - \rho \beta^t (2 + \beta^t)a^{**}_t},
\]

\[
a^{e*}_{t-1} = a^{e*}_t - a^{p*}_t.
\]

with initial values as \( a^{e*}_T = 0 \) and \( a^{p*}_T = \frac{1}{2} \). Furthermore, The contract achieves first best solution that maximizes expected long-term discounted supply chain profit.

We then compare the decisions with and without information asymmetry. The contracts cannot achieve first best solution that maximizes supply chain profit due to information asymmetry and an information rent is paid to the CM for inducing truth telling. We define \( |E(Q_t) - E(Q^*_t)| \) as the absolute mismatch and \( \frac{|E(Q_t) - E(Q^*_t)|}{E(Q^*)} \) as the relative mismatch between the quantities. Next proposition characterizes the absolute mismatch between the order quantity in the dynamic mechanism design and the first best solution.

Proposition 3.3. \( E(Q_t) \leq E(Q^*_t) \) and \( E(Q^*_t) - E(Q_t) = \sum_{i=1}^t \left( a^{e*}_i \prod_{j=i+1}^t a^{u*}_j \right) \sigma \).

Proof of Proposition 3.3: For a realization of the random shocks \( (\varepsilon_1, \cdots, \varepsilon_T) \), we have a realization of order quantities \( (Q_1, \cdots, Q_T) \) determined by the recursive formulation as below:

\[
Q_t = \sum_{i=1}^t \left[ \left( a^p_i \prod_{j=i+1}^t \beta^j a^{u*}_j \right) \right] p_0 + \sum_{i=1}^t \left[ \left( a^{u*}_i \prod_{j=i+1}^t \beta^j a^{e*}_j \right) \right] \mu - \sum_{i=1}^t \left[ \left( a^{e*}_i \prod_{j=i+1}^t \beta^j a^{u*}_j \right) \right] \sigma - \sum_{i=1}^t \left[ \left( a^{e*}_i \prod_{j=i+1}^t \beta^j a^{u*}_j \right) \varepsilon_i \right],
\]
The first best solution $Q^*_t$ is determined as below:

\[
Q^*_t = \sum_{i=1}^{t} \left[ \left( a^p_i \prod_{j=i+1}^{t} \beta^j a^*_{j} \right) \right] p_0 + \sum_{i=1}^{t} \left[ \left( a^{u*}_i \prod_{j=i+1}^{t} \beta^j a^{**}_{j} \right) \right] \mu - \sum_{i=1}^{t} \left[ \left( a^{v*}_i \prod_{j=i+1}^{t} \beta^j a^{**}_{j} \right) \varepsilon_i \right],
\]

Therefore,

\[
E(Q_t) = \sum_{i=1}^{t} \left[ \left( a^p_i \prod_{j=i+1}^{t} \beta^j a^*_{j} \right) \right] (p_0 - \mu) - \sum_{i=1}^{t} \left[ \left( a^{v}_i \prod_{j=i+1}^{t} \beta^j a^{*}_{j} \right) \right] \sigma,
\]

\[
E(Q^*_t) = \sum_{i=1}^{t} \left[ \left( a^{p*}_i \prod_{j=i+1}^{t} \beta^j a^{*}_{j} \right) \right] (p_0 - \mu).
\]

Since $a^v_i > 0$, $a^{p*}_i = a^p_i$ and $a^{**}_i = a^*_{i}$ for all $1 \leq t \leq T$, $E(Q_t) \leq E(Q^*_t)$ and $E(Q^*_t) - E(Q_t) = \sum_{i=1}^{t} \left[ \left( a^v_i \prod_{j=i+1}^{t} \beta^j a^{*}_{j} \right) \right] \sigma$.

Therefore, the average sales will always be lower than the first best solution due to information asymmetry, and the absolute mismatch is proportional to the variability. As the OEM’s uncertainty about the random costs increases, both the absolute mismatch and relative mismatch will be larger.

The next proposition shows the impact of learning rate $\beta$ on both absolute and relative mismatch.

**Proposition 3.4.** $E(Q^*_t) - E(Q_t)$ is increasing in $\beta$, while \( \frac{E(Q^*_t) - E(Q_t)}{E(Q^*_t)} \) is decreasing in $\beta$.

With a higher learning rate, the OEM will have an incentive to order larger quantities with or without information asymmetry, as the overall production costs are lower, leading to a larger absolute mismatch. However, what is more interesting is that the relative mismatch will decrease in the learning rate, although the mismatch is only caused by information asymmetry. The reason is that with a higher learning rate, the incentive to order large quantities reduces the relative mismatch. The result implies that improving learning is a way of enhancing supply chain efficiency.
3.5.3 Numerical Results

In this section, we conduct several numerical study to examine the characteristics of the dynamic contracts, and compare the decisions and outcomes in different scenarios. We consider a 10-period outsourcing game. Let $p(Q) = 1 - Q$, $\varepsilon_t \sim U[0.4,0.6]$, $\rho = 0.8$, and consider the cases when $\beta = 0.25$ and $\beta = 0.5$.

Figures 3.3 and 3.4 show the average order quantities in each period with and without information asymmetry. As proved in Proposition 3.3, $E(Q_t)$, the average order quantity in dynamic mechanism design, is always less than $E(Q^*_t)$, the first best solution when the cost information is observable to both parties after its realization. As $\beta$ increases, the average order quantities increase in both cases. Furthermore, although the average production cost is always decreasing over time due to the learning effect, it is not necessarily true that the order quantity will increase as the production cost decreases. On one hand, the OEM is motivated to order more for the first several periods to accumulate experience that helps reduce future costs, and her demand will decrease for the last few periods, when the learning effect is less significant. On the other, she intends to place a larger order with cost reduction, which results in a desire for increasing order quantities.

![Figure 3.3: The Order Quantities with and without Information Asymmetry, $\beta = 0.25$.](image-url)

Figure 3.3: The Order Quantities with and without Information Asymmetry, $\beta = 0.25$.  

1 2 3 4 5 6 7 8 9 10

0.24 0.26 0.28 0.3 0.32 0.34 0.36 0.38 0.4 0.42 0.44

Order Quantity

Symmetric Information
Asymmetric Information
Figure 3.4: The Order Quantities with and without Information Asymmetry, $\beta = 0.5$.

Figures 3.5 and 3.6 show the mismatch between the dynamic mechanism design and the first best solution. The absolute mismatch increases in $\beta$, while the relative mismatch decreases.

Figure 3.5: Order Quantity Mismatch, $\beta = 0.25$. 

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Figure 3.6: Order Quantity Mismatch, $\beta = 0.5$.

Figure 3.7 demonstrates cumulative profit for (a) the CM, (b) the OEM and (c) the supply chain without information asymmetry.

Figure 3.7: Cumulative discounted Profit, $\beta = 0.25$. 
Due to information asymmetry, the OEM’s expected profit over the T Periods is less than the optimal supply chain profit under the first best solution when there is no information asymmetry, and the CM is able to earn the surplus, known as information rent, for possessing the private cost information. In the figure, the OEM obtains most of the profit as the cost uncertainty is relatively low. Even if the learning capability is significantly enhanced such that $\beta$ has increased from 0.25 (Figure 3.7) to 0.5 (Figure 3.8), an 100% increase, the CM still shares a small proportion of the total profits, although his expected profit decreases. CM’s expected profit over the 10 periods has increased by less than 0.2, while the OEM’s gain has increased by more than 0.7. The comparison indicates that learning improvement may mainly benefit the OEM, while the CM might be the party who invests in the improvement. Given a relatively accurate estimation on the random cost, the OEM is capable of squeezing out almost all the profits obtained from learning ability improvement.

![Graph showing cumulative discounted profit](image)

**Figure 3.8:** Cumulative discounted Profit, $\beta = 0.5$. 

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3.6. Conclusion

When brand name companies outsource their business processes, they intend to build in long-term partnerships with their service providers to maintain sustainable improvement and to realize on-going cost savings. One important question is how to construct the contracts between a brand name company and a contract manufacturer over a long period of time, in the presence of information asymmetry on the production costs due to the learning effect. Our study is the first attempt to study dynamic mechanism design in a supply chain.

In this chapter, we consider a multi-period game in which a brand name company outsources the production to a contract manufacturer. The unit production cost in each period consists of a random shock that is only observed by the contract manufacturer after its realization, and a deterministic cost representing the learning effect on the production cost. We derived the optimal contracts in each period which can be calculated recursively, and extend the result to a game over the infinite time horizon by providing a convergence condition. Our research indicates that information asymmetry in a decentralized supply chain will lead to a lower average order quantity, compared with the first best solution for a centralized system. While the absolute mismatch increases in the learning rate, the relative mismatch decreases in the learning rate.

Future study will allow both parties to invest in quality-improvement efforts in each period, and examine the optimal quality-improvement decisions along with the roles and incentives of different parties in quality improvement.
Chapter 4

Conclusion

This chapter presents the concluding remarks on this thesis and suggests directions for further research. Section 4.1 summarizes the contributions and conclusions of this research and Section 4.2 points out some possible extensions for future research.

4.1. Summary

Outsourcing is increasingly becoming a key business strategy for brand name companies as a means of achieving low costs, concentrating on their core competencies, getting access to specialized services and etc. However, supply chain uncertainties arise when controls over business processes are delegated to an outsourcing company, and the existence of information asymmetry results in supply chain inefficiency. To align the incentives of both parties and to avoid interest conflicts, brand name companies need to design incentive contracts that maximize their profits.

In Chapter 2, we construct incentive contracts in a single-period model to address whether a brand name company should outsource the procurement process along with production as well as evaluating the value of strategic outsourcing in offsetting supply chain risks. We also extend the basic model to different scenarios to analyze the impact of manufacturing process uncertainty, demand variability and control delegation, and propose a quality management scheme as a means of fraud prevention. We find that procurement outsourcing improves the profitability of both the OEM and the CM and motivates quality improvement, if an outsourcing mechanism is appropriately designed and executed. So far as we know, our study is the first attempt to examine analytically the risks
and benefits of strategic procurement outsourcing. This study can complement the existing literature by adding this new dimension. In practice, the outcomes of this research can help managers to improve their decision-making in procurement outsourcing.

In Chapter 3, we construct a dynamic mechanism for brand name companies to design contracts over a long period of time. We derived the optimal quantity-based contracts in each period that can be calculated recursively, and extend the result to the infinite time horizon by providing a convergence condition. We also analyze the impact of learning as well as comparing our contracts and the first best solutions. Our study reveals that the order quantity in each period is positively correlated with past order quantities. The average order quantity is always lower than the first best solution due to information asymmetry, while the relative mismatch can be reduced through increasing the learning rate. Our research contributes to the contracting literature in supply chain management, as it is the first attempt to study the structure of the dynamic contracts in outsourcing decisions.

4.2. Future Work

There are several potential directions for future research. First, we can extend our study on mechanism design to a dynamic information acquisition model and study information revelation in a multi-period game, focusing on information delivery and information distortion over the supply chain. Such an extension will lead to more realistic discussions on contract design. Second, we can allow both parties to invest in quality-improvement in each period, and study the roles of different parties in quality improvement. Third, our models can be extended to more complex supply chains involving multiple players to study the competition and cooperation among different parties.


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Appendix A

1. Modeling the Uncertainty of the CM’s Production System

We model the production system with two states, in-control and out-of-control. Each production run starts in the in-control state but may shift to the out-of-control state after a random amount of time $\tau$, which is exponentially distributed with mean $\frac{1}{t}$, where $t$ is small. During each production run, once the process shifts to the out-of-control state, it will stay in that state until the entire batch is finished, because it is either impossible or too expensive to interrupt a production run. Thus, the lower the $\alpha$ and $t$, the higher quality the purchased material and more robust the CM’s production system, respectively. Since the quality of the material and the production system jointly determine the quality of the final product, the annual quality costs are functions of both $\alpha$ and $t$.

We assume, for simplicity and tractability, that all the units produced with defective material are nonconforming. Those produced with conforming material are conforming if produced in the in-control state and nonconforming if produced in the out-of-control state. If the CM produces in batches of size $q$, each production run lasts $\frac{q}{\delta}$, where $\delta$ is the production rate, and the average number of defective final products in a production run can be computed as

$$\alpha q + \int_0^{\frac{q}{\delta}} (1 - \alpha)(q - \delta \tau)e^{-\tau d}d\tau = \alpha q + \frac{(1 - \alpha)tq - (1 - \alpha)\delta}{t} + \frac{(1 - \alpha)\delta}{t}e^{-tq/\delta}$$

$$\approx \alpha q + \frac{(1 - \alpha)tq^2}{2\delta},$$

where $q - \delta \tau$ is the amount produced when the process is in the out-of-control state, $1 - \alpha$
of which are nonconforming. Because \( t \) is typically very small for a production process, we use the approximation of \( e^{-tq/r} \approx 1 - tq + (1/2!)(tq)^2 \) (Rosenblatt and Lee 1986). Since there is a total of \( \frac{1}{q} \) production runs, the average number of nonconforming final products is \( \alpha + \frac{(1 - \alpha) tq}{2\delta} \). Denote the quality level of the production system by \( \mu = \frac{tq}{2\delta} \) and \( \alpha + (1 - \alpha) \mu \) determines the overall quality of the outputs.

\[ \square \]

### 2. Proof of Lemma 3.1

Suppose for \( i = t+1, \cdots, N \), we have:

\[
\begin{align*}
    h_i(Q_i^{i-1}) &= h_i^p p_0 + h_i^s Z_i + h_i^u \mu - h_i^v \sigma, \\
    g_i(Q_i^{i-1}) &= g_i^p p_0 + g_i^s Z_i + g_i^u \mu - g_i^v \sigma.
\end{align*}
\]

In period \( t \), the Incentive Compatibility constraint IC\((t)\) is as below:

\[
\frac{\partial R_t}{\partial \varepsilon} - (\varepsilon - Z_t) \frac{\partial Q_t}{\partial \varepsilon} + \rho g_{t+1}(Q^t)Q'_t(\varepsilon_t) = 0
\]

The Hamiltonian is constructed as below:

\[
H = (p_0 - Q_t) Q_t - R_t + \rho E_{\varepsilon_{t+1}} V_{\Delta t}^{t+1} + \lambda_R [\varepsilon - Z_t - \rho g_{t+1}(Q^t)] u + \lambda_Q u
\]

An optimal solution thus satisfies the following conditions.

\[
\begin{align*}
    \frac{\partial H}{\partial \varepsilon} &= \lambda_R(\varepsilon - Z_t) + \lambda_Q = 0, \quad (1) \\
    \lambda'_R(\varepsilon) &= -\frac{\partial H}{\partial R_t} = 1, \quad (2) \\
    \lambda'_Q(\varepsilon) &= -\frac{\partial H}{\partial Q_t}. \quad (3)
\end{align*}
\]
From (2) we obtain $\lambda_R = \varepsilon - \bar{\varepsilon}$.

From (3) we have

$$\frac{\partial Q_t}{\partial \varepsilon} = - [p_0 - 2Q_t + \rho h_{t+1}(Q^t) - \lambda_R \rho g_{t+1}(Q^t) u]$$  \hspace{1cm} (4)$$

From (1) we have

$$\lambda_Q = -\lambda_R (\varepsilon - Z_t - \rho g_{t+1}(Q^t))$$  \hspace{1cm} (5)$$

Taking derivative with respect to $\varepsilon_t$ of (5) and substituting the result to (4), we obtain

$$p_0 - 2Q_t + \rho [h_{t+1}(Q^t) + g_{t+1}(Q^t)] = 2\varepsilon - \bar{\varepsilon} - Z_t,$$

which is

$$[2 - \rho \beta^t(h^*_t + g^*_t)][Q_t] = [1 + \rho(h^*_t + g^*_t)]p_0 + [1 + \rho(h^*_t + g^*_t)]Z_t + [1 + \rho(h^*_t + g^*_t)]\mu - [1 + \rho(h^*_t + g^*_t)]\sigma - 2\varepsilon.$$

Therefore, we have

$$Q_t = a_t^p p_0 + a_t^s Z_t + a_t^u \mu - a_t^e \sigma - a_t^e \varepsilon,$$

where

$$a_t^p = \frac{1 + \rho(h^*_t + g^*_t)}{2 - \rho \beta^t(h^*_t + g^*_t)},$$

$$a_t^s = \frac{1 + \rho(h^*_t + g^*_t)}{2 - \rho \beta^t(h^*_t + g^*_t)},$$

$$a_t^u = \frac{1 + \rho(h^*_t + g^*_t)}{2 - \rho \beta^t(h^*_t + g^*_t)},$$

$$a_t^e = \frac{2}{2 - \rho \beta^t(h^*_t + g^*_t)}.$$
We obtain
\[ \varepsilon = \frac{1}{a_t} (a_t^p p_0 + a_t^e Z_t + a_t^a \mu - a_t^v \sigma - Q_t) \]

Substituting the above equation to the IC(t) constraint, we have
\[ R_t' (Q_t) = \varepsilon - Z_t - \rho g_{t+1} (Q_t) \]
\[ = \frac{1}{a_t} (a_t^p p_0 + a_t^e Z_t + a_t^a \mu - a_t^v \sigma - Q_t) - Z_t - \rho \left( g_{t+1}^p p_0 + g_{t+1}^e \sum_{i=1}^t Q_i + g_{t+1}^a \mu - g_{t+1}^v \sigma \right) \]
\[ = \left( \frac{a_t^p}{a_t^e} - \rho g_{t+1}^p \right) p_0 + \left( \frac{a_t^a}{a_t^e} - 1 - \rho g_{t+1}^a \right) Z_t - \left( \frac{1}{a_t^e} + \rho g_{t+1}^a \beta^t \right) Q_t + \left( \frac{a_t^v}{a_t^e} - \rho g_{t+1}^v \right) \mu - \left( \frac{a_t^v}{a_t^e} - \rho g_{t+1}^v \right) \sigma. \]

Solving the above differential equation we have
\[ R_t (\varepsilon, Z_t) = [b_t^p p_0 + b_t^e Z_t + b_t^a \mu - b_t^v \sigma] Q_t - b_t^2 Q_t^2 + R_t^0 (Z_t), \]
where
\[
\begin{align*}
b_t^p &= \frac{a_t^p}{a_t^e} - \rho g_{t+1}^p, \\
b_t^e &= \frac{a_t^e}{a_t^e} - 1 - \rho g_{t+1}^e, \\
b_t^a &= \frac{a_t^a}{a_t^e} - \rho g_{t+1}^a, \\
b_t^v &= \frac{a_t^v}{a_t^e} - \rho g_{t+1}^v, \\
b_t^q &= \frac{1}{2} \left( \frac{1}{a_t^e} + \rho g_{t+1}^a \beta^t \right),
\end{align*}
\]
and \( R_t^0 (Z_t) \) is a constant such that \( V_{CM}^t (Q_t, Z_t), R_t (\varepsilon, Z_t), Z_t, \varepsilon) = 0. \)

Next we will derive \( g_t (Q_t^{i-1}) \) and \( h_t (Q_t^{i-1}) \).
Since \( g_t(Q^{t-1}) = \frac{d}{dQ_{t-1}} E_{\varepsilon_t} \left( R_t - c_t Q_t + \rho E_{\varepsilon_t+1} V_{CM}^{t+1} \right) \), we have:

\[
g_t(Q^{t-1}) = \frac{d}{dQ_{t-1}} E_{\varepsilon_t} \left[ R_t(\varepsilon_t) - R_t(\bar{\varepsilon}) \right] + \rho \frac{d}{dQ_{t-1}} E_{\varepsilon_t} \left[ V_{CM}^{t+1}(Q_t(\varepsilon_t)) - V_{CM}^{t+1}(Q_t(\bar{\varepsilon})) \right] - \frac{d}{dQ_{t-1}} E_{\varepsilon_t} (c_t Q_t)
\]

\[
= \beta^{t-1} a_t^e \rho (b_t^e - 2b_t^s a_t^e) + \rho \beta^{t-1} (1 + \beta^t a_t^s) g_{t+1}^* a_t^e \sigma - \beta^{t-1} a_t^e \mu + \beta^{t-1} a_t^s Z_t + \\
\beta^{t-1} E_{\varepsilon_t} Q_t
\]

\[
= \beta^{t-1} (a_t^s - a_t^e - 1) \sigma - \beta^{t-1} a_t^e \mu + \beta^{t-1} a_t^s Z_t + \\
\beta^{t-1} [a_t^p p_0 + a_t^s Z_t + (a_t^u - a_t^e) \mu - a_t^v \sigma]
\]

\[
= \beta^{t-1} a_t^p p_0 + 2\beta^{t-1} a_t^s Z_t - \beta^{t-1} (a_t^p + a_t^s) \mu + \beta^{t-1} (a_t^s - a_t^e - 1 - a_t^v) \sigma.
\]

For \( i = t \), \( g_i(Q^{t-1}) \) is still a linear combination of \( p_0, Z_t, \mu \) and \( \sigma \). As \( g_t(Q^{t-1}) + h_t(Q^{t-1}) \) is also a linear combination, it is straightforward that \( h_t(Q^{t-1}) \) is also a linear combination. By induction, we can prove the result is true for any period. We also have

\[
b_{t}^p = \frac{a_t^p}{a_t^e} - \rho \beta^{t} a_{t+1}^p,
\]

\[
b_{t}^s = \frac{a_t^s}{a_t^e} - 1 - 2\rho \beta^{t} a_{t+1}^s,
\]

\[
b_{t}^u = \frac{a_t^u}{a_t^e} + \rho \beta^{t} (a_{t+1}^p + a_{t+1}^s),
\]

\[
b_{t}^v = \frac{a_t^v}{a_t^e} - \rho \beta^{t} (a_{t+1}^s - a_{t+1}^e - 1 - a_{t+1}^v),
\]

\[
b_{t}^q = \frac{1}{2} \left( \frac{1}{a_t^e} + 2\rho \beta^{2t} a_{t+1}^s \right),
\]

\[
\square
\]